FUNDAMENTA ARITHMETICA ET GEOMETRICA CUM EORUNDEM USU IN VARIJ...

Ludolf : van Ceulen, Willebrord Snell

FVNDAMEMTA

ARITHMETICA ET GEOMETRICA

cum eorundem usu

In varij problematis, Geometricis, partim solo linearum, ductu, partim per numeros irrationales, & tabulas sinuum, & Algebram solutis.

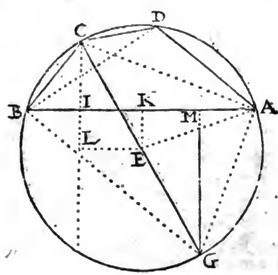
AVTHORE

LVDOLPHO A CEVLEN Hildesheimensi.

E vernaculo in Latinum translata

A

WIL. SN. R. F.



LVGDVNI BATAVORVM,
Apud IACOBVM MARCVM Bibliopolam,
Anno CIↃ IↃ. CXV.

LVDOLPHI à CEVLEN
Surdorum Arithmetica.

Cap. I.

De analysi lateris quadrati.

Geometricorum problematum solutionem non tantum Geometricé in solis lineis, sed per numeros quoque earundem linearum mensuram in assumptis partibus exhibere summam oblectationem habet cum utilitate cõiunctam. Ideóque surdorum numerorum tractationem (quia huic instituto maximè serviunt) pro nostro modulo ei tanquam præviam præmittere necessarium duxi. Et quidem primum

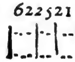
Quadrati Lateris analysin, quam radicis extractionem vulgò indigetant.

Exemplum.

Esto ager quadratus, hoc est, angulis normalibus & lateribus quaquaversum æqualibus comprehensus, continens pedes 622521 quadratos, quæritur eiusdem latus, sive agri hujus cum longitudo, tum latitudo: sunt enim æquales. Respondeo 789 pedibus in longitudinem patere

Latera.	Quadrati.
1	4
2	8
3	9
4	16
5	25
6	36
7	49
8	64
9	81

Dato numero lineam proxime subscribito itēque alteram exiguo intervallo priori parallelam, deinde binas quasque notas, initio à novissimis facto, lineis priores ad perpendiculū secantibus secernito, quibus tot spatia tanquā colmubaria comprehendes, quot notis optati quadrati latus constabit, quemadmodum hic expressum vides 622521.



 Vt

Vt itaque initium operis facias videndum tibi, quis numerus primæ cellulæ sub 62 inscribendus, ut eius quadratus saltem non maior sit numero 62. hic igitur, quia 8 huncmodum excedat, nam quadratus ejus est 64, sumes 7, cuius quadratus 49 de 62 subductus relinquet 13. hâc formula

1 3
~~6~~ ~~2~~ | 25|21
7
~~4~~ ~~9~~

Atque ista via semper prima nota eruenda est. Ad secundæ notæ investigationem ita porro procedes. Inventum latus 7 semper duplicato, factus erit 14, quem ita subscribes vt vltima nota congruat primæ notæ cellulæ sequentis, vt hic vides, characterem 4 notatum sub 2, & 1 sinistrorsum sub 3.

13
~~6~~ ~~2~~ 25 21
7| 8|
14

Iam 132 per 14 dividito, quotum cellulæ secundæ sub novissima ejus nota 5 inscribito, totumque hunc numerum, qui 148 sunt, (nam post 14 jam ordine consequitur 8) totum inquam 148 per eundem vltimum, seu quotum 8 multiplicato, factumque 1184 de 1325 subducito, reliquus erit 141, qui ordine suis locis notati ista formula consistent.

1
~~1~~ ~~3~~ 41
~~6~~ ~~2~~ ~~2~~ ~~5~~ 21
7| 8|
14

Atque hæc secundæ notæ investigatio fuit: sequitur tertia huic simillima, uterque enim numerus jam inventus duplicatus vt ante dabit 156, cujus ultimum characterem sub prima tertiæ cellulæ nota subscribes, reliquosque ordine sinstrorsum suis locis, vt hic vides.

~~1~~
6 ~~1~~ ~~3~~ 41
~~6~~ ~~2~~ ~~2~~ ~~5~~ 21
7| 8|
15 6

Tumque numero 1112 per 156 diviso habebis in quoto 9, eumque in tertia cellula novissimo characteri subscribes, totum numerum ex eo & duplo continuatum 1569 per eundem quotum 9 multiplicabis, & factum 14121 de numero reliquo 14121 subduces, qui cum nihil faciat reliqui argumento est 789 verum & accuratum latus esse dati numeri quadrati 622524. En tibi aliquot exẽpla cum suis δοκιμασίαις, quas probas sive examina vocant.

```
        111  ÷                               1 129  ÷
      2354610                            1 111337 6121  ÷
    1388656660                            83719324462 57
  68632612164                          376200786346 78
  ___________                          ________________
  3/ 1| 2|4| 6| 2|                     7/ 5| 9/ 0/ 7| 8| 9|
  ___________                          ________________
    662244891                          145018801456
      6624                               111155881
       62                                   118
                 1505                       15
               5337758                      ÷
             65757156411
          12613168360907 13
          7630602345896874 20
          _____________________
           8/ 7| 3| 5| 3/ 3| 1/ 9| 0
          _____________________
            16744670066662 38
              11774477006666
               117744706 70
                  1174
```

Operis facti examen tale eſt, numerum novenarium vt in diviſione fieri ſolet ex latere invento ſubdicito quoties poteſt, reliquum quadrato, ex hoc facto rurſum novem quoties poterit ſubmoveto, reliquumque hunc cum numero poſt in analyſi reſiduo componito & novenarium quoque hinc ejicito, numerus poſt novenarios exemptiles tandem reliquus, æquabitur numero qui é quadrato ipſo relinquetur cum inde novenarium eodem modo exemeris. Exemplum nobis eſto in ultimo. é latere 873533190 novenarius quoties poteſt exemptus relinquet 3, cujus quadratus 9, inde novenarius demptus relinquet 0, qui ad reliquum 558111320 additus, eum nihilo auget, unde novenarij exemptiles ejecti relinquunt 8 tantundem ſupereſt ſi omnes novenarios ex toto quadrato dato 763060234589687420 ſubducas.

Hic illud obſervatione dignum occurrit, in primo quidem exemplo poſt lateris quadrati analyſin nihil redundare, ideoque numerum 622521 vere quadratum eſſe, quippe qui factus ſit á 789 in ſeipſum multiplicato: ideoque contra verum & exactum ejuſdem latus in numeris dari. At vero tres reliqui exactum latus numeris explicabile nullum habent. ideoque iſta inexplicabilia & ſurda cum ſint, ita vocantur, cujus generis à logiſtis quoque *irrationales* appellantur, quales ſequentes hi ſunt 2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 17, 18, alijque, preterea infiniti. Ex iſtis igitur, ceteriſque id genus accuratum & verum latus erui non poteſt: at vero tam propinquum tamen quam cuique erit collibitum in libro de circulo olim à me edito invenire docui, & hic quoque paulo infra iterum docebitur.

Vt itaque initium operis facias videndum tibi, quis numerus primæ cellulæ ſub 62 iuſcribendus, ut eius quadratus ſaltem non maiorſit numero 62. hic igitur, quia 8 huncmodum excedat, nam quadratus ejus eſt 64, ſumes 7, cuius quadratus 49 de 62 ſubductus relinquet 13 .hâc formula

1 3
~~6~~ ~~2~~ | 25|21
7|
~~49~~

Atque iſta via ſemper prima nota eruenda eſt. Ad ſecundæ notæ inveſtigationem ita porro procedes. Inventum latus 7 ſemper duplicato, factus erit 14, quem ita ſubſcribes vt vltima nota congruat primæ notæ cellulæ ſequentis, vt hic vides, characterem 4 notatum ſub 2, & 1 ſiniſtorſum ſub 3.

13
~~6~~ ~~2~~ 2521
7| 8|
14

Iam 132 per 14 dividito, quotum cellulæ ſecundæ ſub noviſſima ejus nota 5 inſcribito, totumque hunc numerum, qui 148 ſunt, (nam poſt 14 jam ordine conſequitur 8) totum inquam 148 per eundem vltimum, ſeu quotum 8 multiplicato, factumque 1184 de 1325 ſubducito, reliquus erit 141, qui ordine ſuis locis notati iſta formula conſiſtent.

1
~~1~~ ~~3~~ 41
~~6~~ ~~2~~ ~~2~~ ~~5~~ 21
7| 8|
14

Atque hæc ſecundæ notæ inveſtigatio fuit : ſequitur tertia huic ſimillima, uterque enim numerus jam inventus duplicatus vt ante dabit 156, cujus ultimum characterem ſub prima tertiæ cellulæ nota ſubſcribes, reliquoſque ordine ſiniſtrorſum ſuis locis, vt hic vides.

~~1~~
6 ~~1~~ ~~3~~ 41
~~6~~ ~~2~~ ~~2~~ ~~5~~ 21
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15 6

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~~1~~11 ÷
~~23846~~10
~~158891966~~0
~~96632917194~~

3/ 1| 2|4| 6| 2|

~~662244891~~
~~6624~~
~~62~~

~~1~~ 129 ÷
~~1~~ ~~1133796~~121 ÷
~~83716324462~~57
~~87620078934678~~

7/ 5| 9/ 0/ 7| 8| 9|

~~1450188014~~56
~~111111~~881
~~111~~
15 ÷

~~110~~5
~~13377~~58
~~6878712894~~11
~~11613196836060~~13
~~7630602345896874~~20

8/ 7| 3| 5| 3/ 3| 1/ 9| 0

~~16744670006666~~38
~~113377447700~~66
~~117744706~~70
~~1~~174

Operis facti examen tale eſt, numerum novenarium vt in diviſione fieri ſolet ex latere invento ſubdicito quoties poteſt, reliquum quadrato, ex hoc facto rurſum novem quoties poterit ſubmoveto, reliquumque hunc cum numero poſt in analyſi reſiduo componito & novenarium quoque hinc ejicito, numerus poſt novenarios exemptiles tandem reliquus, æquabitur numero qui é quadrato ipſo relinquetur cum inde novenarium eodem modo exemeris. Exemplum nobis eſto in ultimo. é latere 873533190 novenarius quoties poteſt exemptus relinquet 3, cujus quadratus 9, inde novenarius demptus relinquet 0, qui ad reliquum 558111320 additus, cum nihilo auget, unde novenarij exemptiles ejecti relinquunt 8 tantundem ſupereſt ſi omnes novenarios ex toto quadrato dato 763060234589687420 ſubducas.

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 Attamen

Attamen non rarò in Geometricis & Algebricis zetemasi evenit, vt latus verum é dato numero quadrato eruendum nobis sit, quam ad rem hoc signum $\sqrt{}$ usurpamus, quod dato numero præfixum indicio est istius radicem sive latus numero esse in explicabile, quod propterea *surdum* & irrationale vocari diximus, vt hic $\sqrt{3}$, $\sqrt{7}$, $\sqrt{5}$, $\sqrt{8}$, $\sqrt{17}$, Horum numerorum tractione subtilissima quæque zetemata tum in Arithmeticis tum in Geometricis argutissime explicari possunt.

Sunt verò duûm generum, seu potius inter se comparati aliter atque aliter affecti sunt. Nam alij eorum inter se asymmetri sive incommeusurabiles sunt, vt $\sqrt{7}$ & $\sqrt{5}$, omnesque illi supra à nobis propositi: secus autem se habent latus quadratum é 12 & 27, sive quod idem est $\sqrt{12}$ & $\sqrt{27}$. ita enim inter se affecti sunt hi numeri, vt si ipsorum quadrata ad minimos in eadem ratione terminos revocentur, sint numeri veré quadrati, vt hic 12 & 27 per 3 reducti dabunt 4 & 9, atque horum latera sunt 2 & 3, Ideoque $\sqrt{12}$ & $\sqrt{27}$ vocantur symmetri vel commensurabiles vulgó communiantes, ita $\sqrt{125}$ & $\sqrt{80}$ revocantur pe 5, divisorem communem ad $\sqrt{25}$ & $\sqrt{16}$, quorum latera 5 & 4, ac propterea $\sqrt{125}$ & $\sqrt{80}$ symmetri. Numeri prró ommnes symmetri in unam summam seu numerum additione redigi facile possunt. Ita totus á $\sqrt{125}$ & $\sqrt{80}$ est $\sqrt{405}$, item post subductionem reliquus erit numerus surdus $\sqrt{5}$. Præterea totus & reliquus eorundem nnmerorum per se mutuo multiplicati aut divisi dabunt numerum vere quadratum. Cæteri asymmetri additi, vel de se mutuo subducti numerum é binis nominibus conflatum producent; & additi quidem *binomium*, subducti veró *residuum*: & propterea sive inter se multiplicentur sive dividantur prodibit numerus veré inexplicabilis & surdus, quorum numerationem deinceps tractare instituì.

CAP. II.

De Additione irrationalium simplicium.

QAadrati ipsi seorsim addantur, ijdemque inter se multiplicati quadruplicentur, facti latus ad supra scriptam summam addatur, hujus latus erit optata datorum numerorum summ a.

Idem eodem sensu paulò aliter enuntiatum. Si latus quadrupli ádatis numeris facti ad eorundem quadratorum summam addas, simultitriusque latus erit optata datorum numerorum summa.

Exemplum.

Addantur $\sqrt{75}$ & $\sqrt{12}$, totus erit $\sqrt{147}$. adde 12 (quadratum à $\sqrt{12}$) ad 75 quadratum de $\sqrt{75}$ (numeri enim isti sublato tantum signo quadrantur) eorum summam 87 seorsum nota: tum 75 per 12 multiplicato factus erit 900, hujus quadruplum 3600, cujus latus quadratum 60, qui numerus ad summam 87 additus couflabit 147, cujus latus $\sqrt{147}$ exibet optatam datorum numerorũ sũmam.

Exempla

Exempla duo ad expositam formulam expressa.

Addantur $\sqrt{512}$ ad $\sqrt{288}$

512	512	
288	288	
800	4096	1
768	4096	512
Sūma $\sqrt{1568}$ quæsita.	1024	589824
	147456	7| 6/ 8
	4	1428
	589824	1

Addantur $\sqrt{50}$ ad $\sqrt{8}$ summa erit $\sqrt{98}$

50	50	
8	8	
58	400	
40	4	1600
$\sqrt{98}$ Summa quæsita.	1600	4/ 0/

Simili via si addantur. $\begin{cases} \sqrt{27} \ \& \ \sqrt{48} \\ \sqrt{7} \ \& \ \sqrt{28} \\ \sqrt{75} \ \& \ \sqrt{27} \\ \sqrt{10} \ \& \ \sqrt{1000} \\ \sqrt{176} \ \& \ \sqrt{396} \\ \sqrt{50} \ \& \ \sqrt{162} \end{cases}$ Conflabunt summam $\begin{cases} \sqrt{147} \\ \sqrt{63} \\ \sqrt{192} \\ \sqrt{1210} \\ \sqrt{1100} \\ \sqrt{392} \end{cases}$

Non aliter etiam complures symmetri in unam summam contrahentur, si duobus quibuslibet additis, eorum sūma cum tertio cōponatur, vt $\sqrt{12}$ $\sqrt{27}$ $\sqrt{48}$ dabunt $\sqrt{243}$. Namque $\sqrt{12}$ & $\sqrt{27}$ additi dant $\sqrt{75}$, qui cum $\sqrt{48}$ compositus dabit omnium summam $\sqrt{243}$.

Sic $\sqrt{18}$, $\sqrt{50}$, $\sqrt{72}$, & $\sqrt{200}$ dant summam $\sqrt{1152}$

Item $\sqrt{24}$ & $\sqrt{66\frac{2}{3}}$ dant summam $\sqrt{170\frac{2}{3}}$.

24 $\}$ quadrati	$66\frac{2}{3}$ $\}$ quadrati
$66\frac{2}{3}$ $\}$ addendi.	24 $\}$ multiplicandi.
$92\frac{2}{3}$ Summa	264
80	132
	16
$\sqrt{170\frac{2}{3}}$ Optata summa	1600 Factus.
	4
	7400 Quadruplum.
	6400
	8/ 0 Latus.

Addantur $\sqrt{4\frac{1}{2}}$ ad $3\frac{5}{9}$

$$\begin{array}{ll} 4\frac{1}{2} & 4\frac{1}{2} \\ 3\frac{5}{9} & 3\frac{5}{9} \\ \hline 8\frac{1}{18} & 12 \\ 8 & 1\frac{1}{2} \\ \cline{1-1} & 2\frac{2}{9}\quad\frac{5}{18} \\ \cline{2-2} & 16 \\ & 4 \\ \cline{2-2} & 64 \end{array}$$

Summa optata $\sqrt{16\frac{1}{18}}$

$$\begin{array}{c|c} 64 & \\ \hline 8 & \end{array}$$

Addantur $\sqrt{\frac{5}{8}}$ ad $\sqrt{\frac{2}{5}}$

add. $\left\{\begin{array}{c}\frac{5}{8}\\ \frac{2}{5}\end{array}\right\}$ Mult. $\frac{5}{8}$ factus $\frac{1}{4}$

$$\begin{array}{c} 1\frac{1}{40} \\ 1 \\ \hline \end{array}$$

4 quadruplicatum
1 hujus latus 1 it idem unitas

$$\begin{array}{c|c} 1 & \\ \hline 1 & \end{array}$$

$\sqrt{2\frac{1}{40}}$ Optata summa.

Item $\sqrt{12}$, $\sqrt{21\frac{1}{3}}$, $\sqrt{33\frac{1}{3}}$, $\sqrt{61\frac{1}{3}}$ dabunt summam. $\sqrt{481\frac{1}{3}}$:

Ad symmetrorum numerorum additionem via magis compendiosa.

Dati numeri symmetri per eundem numerum aliquem multiplicati aut divisi ad numeros verè quadratos revocentur, horum latera deinde addito summam iterum per eum numerum multiplicato vel dividito, per quem ante ad verè quadratos reducti sunt.

Exemplum.

Addantur $\sqrt{32}$ ad $\sqrt{98}$

$$\begin{array}{lll} & \sqrt{32} & \sqrt{98} \\ \sqrt{2} & \hline & \sqrt{16} & \sqrt{49} \\ & 4 & 7 \end{array}$$

$$\begin{array}{c} 7 \\ 4 \\ \hline 11 \\ 11 \\ \hline 11 \\ 11 \\ \hline \sqrt{121} \\ \sqrt{2} \\ \hline \end{array}$$

Datorum $\sqrt{242}$ summa.

Vel multiplicatione ad numeros quadratos revocabuntur hoc modo.

$$\begin{array}{lll} & \sqrt{48} & \sqrt{98} \\ \sqrt{2} & \hline & \sqrt{64} & \sqrt{196} \\ & 8 & 14 \end{array}$$

$$\begin{array}{c} 14 \\ 8 \\ \hline 22 \\ 22 \\ \hline 44 \\ 44 \\ \hline \sqrt{484} \\ \sqrt{2} \\ \hline \end{array}$$

$\sqrt{242}$ Optata summa vt supra.

Addantur

Addantur $\sqrt{4\frac{1}{2}}$ ad $\sqrt{12\frac{1}{2}}$

$\sqrt{2}$

$\sqrt{9}$ $\sqrt{25}$

3 5 5

3

8

8

$\sqrt{64}$
$\sqrt{\ 2}$ $\Big\}\sqrt{32}$ Summa.

Idem dividendo

$\sqrt{4\frac{1}{2}}$ $\sqrt{12\frac{1}{2}}$

$\sqrt{2}$

$\sqrt{2\frac{1}{4}}$ $\sqrt{6\frac{1}{4}}$

$1\frac{1}{2}$ $2\frac{1}{2}$

$2\frac{1}{2}$

$1\frac{1}{2}$

4

4

$\sqrt{16}$

$\sqrt{2}$

$\sqrt{32}$ Summa eadem quæ prius.

Addantur $\sqrt{32}$. $\sqrt{128}$. $\sqrt{200}$

$\sqrt{2}$

$\sqrt{16}$ $\sqrt{64}$ $\sqrt{100}$

4 8 10

8

10

22

22

44

44

$\sqrt{484}$

$\sqrt{\ \ 2}$

Optata $\sqrt{968}$ Summa.

Add.. $\sqrt{243}$. $\sqrt{300}$ $\sqrt{147}$. $\sqrt{192}$

$\sqrt{3}$

$\sqrt{81}$ $\sqrt{100}$ $\sqrt{49}$ $\sqrt{64}$

9 10 7 8

10

7

8

34

34

136

102

$\sqrt{1156}$

$\sqrt{\ \ 3}$

$\sqrt{3468}$ Optata summa.

Addantur

Addantur $\sqrt{24}$. $\sqrt{54}$. $\sqrt{66\frac{2}{3}}$. $\sqrt{42\frac{2}{3}}$. $\sqrt{130\frac{2}{3}}$.

$\sqrt{6}$

Veri quadrati horum latera.	$\sqrt{4}$	$\sqrt{9}$	$\sqrt{11\frac{1}{9}}$	$\sqrt{7\frac{1}{9}}$	$\sqrt{21\frac{7}{9}}$
	2	3	$3\frac{1}{3}$	$2\frac{1}{3}$	$4\frac{2}{3}$

2
3
$3\frac{1}{3}$
$2\frac{2}{3}$ $15\frac{2}{3}$ Hanc laterum ſumam quadrato.
$4\frac{2}{3}$ $15\frac{2}{3}$

Summa laterum $15\frac{2}{3}$ 75

15

$20\frac{4}{9}$

$\sqrt{245\frac{4}{9}}$
$\sqrt{6}$ Iſto enim diviſore ad quadratos ſunt reducti.

$\sqrt{1472\frac{2}{3}}$ Summa datorum numerorum.

Ilud porró notandum cum numeri proponentur aſymmetri, hoc eſt cujuſmodi ſunt $\sqrt{11}$ & $\sqrt{3}$ iſtos non in unum numerum contrahi poſſe, ſed ſigno $+$ intermedio conjungi, atque id additionis inſtar eſſe, idque ſignum pluris vocari, hoc modo $\sqrt{11}+\sqrt{3}$. id eſt $\sqrt{11}$ plus $\sqrt{3}$. ſeu latus numeri 11 plus latere ternarij.

CAP. III.

De irrationalium ſimplicium ſubductione

QVadratos datorum numerorum addito, eoſdemque inter ſe multiplicatos quadruplicato, facti latere de ſupra ſcripta ſumma ſubducto reliqui latus quadratum erit optata datorum numerorum differentia.

Idem eodem ſenſu.

Si latus quadrupli à datis numeris facti ab eorundem quadratorum ſumma demas, reliqui lauts erit optata datorum numerorum differentia.

Exemplum.

Subduc.

Sbduc. $\sqrt{48}$ de $\sqrt{300}$

quad add.	48	48 multip.
	300	300
	348	14400
Subd.	240	4
Reliquus.	$\sqrt{108}$	57600
		2|4|

Subduc. $\sqrt{32}$ de $\sqrt{60\frac{1}{2}}$

	32	
	$60\frac{1}{2}$	32
		$60\frac{1}{2}$
	$92\frac{1}{2}$	
Subd.	88	1920
		16
Reliq.	$\sqrt{4\frac{1}{2}}$	
		1936
		4
		7744
		8| 8/ Latus.

Vtrumque hoc theorema additionis & ſubductionis paulo infra nobis demonſtrabitur: interim vt te in hoc abaco exerceas exempla aliquot numeroum ſymmetrorum ſubjeci.

Subducito		de	Relinquetur
	$\sqrt{2}$	$\sqrt{18}$	$\sqrt{8}$
	$\sqrt{5}$	$\sqrt{45}$	$\sqrt{20}$
	$\sqrt{3}$	$\sqrt{108}$	$\sqrt{75}$
	$\sqrt{12}$	$\sqrt{147}$	$\sqrt{75}$
	$\sqrt{2\frac{2}{5}}$	$\sqrt{29\frac{2}{5}}$	$\sqrt{25}$
	$\sqrt{33\frac{1}{5}}$	$\sqrt{85\frac{1}{5}}$	$\sqrt{12}$
	$\sqrt{57\frac{3}{5}}$	$\sqrt{102\frac{2}{5}}$	$\sqrt{6\frac{2}{5}}$
	$\sqrt{28\frac{4}{5}}$	$\sqrt{457\frac{1}{5}}$	$\sqrt{257\frac{3}{5}}$
	$\sqrt{115\frac{1}{5}}$	$\sqrt{204\frac{4}{5}}$	$\sqrt{12\frac{4}{5}}$
	$\sqrt{28\frac{1}{2}}$	$\sqrt{36\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$

Idem aliter & brevius.

SYmmetros numeros ad veré quadratos revocato, horum laterum minus de majore ſubducito, reliquum iterum quadrato, factum per communem ſymmetriæ menſuram multiplicato vel dividito (ſi videlicet initio diviſione uſus es, nunc multiplicabis, & contra) inde optata numerorum differentia prodibit.

Exemplum.

Exemplum.

Subducito $\sqrt{16\frac{2}{3}}$ de $42\frac{2}{3}$
Divide per $\sqrt{6}$
Quoti $\sqrt{2\frac{7}{9}}$ & $\sqrt{7\frac{1}{9}}$

Latera $1\frac{2}{3}$ & $2\frac{2}{3}$ Subducito

Relinquitur 1
1

$\sqrt{1}$
$\sqrt{6}$ multipl.

Reliquus $\sqrt{6}$ quæsitus

Aliter, ubi per multiplicationem ad verè quadratos revocantur.

$\sqrt{16\frac{2}{3}}$ de $42\frac{2}{3}$ Multip. per $\sqrt{6}$

$\sqrt{100}$ & $\sqrt{256}$ Facti

10 16 Latera
10 Subducito.

6 relinquitur
6

$\sqrt{36}$ Hunc dividito per $\sqrt{6}$.
$\sqrt{6}$ dabitur in quoto vt supra pro optata datorum numerorum differentia.

Sequentia exempla sunt numerorum asymmetrorum.

Subducantur $\sqrt{7}$ de $\sqrt{13}$, relinquetur $\sqrt{13}$ minus $\sqrt{7}$. ideoque præponito $\sqrt{13}$ eique postponito $\sqrt{7}$ inter medio hoc signo —, quod minus seu subductionem notat, hac formula $\sqrt{13}$ — $\sqrt{7}$. Sed eosdem quoque secundum primi præcepti doctrinam subducere licebit, eo qui sequitur modo. Quadrata datorum numerorum 13 & 7 addita sunt 20, eorundem factus 91 quadruplicatus dabit 364. hujus latus est $\sqrt{364}$, quod de 20 subductum reliquum faciet 20 — $\sqrt{364}$ cujus latus ita notatur $\sqrt{.20 - \sqrt{364}}$. puncto primo characteri affixo modo $\sqrt{.}$ quod indicium est latus ex utroque juncto eruendum Istum autem numerum tantundem valere quantum $\sqrt{13}$ — $\sqrt{7}$ infra demonstrabitur.

Numeros hos paria facere levi momento ita experiri licebit. $\sqrt{13} - \sqrt{7}$ in se ipsum multiplicato, factus erit $\sqrt{20} - \sqrt{364}$. atque hujus latus $\sqrt{.20 - \sqrt{364}}$. qui cum ante invento idem est.

$$\text{Subd.}\begin{Bmatrix} \sqrt{8} \\ \sqrt{4\frac{1}{2}} \\ \sqrt{17} \\ \sqrt{20\frac{2}{3}} \end{Bmatrix} \text{De} \begin{Bmatrix} \sqrt{12} \\ \sqrt{19} \\ \sqrt{23\frac{1}{2}} \\ \sqrt{36\frac{1}{3}} \end{Bmatrix} \text{Relin.} \begin{Bmatrix} \sqrt{12} - \sqrt{8} \\ \sqrt{19} - \sqrt{4\frac{1}{2}} \\ \sqrt{23\frac{1}{2}} - \sqrt{17} \\ \sqrt{36\frac{1}{3}} - \sqrt{20\frac{2}{3}} \end{Bmatrix} \text{Vel} \begin{Bmatrix} \sqrt{.20 - \sqrt{384}} \\ \sqrt{.23\frac{1}{2} - \sqrt{342}} \\ \sqrt{.40\frac{1}{2} - \sqrt{1598}} \\ \sqrt{.57 - \sqrt{3003\frac{1}{3}}} \end{Bmatrix}$$

Plane ad eundem modum in additione secundum primum ejusdem præceptum duo numeri asymmetri in unam summam componi possunt.

Esto addendum $\sqrt{6}$ ad $\sqrt{7}$, totus erit $\sqrt{7} + \sqrt{6}$ vel $\sqrt{.13 + \sqrt{168}}$. atque ita deinceps in cæteris.

CAP

CAP. IIII.

De irrationalium simplicium multiplicatione

Datorum numerorum quadrata inter se multiplicentur, facti latus erit numerus optatus.

Exemplum multiplicato $\sqrt{5}$ per 3 quadrata 5 & 9 inter se multiplicentur, unde 45 existent, cujus $\sqrt{}$ erit $\sqrt{45}$ numerus optatus á datis factus.

Est autem istud accuraté observandum cum duo numeri quorum alter numerus absolutus vel rationalis sit, alter veró irrationalis, hoc est notam hanc $\sqrt{}$ sibi habet præfixam, túm numerum rationalem & absolutum ante quadrari debere, ut dato ita fiat homogeneus, vt 3 in se ducta dabunt 9. est igitur $\sqrt{9}$ idem quod 3, & datorum uterque signis homogeneis afficitur vt pote $\sqrt{5}$ & $\sqrt{9}$. ideoque ad multiplicationem vel divisionem quoque apti, quæ ratio in cæteris omnibus similibus locum habet.

Exempla symmetrorum.

$$\text{Multiplicato}\left\{\begin{matrix}\sqrt{72}\\ \sqrt{300}\\ \sqrt{60\frac{1}{2}}\\ \sqrt{31\frac{1}{4}}\end{matrix}\right\}\text{Cum}\left\{\begin{matrix}\sqrt{32}\\ \sqrt{192}\\ \sqrt{50}\\ \sqrt{24\frac{1}{5}}\end{matrix}\right\}\begin{matrix}\text{Factus}\\ \text{erit.}\end{matrix}\left\{\begin{matrix}48\\ 240\\ 55\\ 27\frac{1}{2}\end{matrix}\right.$$

In istis illud notatu dignum occurrit, duos numeros symmetros (cujus generis sunt suprascripti) inter se multiplicatos facere numerum rationalem. Ideoque nihil necesse esse datos numeros inter se multiplicare, sed satis esse ipsorum latera, cum ad veré quadratos per communem divisorem erunt reducti inter se multiplicare, & numerum ab his lateribus factum per communem illum diuisorem iterum multiplicare, ab his enim factus erit numerus optatus.

Exemplum			Idem aliter		Idem altiter
Multipli. sunto	$\sqrt{128}$ & $\sqrt{32}$		$\sqrt{128}$ cum $\sqrt{32}$		$\sqrt{128}$
	$\sqrt{128}$	$\sqrt{32}$	$\sqrt{2}$		$\sqrt{32}$
Per $\sqrt{2}$			$\sqrt{256}$	$\sqrt{64}$	
	$\sqrt{64}$	$\sqrt{16}$	Latera $\{$ 16	8	256
Ltera $\{$	8	4	8	8	384
	4		128		$\sqrt{4096}$
	32		2 quadr. á $\sqrt{2}$		4096
	2 quadratus á $\sqrt{2}$				
	64 factus optatus.		64 vt supra		64 Idem cum antecedentibus.

Sequentia

Sequentia sunt numerorum asymmetrorum exempla.

$$\text{Multiplicato}\left\{\begin{matrix}\sqrt{12}\\ \sqrt{17}\\ \sqrt{64}\\ \sqrt{1\frac{4}{5}}\end{matrix}\right\}\text{Cum}\left\{\begin{matrix}\sqrt{38}\\ \sqrt{23\frac{1}{2}}\\ \sqrt{42}\\ \sqrt{2\frac{1}{2}}\end{matrix}\right\}\text{Factus erit}\left\{\begin{matrix}\sqrt{456}\\ \sqrt{399\frac{1}{2}}\\ \sqrt{2688}\\ \sqrt{4\frac{1}{2}}\end{matrix}\right\}$$

Mult. $\sqrt{38}$	Mult. $\sqrt{23\frac{1}{2}}$	Mult. 8. hoc est $\sqrt{64}$
Cum $\sqrt{12}$	Cum $\sqrt{17}$	Cum $\sqrt{42}$
76	161	128
38	23	256
	$8\frac{1}{2}$	
Factus $\sqrt{456}$	Fact. $399\frac{1}{2}$	Factus 2688

Cap. V.

De Irrationalium simplicium divisione

QVadratos datorum numerorum inter se dividito, hujus quoti latus erit quotus quæsitus.

Exempla symmetrorum.

$$\text{Dividito}\left\{\begin{matrix}\sqrt{288}\\ \sqrt{2592}\\ \sqrt{1728}\\ \sqrt{18}\\ \sqrt{12}\end{matrix}\right\}\text{Per}\left\{\begin{matrix}\sqrt{18}\\ \sqrt{72}\\ \sqrt{12}\\ \sqrt{4\frac{1}{2}}\\ \sqrt{27}\end{matrix}\right\}\text{Quotus erit}\left\{\begin{matrix}4\\ 6\\ 12\\ 2\\ \frac{2}{3}\end{matrix}\right.$$

~~10~~
~~288~~ {16 hujus latus
18 { 4 est quotus
quæsitus.

~~43~~
~~2592~~ {36
~~77~~
quotus 6
quæsitus.

Divide $\sqrt{12}$ per $\sqrt{27}$ quotus erit $\sqrt{\frac{12}{27}}$ hoc est ob terminorum reductionem sunt enim compositi per 3 $\sqrt{\frac{4}{9}}$ cujus latus $\frac{2}{3}$.

Exempla asymmetrorum.

$$\text{Dividito}\left\{\begin{matrix}\sqrt{832}\\ \sqrt{796}\\ \sqrt{27}\\ \sqrt{2\frac{1}{2}}\\ \sqrt{\frac{8}{9}}\end{matrix}\right\}\text{Per}\left\{\begin{matrix}\sqrt{32}\\ \sqrt{24}\\ \sqrt{2\frac{2}{3}}\\ \sqrt{1\frac{1}{2}}\\ \sqrt{\frac{5}{6}}\end{matrix}\right\}\text{Quotus erit}\left\{\begin{matrix}\sqrt{26}\\ \sqrt{33\frac{1}{6}}\\ \sqrt{10\frac{1}{8}}\\ \sqrt{1\frac{2}{3}}\\ \sqrt{1\frac{1}{15}}\end{matrix}\right.$$

Cum

Cum numerus irrationalis per absolutum seu rationalē dividendus proponitur, numerus hic ante quadrandus tibi erit vt reliquo sit homogeneus, & inde secundum exposita præcepta dividendus.

Notabis divisionis δοκιμασίαν contraria multiplicatione probari, & multiplicationem contrariâ divisione.

Additionis δοκιμασίαν in subductione esse: subductionis autem periculum additione fieri.

CAP. VI.

De binominorum & residuorum, hoc est irrationalum compositorum notatione ac numeratione.

Binomium est numerus é binis nominibus affirmationis signo $+$ junctis compositus, vt $9 - \sqrt{7}$, & $\sqrt{13} + 5$, vel $\sqrt{15} + 2$.

Residuum est numerus é binis nominibus negationis signo junctis compositus, vt $\sqrt{7} + \sqrt{3}$. & $12 - \sqrt{6}$. ac $\sqrt{19} - 3$ aliaque id genus.

Irrationalium compositorum Additio.

Additio irrationalium compositorum sequitur leges simplicium. absoluti enim absolutis, irrationales irrationalibus adduntur, signorum autem affirmati & negati, seu $+$ & $-$ leges ita habent.

Additio in ijsdem signis retinet idem sigum, vt affirmatum $+$ affirmato $+$ additum facit totum affirmatū $+$: negatum $-$ negato $-$ facit totum negaum $-$. In diversis signis additio est subductio & reliquus cum signo majoris erit eorum summa

Exempla.

$$\text{Addito}\left\{\begin{array}{l} 8+\sqrt{50} \\ 7-\sqrt{12} \\ 10+\sqrt{32} \\ 12-\sqrt{98} \\ \sqrt{243}+8 \\ \sqrt{50}-\sqrt{28} \end{array}\right\}\text{Ad}\left\{\begin{array}{l} 11+\sqrt{18} \\ 9-\sqrt{27} \\ 6-\sqrt{2} \\ 8-\sqrt{4\tfrac{1}{2}} \\ 20-\sqrt{12} \\ \sqrt{63}-\sqrt{8} \end{array}\right\}.\ \begin{array}{c}\text{Summa} \\ \text{erit.}\end{array}\left\{\begin{array}{l} 19+\sqrt{128} \\ 16-\sqrt{75} \\ 16+\sqrt{18} \\ 20-60\tfrac{1}{2} \\ 28+\sqrt{147} \\ \sqrt{18}+\sqrt{7} \end{array}\right.$$

Per $\sqrt{2}$

$8 + \sqrt{50}$	25	5	
$11 + \sqrt{18}$	9	3	
Sum. $19 + \sqrt{128}$		8	
		8	
		$\sqrt{64}$	
		$\sqrt{2}$	
		$\sqrt{128}$	

Per $\sqrt{2}$

$10 + 34\sqrt{32}$	16	3
$6 - \sqrt{2}$	1	1
$16 - - \sqrt{18}$		3
		3
		$\sqrt{9}$
		$\sqrt{2}$
		$\sqrt{18}$

$\sqrt{50} - \sqrt{28}$
$\sqrt{63} - \sqrt{8}$

In hoc exemplo quia alternis sunt symm. $\sqrt{50}$. & $\sqrt{8}$. itē $\sqrt{28}$ & $\sqrt{63}$ potius ita transponatur

Per $\sqrt{2}$ $\sqrt{}$

$+5$	25	$\sqrt{50} --- \sqrt{28}$	4	$+2$
-2	4	$\sqrt{8} + \sqrt{63}$	9	$+3$
$+3$				$1+$
3				1
$\sqrt{9}$				$\sqrt{1}$
$\sqrt{2}$				$\sqrt{7}$
$+\sqrt{18}$		Summa.		$+\sqrt{7}$

$\sqrt{18} + \sqrt{7}$

Huc accedant & ista.

Addito		Ad		Summa erit
	$21 + \sqrt{5}$		$17 - \sqrt{8}$	$38 - \sqrt{8} + \sqrt{5}$
	$15 - \sqrt{10}$		$8 - \sqrt{7}$	$13 - \sqrt{10} - \sqrt{7}$.
	$\sqrt{32} - \sqrt{12}$		$\sqrt{300} - \sqrt{32}$	$\sqrt{192}$
	$\sqrt{18} + \sqrt{12}$		$\sqrt{12} - \sqrt{8}$	$\sqrt{48} + \sqrt{2}$.

Irrationalium subductio.

SVbductio haud obscura erit additionis præcepta edocto, quemadmodum in exemplis infra scriptis perspicere est. Subducito $\sqrt{2}$ de $\sqrt{8}+2$, relinquetur $2+\sqrt{2}$. Et $\sqrt{12}+2$ de $\sqrt{27}+3$, reliquus erit $\sqrt{3}+1$. Item $21-\sqrt{3}$ de $28+\sqrt{48}$, relinquetur $7+\sqrt{75}$. In novissimo hoc exemplo notabis 21 non integrum de reliquo numero esse subducendum, nam 21 afficitur multa $\sqrt{3}$, tanto igitur minus erit subducendum: cum igitur 21 de 28 subduxeris, necessum erit vt illud damnum compenses addendo $\sqrt{3}$ ad $\sqrt{48}$, unde conflabitur numerus $\sqrt{75}$. Ideoque relinquetur in universum $\sqrt{75}+7$.

Subductis $23-\sqrt{45}$ de $33-\sqrt{20}$, relinquetur $10+\sqrt{55}$, Ex superioris exempli ἀναλογία intellexisti, cum 22 de 33 integré subducentur tum illum defectum $-\sqrt{45}$ compensandum additione, atqui numero isti unde subductio fieri debet desunt $\sqrt{20}$. quare hæc $\sqrt{20}$ de $\sqrt{45}$ subducta relinquent $\sqrt{5}$ quæ ad 10 addita dabunt $10+\sqrt{5}$ quæsitam datorum numerorum differentiam.

Subducito

Subducito $\sqrt{128}+\sqrt{5}$ de $\sqrt{512}-\sqrt{45}$, relinquetur $\sqrt{128}-\sqrt{80}$. Hic de $\sqrt{512}-\sqrt{45}$ ſubducendi ſunt $\sqrt{128}+\sqrt{5}$, quare $\sqrt{128}$ ſubducto de $\sqrt{512}$ reliquus erit $\sqrt{128}$. ſed inſuper quoque $\sqrt{5}$ illinc eſt deducendus: quare $\sqrt{5}$ additus ad $-\sqrt{45}$ conflabit $\sqrt{80}$, qui numerus de $\sqrt{128}$ de tractus dabit optatam differentiam $\sqrt{128}-\sqrt{80}$, vt ſupra.

$$\text{Subducito}\left\{\begin{array}{l} 13-\sqrt{72} \\ 17-\sqrt{27} \\ 16+\sqrt{8} \\ 29+\sqrt{3} \end{array}\right\}\text{De}\left\{\begin{array}{l} 37+\sqrt{128} \\ 29-\sqrt{12} \\ 25-\sqrt{2} \\ 38-\sqrt{19} \end{array}\right\}\text{Relinquetur}\left\{\begin{array}{l} 24+\sqrt{392} \\ 12+\sqrt{3} \\ 9+\sqrt{18} \\ 9-\sqrt{19}-\sqrt{3} \end{array}\right.$$

Irrationalium compoſitorum multiplicatio.

MVltiplicato affirmato $+$ per affirmatum factus erit affirmatus $+$. item negato $-$ per negatum $-$ factus erit $-$. Verum ſi affirmatus per negatum, vel negatus $-$ per affirmatum multiplicetur factus erit negatus.

Hoc eſt vt verba in compendium conferam. multiplicatio in ſignis ijſdem habet ſignum affirmatum $+\surd$ in diverſis veró negatum.

Præcepti veritatem hujuſmodi exemplo planiſſimé perſpicies. ſi multiplices $11-3$, hoc eſt 8, per $7+2$, hoc eſt per 9. Neceſſe itaque eſt horum mutua multiplicatione 72 exiſtere.

$$\begin{array}{rr} 11 & -3 \\ 7 & +2 \\ \hline +77 & -21 \\ +22 & +6 \\ \hline +99 & -27 \end{array}$$

Vtere formula præſcripta, primum 11 per 7 hoc eſt affirmatum $+$ cum affirmato $+$ (namque ubi nullum ſignum præfixum is numerus pro affirmato eſt) factus erit $+77$. Deinde multiplica $+7$ cum -3; dabunt -21, quia ſigna diuerſa. Et $+2$ cum $+11$ fient $+22$ denique $+2$ cum -3 exiſtent -6. Omnes hi numeri pro ſignorum affectione additi dabunt $+99-27$. Hoc eſt ſi hunc de illo ſubducas $+72$ quemadmodum oportuit.

Multiplicato $\sqrt{7}+\sqrt{3}$ cum $\sqrt{28}+\sqrt{12}$. Primum multiplicato $\sqrt{28}$ per $\sqrt{7}$ fiunt $\sqrt{196}$, vel 14: hinc $\sqrt{12}$ per $\sqrt{7}$ exiſtent $\sqrt{84}$, deinde $\sqrt{12}$ per $\sqrt{3}$, facies $\sqrt{36}$ ſeu 6: deinque $\sqrt{28}$ per $\sqrt{3}$ fiunt $\sqrt{84}$. Hinc ad extremum colligito in unam ſummã 14 & 6 totus erit 20: itemq; $\sqrt{84}$ $\sqrt{84}$, (id eſt duplicato $\sqrt{84}$ ſeu quod idem eſt per $\sqrt{4}$ multiplicatio) totus erit $\sqrt{336}$ Et numerus integer datorum multiplicatione factus hic erit $20+\sqrt{336}$.

En tibi

En tibi operis formulam.

$\sqrt{28}+\sqrt{12}$	196	14
$\sqrt{7}+\sqrt{3}$	—	6
	1|4	—
$+\sqrt{196}+\sqrt{84}$	—	20
$+\sqrt{36}+\sqrt{84}$	36	
Summa $20+\sqrt{336}$	6	
	—	

Idem brevius.

per $\sqrt{7}$ per $\sqrt{3}$ reducti

2	$\sqrt{4}$	$\sqrt{28}+\sqrt{12}$	$\sqrt{4}$	2	2
1	$\sqrt{1}$	$\sqrt{7}+\sqrt{3}$	$\sqrt{1}$	1	2

2	2	4
7	3	4
—	—	—
$+14$	$+6$	16
$+6$		21
$20+\sqrt{336}$		—
		16
		32
		—
		$\sqrt{336}$

In exempli numeratione iterata ſymmetros utrimque numeros ad quadratos veros revocavi, eorumque latera á latere ſingulis adſcripſi, nam $\sqrt{28}$ & $\sqrt{7}$ ſymmetros per $\sqrt{7}$ ad veré quadratos 4 & 1 diviſione reduxi, quorum latera 2 & 1, qui numeri inter ſe multiplicati faciunt 2, factus hic per 7 multiplicatus dabit $+14$. Eodem modo altrinſecus quoque $\sqrt{12}$ & $\sqrt{3}$ per 3 ad quadratos 4 & 1 revocati, eorumque latera 2 & 1 inter ſe multiplicata dabunt 2, hæc per 3 (quadratum communis antea diviſoris $\sqrt{3}$) multiplicata faciunt $+6$, qui numerus ad ante inventum 14 additus dabit totum 20. hoc prius eſt quæſiti numeri membrum. Vt ſecundum quoque invenias numerorum ſymmetrorum illa quæ ſupra diximus latera decuſſatim inter ſe multiplicato, quippe ſemel bina ſũt 2, & bis ſingula ſunt 2, hic illi additus dabit 4, & quia prior multiplicantium ſeries per $\sqrt{7}$ & poſterior per $\sqrt{3}$ ad quadratos numeros reducti ſunt, iſti numeri inter ſe multiplicentur, factus $\sqrt{21}$, qui multiplicandus per illa 4, hoc eſt per $\sqrt{16}$, ex quorum multiplicatione exiſtet $\sqrt{336}$ pars optati numeri altera, eritque integer á datis numeris factus $20+\sqrt{336}$. vt ſupra.

En tibi exempla item alia ad noviſſimam hanc formulam expreſſa.

			$\sqrt{2}$
6	3	9	$\sqrt{18}+3$
6	2	4	$\sqrt{8}+2$
—	—	—	—
12	6		$+6$
12	2		
—	—		
24	$12+$		
12	$6+$		
—	—		
$\sqrt{144}$	$18+\sqrt{288}$ Factus		
$\sqrt{2}$			
—			
$\sqrt{288}$			

$\sqrt{3}$			
$8+\sqrt{12}$	4	2	14
$7+\sqrt{3}$	1	1	8
—	—	—	—
56		2	22
6		3	22
—		—	—
62		6	44
			44
			—
			$\sqrt{484}$
			$\sqrt{3}$
			—
Factus		$62+$	$\sqrt{1452}$

Multiplicatis $8+\sqrt{2}$ cum $8-\sqrt{2}$ factus erit 62. Itemque $\sqrt{20}+4$ cum $\sqrt{20}-4$. factus est 4. Hujus generis numeri absolutos semper faciunt, quia numeri alterni seu ad decussim multiplicati eundem numerum faciunt, qui facti ob signa diversa + & — mutuó sese tellunt.

$\sqrt{7}$

Mult. $21-\sqrt{63}$ | $\sqrt{9}$ | 3 | 39

Cum $13-\sqrt{28}$ | $\sqrt{4}$ | 2 | 42

63   6 | 81

21   7 | 81

$+273$ $+42$ | 81

$+42$ 648

$+315$ $\sqrt{6561}$

$\sqrt{7}$

Factus. $315-\sqrt{45927}$ $-\sqrt{45927}$

Mult. $\sqrt{128}+5$

Cum $14-\sqrt{50}$

Numeris transpositis vt homogenei homogeneis respondeant, ita erit.

Per $\sqrt{2}$

$5+\sqrt{128}$ | 64 | 8 | $112+$

$14-\sqrt{50}$ | 25 | 5 | $25-$

$+70$ -40 | $87+$

-80 2 | 87

-10 -80 | 609

Factus   696

$\sqrt{15138}-10.$ $\sqrt{7569}$

$\sqrt{2}$

$+\sqrt{15138}$

Multiplicato $12\frac{1}{2}-\sqrt{60\frac{1}{2}}$

Cum $\sqrt{26\frac{8}{9}}-3\frac{2}{3}$

$\sqrt{2}$

Homogenei homogeneis subscribantur, hoc modo.

$12\frac{1}{2}-\sqrt{60\frac{1}{2}}$ | $\sqrt{121}$ | 11 | $40\frac{1}{3}$

$-3\frac{2}{3}+\sqrt{26\frac{8}{9}}$ | $\sqrt{53\frac{7}{9}}$ | $7\frac{1}{3}$ | $91\frac{2}{3}$

36   77 $132+$

$1\frac{1}{2}$ $3\frac{2}{3}$ $132+$

$8\frac{1}{3}$ $-80\frac{2}{3}$ 264

$-45\frac{5}{6}$ 2   396

$-40\frac{1}{3}$ $40\frac{1}{3}$ 132

$-86\frac{1}{6}$ $\sqrt{17424}+$

$\sqrt{2}+$

Factus. $+$

$\sqrt{8712}-89\frac{1}{6}$ $\sqrt{8712}$

Irrationalium compositorum divisio.

Si numerus é binis nominibus compositus per singularem dividendus proponatur, utrumq; membrum sigillatim dividatur. atque eo casu si divisor dividendo non sit homogeneus, ante ad homogeniam erunt reducendi, & qnotus uterque signo suo iterum junctus dabit numerum optatum.

Exempla ista sunto.

$$\text{Dividito}\left\{\begin{array}{l} \sqrt{18}+\sqrt{10} \\ \sqrt{27}+\sqrt{23} \\ \sqrt{21}+\sqrt{12} \\ \sqrt{128}-8 \\ \sqrt{312\frac{1}{2}}-\sqrt{27} \end{array}\right\}\text{Per}\left\{\begin{array}{c} \sqrt{3} \\ 2 \\ \sqrt{3} \\ \sqrt{8} \\ \sqrt{72} \end{array}\right\}\begin{array}{c}\text{Quotus} \\ \text{erit}\end{array}\left\{\begin{array}{l} \sqrt{6}+\sqrt{3\frac{1}{3}} \\ \sqrt{6\frac{3}{4}}+\sqrt{5\frac{3}{4}} \\ \sqrt{147}+2 \\ 4-\sqrt{8} \\ 4\frac{49}{144}-\sqrt{\frac{3}{8}} \end{array}\right.$$

Cum autem & divisor & dividendus numerus erit é binis nominibus compositus, vt $\sqrt{75}+\sqrt{40}$ per $\sqrt{12}+\sqrt{10}$. Hic illud ante cogitandum erit binomium multiplicatum cum suo residuo facere numerum simplcem. Item factos á duobus numeris per eundem proportionales esse multiplicatis. vt 5 & 6 per 7 multiplicati facunt 35 & 42, qui eam rationem inter se habent quam 5 ad 6. illi enim horum sunt æquæmultipli: ideoque 35 per 5, & 42 per 6 divisi dabunt eundem in quoto.

Hæc via ad divisionem esto prævia, hoc modo, multiplicato semper utrumq; & dividendum $\sqrt{75}+\sqrt{40}$ & divisorem $\sqrt{12}+\sqrt{20}$ per divisoris residuum $\sqrt{12}-\sqrt{10}$, factus illius erit $10-\sqrt{30}$. hujus tantum 2. jam $10-\sqrt{30}$ per 2 diviso (nam vt diximus facti isti factorum suorum æque multiplices eandem quoque rationem inter se servant quam factores. $\sqrt{75}+\sqrt{40}$ & $\sqrt{12}+\sqrt{10}$) quotus $5-\sqrt{7\frac{1}{2}}$ erit optatus.

Operis totius formula ita habet.

$$\begin{array}{c|c|c|c|c|c}
 & & \sqrt{3}\quad\ \sqrt{10} & & & \\
5 & 25 & \sqrt{75}+\sqrt{40} & 4 & 2 & 4+ \\
2 & 4 & \sqrt{12}-\sqrt{10} & 1 & 1 & 5-
\end{array}$$

$$\begin{array}{ccc}
10 & 2 & 1 \\
3 & 10 & 1 \\
+30 & & \sqrt{1} \\
-20 & -20 & \sqrt{30} \\
+10 & & -\sqrt{30}
\end{array}$$

$10-\sqrt{30}$
Dividendus

$$\begin{array}{c}
\sqrt{12}+10 \\
\sqrt{12}-10 \\
\hline
+12 \\
-10 \\
\hline
2 \text{ Divisor}
\end{array}$$

$\frac{10-\sqrt{30}}{2}\Big\{\ 5-\sqrt{7\frac{1}{2}}$ Quotus.

opere contrario multipl.

$$\begin{array}{c}
\sqrt{12}+\sqrt{10} \\
5-\sqrt{7\frac{1}{2}} \\
\hline
-\sqrt{90}-\sqrt{75} \\
+\sqrt{250}+\sqrt{300} \\
\hline
\sqrt{75}+\sqrt{40}
\end{array}$$

Idem qui dabatur.

Dividito $34\frac{2}{7}+\sqrt{80}$ per $\sqrt{135\frac{1}{7}}-7$. quotus erit $28\frac{4}{7}$. Hic vt supra docui multiplicato divisorem $\sqrt{135\frac{1}{7}}-7$ cum suo residuo $\sqrt{135\frac{1}{7}}+7$, factus erit $86\frac{1}{7}$. per eundem residuum multiplicato quoque dividendum $34\frac{2}{7}+\sqrt{80}$ & habebis $\sqrt{213996}$[illegible]$+344\frac{4}{7}$. quibus per $86\frac{1}{7}$ divisis, dabitur quotus optatus $\sqrt{28\frac{4}{7}}+4$. Quod si hic vulgatam insistas viam res esset laboris & tædij plena, quæ hac ratione cum quadam animi oblectatione peragitur.

CAP. VII.

De analysi lateris quadrati in irrationalibus compositis.

INvestigato latus quadratum, siveextrahito radicem quadratam vt vulgó loquuntur é binomio $18+\sqrt{308}$. quadratorum utriusque membri 304 & 308 differentia 16, latus hujus 4, id majori numero 18 additum dabit 22, é cujus dimidio 11 latus erit $\sqrt{11}$ prius quæsiti lateris membrum: & rursum 11 de eodem 18 subducti relinquent 7, cujus latus $\sqrt{7}$ dabit ejusdem membrum alterum, quibus numeris signo eodem junctiб, dabitur propositi numeri quæsitum latus $\sqrt{11}+\sqrt{7}$.

$$\text{Latus é}\left\{\begin{array}{l}
31-\sqrt{600} \\
36-\sqrt{1232} \\
11\frac{1}{4}+\sqrt{125} \\
11\frac{2}{3}+\sqrt{122\frac{2}{3}}
\end{array}\right\}\text{Erit}\left\{\begin{array}{r}
5-\sqrt{6} \\
\sqrt{22}-\sqrt{14} \\
2\frac{1}{2}+\sqrt{5} \\
\sqrt{7\frac{2}{3}}+\sqrt{2}
\end{array}\right\}$$

Latus

Latus numeri $542\frac{1}{2} + \sqrt{289406\frac{1}{4}}$ est $17\frac{1}{2} + \sqrt{236\frac{1}{4}}$

$542\frac{1}{2}$ $+$ $\sqrt{289406\frac{1}{4}}$	$17\frac{1}{2} + \sqrt{236\frac{1}{4}}$	$17\frac{1}{2}$
$542\frac{1}{2}$	$17\frac{1}{2} + \sqrt{236\frac{1}{4}}$	2
1084	119	35
2168	17 $+236\frac{1}{4}$	35
2710	$17\frac{1}{4}$	175
$542\frac{1}{4}$		105
$294306\frac{1}{4}$ } quadrati	$+306\frac{1}{4}$	1225
$289406\frac{1}{4}$ }	$236\frac{1}{4}$	$236\frac{1}{4}$
4900 reliquus	$542\frac{1}{2}$	7350
4900		3675
7|0 Latus		2450
		$306\frac{1}{4}$
Memb. majus $542\frac{1}{4}$		$542\frac{1}{2} + \sqrt{289406\frac{1}{4}}$ Qui supra dabatur.
70		
Summa $612\frac{1}{2}$		

Semiſs. $306\frac{1}{4}$ Hujus latus—$17\frac{1}{2}$ Eſt membrm prius : iterum ſubducito $306\frac{1}{4}$ de $542\frac{1}{2}$ reliqui latus $\sqrt{236\frac{1}{4}}$ erit membrum minus hoc modo $17\frac{1}{4} + \sqrt{236\frac{1}{4}}$.

AD lateris quadrati analyſin inſcholis Geometricis & Algebua P. Rami Theorema tale extat. Binomij vel reſidui latus retexitur primum tollendo quadratum dimidiati minoris ſegmenti á quadrato dimidiati ſegmenti, & reliqui latus addendo ad majoris dimidium, latus totius erit ſegmentum majoris quaſiti lateris. dein de tollendo eundem ab eodem majoris ſegmento propoſiti quadrati, reliqui latus erit minus ſegmentum quaſiti lateris ſigno proprio illi nectendum.

$23 + \sqrt{448}$		
$11\frac{1}{2}$ $\sqrt{112}$		
$11\frac{1}{2}$	$11\frac{1}{2}$	$11\frac{1}{2}$
	$4\frac{1}{2}$	$4\frac{1}{2}$
$132\frac{1}{4}$	16	7
112	$4 + \sqrt{7}$ Latus quæſitum	
$20\frac{1}{4}$		
$4\frac{1}{2}$ Hujus latus.		

De nume-

CAP. VIII.

De numeratione compositorum irrationalium universalium.

CVm numerus é binis nominibus compositus hujusmodi nota præfixa insignietur $\surd.$ videlicet $\surd$ sequente puncto, vt in his $\surd.20+\surd 396$, vel $\surd.20-396$, ubi nota hæc $\surd.$ indicio est latus quadratum eruendũ é datorũ numerorũ $20+\surd 396$ vel $20-\surd 396$, membro utroque ejusque vim utrique esse cummunem. Nam signum id tum præponitur, cum datus numerus compositus latere caret, videlicet latere totidem membris singularibus aut parcioribus, quam datũ quadratum habeat enuntiandum cujus investigationem jam paulo ante edoctus es. Hujus antem generis numeri non minus frequentes quam varij ubique occurrunt in lateribus polygonorum circulo adscriptorum, quod plurimus exemplis, in eo, qnem de circulo & adscriptis edidi libro videre est.

Propositi numeri $\surd.20+396$ & $\surd.20-\surd 396$ in suo genere latus verũ habẽt ideoque pro istis ipsorũ latera usurpari possunt, videlicet $\surd 11+3$ & $\surd 11-3$.

Ejusdẽ generis sunt $\surd.2+\surd 3$ vel $\surd.2-\surd 3$ quibus æquivalent $\surd 1\frac{1}{2}+\surd\frac{1}{2}$ & $\surd 1\frac{1}{2}-\surd\frac{1}{2}$, quorum ille est mensura subtensæ 150 gradum, hic vero 30 gradum, posita diametro partium 2.

Sequentes autem $\surd.2+\surd 2$, & $\surd.2-\surd 2$ in suo genere latus nullum habent totidem nominibus explicabile. Item $\surd.2+\surd.2+\surd 2$, Qui tamen additionis leges, quas supra in irrationalibus initio expressimus, in se admittunt. theorema autem ipsum ita habet.

Facti à datorum universalium quadratis quadrupli ad eorundem quadratorum summam additi latus erit optata datorum numerorum summa

Exemplum addautur $\surd.2+\surd 2$ ad $\surd.2-\surd 2$. Horum quadrata $2+\surd 2$ & $2-\surd 2$ addita dabunt 4. Factus ab ipsis $\surd 2$, hic per 2 seu $\surd 4$ multiplicatus dabit $\surd 8$, is cum quatuor compositus dabit $4+\surd 8$, cujus latus $\surd.4+\surd 8$ optatam numerorum summam exhibet.

Rursum addantnr $\surd.2+\surd.2+\surd 2$ ad $\surd.2-\surd.2+\surd 2$. Quadrata addita dabunt 4. Quadrata inter se multiplicata facient $4-\surd 2+\surd 2$. Hoc est 4 multata 2 & præterea $\surd 2$. seu $2-\surd 2$. hic factus per quatuor multiplicatus dabit $8-\surd 32$. cujus altus $\surd.8-\surd 32$ ad illa 4 addita dabunt $4+\surd.8-\surd 32$ hujus latus $\surd.4+\surd.8-\surd 32$ dabit totum numerum ex addendorum summa compositum. Demonstratio suo loco dicetur, interea tamen contemplator figuram subjectam in qua hujus causam planissime perspicere est.

Nam cum tota A B in duas partes secetur ie C, vt A C sit $\surd.2-\surd.2+\surd 2$, & C B $\surd.2+\surd.2+\surd 2$, horum segmentorum quadrata sunt $2-\surd.2+\surd 2$, & $2+\surd.2+\surd 2$ quemadmodum in ipsis notatum vides. Deinde complementa A D B & E F G D $\surd 2-\surd 2$ & $\surd.2-\surd 2$ Illis addita constituent quadratum A B H G super A B lunea descriptum; videlicet $4+\surd 8-\surd 32$. tantum inquam

 est,

eft, integrum quadratum A B H F, cum totum fuis partibus æquale fit Quare hujus latus erit quantitas lineæ A B é datis fegmentis compofitæ, vt pote $\surd.4+\surd.8-\surd 32$.

Caufa inquam tota vel in numerorũ abfolutorum multiplciatione perfpici poterit. Sunto addendi 4 & 5, eadem formula. Hic cum 4 & 5 fint 9. quadrata partium 4 & 5 facient 16 & 25, feu 41, tum rectangulum fub fegmentis 4 & 5 comprehenfum 20, erit duplicandum feu per 2 multiplicandum, unde exiftent 40, ea cum 41 compofita dabunt 81 quadratum æquale quadrato á 9. itaque illius 81 latus reddet 9 datorum numerorum fummam. Hoc ideo paradigmate explicare vifum eft opere pretium, vt caufa additionis magis perfpicua evadat, ne dictio illa autoris de facto datorum numerorum per 4 multiplicando lectori incauto imponat. Quare ad hanc formulam noviffimum illud autoris exemplum revocare non inutile fuerit. Addendi funto numeri $\surd.2+\surd.2+\surd 2$. & $\surd.2-\surd.2+\surd 2$.

$$\begin{array}{rl} & \surd.2+\surd.2+\surd 2 \\ & \surd\ 2+\surd.3+\surd 2 \\ \hline \text{quadr.} & 2+\surd.2+\surd 2 \\ & 2-\surd.2+\surd 2 \\ \hline \text{quad fum.} & 4 \\ & 4+\surd.8-\surd 32 \end{array} \qquad \begin{array}{rl} \surd.2-\surd.2+\surd 2 & \\ \surd.2-\surd.2+\surd 2 & \\ \hline 2-\surd.2+\surd 2 & \text{quadrat.} \end{array}$$

$$\begin{array}{r|l} \surd. & 2+\surd.2+\surd 2 \\ \surd. & 2-\surd.2+\surd 2 \\ \hline \multicolumn{2}{c}{+\ 4 - - 2+\surd 2} \\ \multicolumn{2}{c}{\text{ideft}} \\ \multicolumn{2}{r}{+\surd.2-\surd 2} \\ \multicolumn{2}{r}{4 \quad 16} \\ \hline \multicolumn{2}{r}{+\surd.8-\surd 32} \end{array}$$

Factus á fegmentis duplicandus; ergo hoc modo pro fignorum affectione prior per 4 pofterior per 16 erit multiplicandus.

Hujus igitur latus eorum fumma. $\surd.4-\surd.8-+\surd 32$

Si quam proxumé in abfolutis numeris longitudinẽ A B $\surd.4+\surd.8-\surd 32$ cognofcere libeat. initium tibi erit á fine faciendum, & latus é numero 32 eruendum, idque ipfum (ob figni fui affectionem) de 8 deducendum, & reliqui latus iterum eruendum, atque hoc quoque ob fui figni affectionem ad 4 addẽdum, ejus latus ad extremum quantitatem lineæ A B definiet, pro ἀκϕιβεία & accuratione quam hæc lateris analyfis admittet.

Operis

Operis formula hæc eſt.

32000000000000000	2343145760000000
5|6|5| 6|8|5| 4| 2| 4	1|5|3|0|7|3|3|7| 2
800000000	4
565685424	153073372
234314576	553073372
55307337200000000	
2|2|3| 5|1|7|5| 1|2	

Quæſita A B $2\frac{23517512}{100000000}$

ATque hinc appellantur univerſales, quod nota poteſtatis eruendæ $\surd$. *non unum tantum numerum afficiat, vt in ſimplicibus* $\surd 5$, *& compoſitis quoque* $\surd 7 + \surd 5 + 3$ *vel* $\surd 7 - \surd 5 + \surd 3$. *cæteriſque id genus infinitis, ubi ſingulæ notæ ſingulos numeros afficiunt: ſed vt minimum ad duos ſeſe didat, vt in* $\surd . 7 + \surd 5$. *indicat enim latus vltimi numeri 5, eruendum & antecedenti 7 addendum, ex eoque latus erutum eſſe numerum quæſitum ſub his notarum & affectionum involucris reconditum. Atque ita* $\surd . 9 + 49$ *nihil aliud eſt quam 4. &* $\surd . 4 + \surd . 21 + \surd 16$ *idem quod 3. Nolui igitur appellationem & nomen mutare, cum non ab autore noſtro ſolum, ſed jam olim á præſtantibus in hâc arte viris iſta notione uſurpatum videam, idque ne eſſes neſcius viſum eſt monere.*

VNIVERSALIVM SVBDVCTIO.

QVadrata datorum univerſalium addito, atq; eadem inter ſe multiplicato, factum per 4 multiplicato, hujus latus de quadratorum ſumma ſubducito, reliqui latus optatam datorum numerorum differentiam exhibebit.

Brevius.

Facti á datorum univerſalium quadratis quadrupli ab eorundem quadratorum ſumma ſubducti latus, erit optata datorum numerorum differentia.

Vbi iſta quæ ad additionis theorema á nobis annotata ſunt, quoque locum habere memineris.

Subducito $\surd . 2\frac{1}{2} - \surd 1\frac{1}{4}$ de $\surd . 2\frac{1}{2} + \surd 1\frac{1}{4}$. reliquus erit $\surd . 5 - \surd 20$. ſequere præcepti normam, addito primum quadrata, ſumma erit 5; eadem quadrata inter ſe multiplicato, factus itidem erit 5, hujus quadruplum 20. ejuſque latus $\surd 20$, idque de ſumma 5 ſubductum dabit $5 - \surd 20$, cujus latus dabit optatam differentiam $\surd . 5 - \surd 20$.

Subducantur

Subducantur $\surd.\surd 490 — 20$ de $\surd.\surd 321\frac{9}{16} — \surd 8\frac{3}{4}$, reliquus erit $\surd.\surd 4784\frac{1}{16} — 63\frac{3}{4}$

Addito Quadr. $\left\{\begin{array}{l} \surd 420 —— 20 \\ \surd 321\frac{9}{16} —— 8\frac{3}{4} \end{array}\right.$

Summa $\surd 1476\frac{9}{16} —— 28\frac{3}{4}$

Deinde multiplicato eorum quadrata.

$$\begin{array}{c|c|l} 1\frac{3}{4} & 3\frac{1}{16} & \surd 321\frac{9}{16} —— 8\frac{3}{4} \\ 2 & 4 & \surd 420 —— 23 \end{array}$$

3
3

9
2
$2\frac{1}{4}$

Súbducito $\surd.\surd 420 - \surd 20$ de $\surd.\surd 321\frac{9}{16} - 8\frac{3}{4}$ relinquetur $\surd.\surd 4784\frac{1}{16} - 63\frac{3}{4}$

Quadra. addenda $\left\{\begin{array}{l} \surd 420 —— 20 \\ \surd 321\frac{9}{16} —— 8\frac{3}{4} \end{array}\right.$

Eorum summa. $\surd 1476\frac{9}{16} —— 28\frac{3}{4}$

Eadem quad inter se multiplicatio.

$\surd 105$

$$\begin{array}{cc|l|l} 1\frac{3}{4} & 3\frac{1}{16} & \surd 321\frac{9}{16} — 8\frac{3}{4} & 17\frac{1}{2} \\ 2 & 4 & \surd 420 — 20 & 35 \end{array}$$

$3\frac{1}{2}$
105

315
$52\frac{1}{2}$

$+267\frac{1}{2}$
875

$+542\frac{1}{2}$

$+175$

$52\frac{1}{2}$
$52\frac{1}{2}$

104
260
$52\frac{1}{4}$

$2756\frac{1}{4}$
105

13780
2756
$26\frac{1}{4}$

$\surd 289406\frac{1}{4}$

Factus erit $542\frac{1}{2} —— \surd 289406\frac{1}{4}$
Mult. per 4 16

Hujus latus erit.	$2170 —— \surd 4630500$
	$35 —— \surd 945$
Idque subductum de	$\surd 1476\frac{9}{16} --- 28\frac{3}{4}$
Relinqnet	$\surd 4784\frac{1}{16} --- 63\frac{3}{4}$
Ejus latus	$\surd.\surd 4784\frac{1}{16} --- 63\frac{3}{4}$

VNIVERSALIVM MVLTIPLICATIO.

QVadratis universalium inter se multiplicatis facti latus erit numerus ab ipsis factus.

Multiplicentur $\surd.2 - \surd 2$ cum $\surd.2\frac{1}{2} - \surd 1\frac{1}{4}$ factus eorum erit $\surd.5 + \surd 2\frac{1}{2} - \surd 12\frac{1}{2} - \surd 5$. Hic quadrata $2 - \surd 2$ & $2\frac{1}{2} - \surd 1\frac{1}{4}$ inter se multiplicētur & facti latus eruatur præfixo charactere $\surd.$, vt supra vides.

Quam vim præposita nota $\surd.$ in isto exemplo atque alijs consimilibus habeat, obiter indicavisse operæ videtur pretium. Vt istum numerum $\surd.5 + \surd 2\frac{1}{2} - \surd 12\frac{1}{2} - \surd 5$ ad numerum absolutum reducas, primo è singulis $2\frac{1}{2}$ & $12\frac{1}{2}$ & 5 sigillatim latera erues, hinc pro signorum affectione latus numeri $2\frac{1}{2}$ ad primum 5 addes, reliquorum autē duorum $12\frac{1}{2}$ & 5 latera ex ea summa deduces, hujus reliqui latus quæsitum numerū his characteribus & signis expressum exhibebit, atque ita in cæteris, vide autorem sub hujus capitis initium.

Multiplicato $\surd.2 - \surd 3$ cum $\surd 1\frac{1}{2} + \surd\frac{1}{2}$ factus erit 1. Hic quadratus prioris numeri est $2 - \surd 3$. posterior autem $\surd 1\frac{1}{2} + \surd\frac{1}{2}$ quoque quadratus dabit $2 + \surd 3$, numerus ab his quadratis factus est 1, cujus latus itidem 1.

Item multiplicato $\surd.2 + \surd.2 + \surd 2$ per 3. horū quadrati sūt $2 - \surd.2 + \surd 2$ & 9, quorum factus $18 - \surd.162 + \surd 13122$, ejus latus prefixa nota $\surd.$ erutum dabit $\surd.18 - \surd.162 + \surd 13122$.

Denique si $\surd.2 - \surd.2 + \surd 3$ multiplices cum $\surd.2 + \surd.2 + \surd 3$ secundū epilogismum præscriptum invenies factum $\surd.2 - \surd 3$ vel $\surd 1\frac{1}{2} - \surd\frac{1}{2}$.

VNIVERSALIVM DIVISIO

DAtorum universalium quadratis inter se divisis, ejus quod inde existet latus erit quotus optatus.

Dividito $\surd 5$ per $\surd\ 2\frac{1}{2} - \surd.1\frac{1}{4}$ - hic 5 (quadratus scilicet à 5) per $2\frac{1}{2} - \surd 1\frac{1}{4}$ (qui est quadratus numeri $\surd.2\frac{1}{2} - \surd 1\frac{1}{4}$) dividatur itaque divisor $2\frac{1}{2} - \surd 1\frac{1}{4}$ per suum binomium $2\frac{1}{2} + \surd 1\frac{1}{4}$ multiplicatus dabit factum 5. & 5 dividendus per eundem numerum $2\frac{1}{2} + \surd 1\frac{1}{4}$ multiplicatus dabit factum $12\frac{1}{2} + \surd 31\frac{1}{4}$, qui per superiorem factum 5 divisus dabit quotum $2\frac{1}{2} + \surd 1\frac{1}{4}$: cujus latus $\surd.2\frac{1}{2} + \surd 1\frac{1}{4}$ erit quotus optatus

Ita divisis $\surd.2\frac{1}{2} + \surd 1\frac{1}{4}$ per 2, quotus erit $\surd.$ [illegible] $+ \surd$ [illegible]. Eodem modo divisis $\surd.2 - \surd.2 + \surd 2$ per $\surd.2 + \surd.2 + \surd 2$, quotus erit $\surd.4 + \surd 82. - \surd 2 + 1$ qui tangens est 11 gradum 15 minutorum.

Hic divisoris $\surd.2 + \surd.2 + \surd 2$ quadratus $2 + \surd.2 + \surd 2$ multiplicetur per suū residuum $2 - \surd.2 + \surd 2$ factus erit $2 - \surd 2$ hic multiplicatus per suū binomiū $2 + \surd 2$ faciet 2, ejus latus $\surd 2$ dabit divisorem optatum. eademq; ratione dividendus $\surd.2 - \surd.2 + \surd 2$ per suprascripti divisoris residuū $\surd.2 - \surd.2 + \surd 2$ multiplicatus faciet $6 + \surd 2 - \surd.32 + \surd 512$. Hic factus cum $2 - \surd.2$ multiplicatus

 dabit

dabit $14 + \surd 128 - \surd .320 - \surd 100352$. cujus latus $\surd .14 + \surd 128 - \surd .320 - \surd 100352$ id per $\surd 2$ divisum dabit quotum suprascriptum, totius numerationis formula hæc est.

Divisor $\surd .|2 + \surd .2 + \surd 2$
$\surd .|2 - \surd .2 + \surd 2$

4
$-.2 + \surd 2$

$\surd .|2 - \surd 2$
$\surd .|2 + \surd 2$

4
-2

2

$\surd 2$

Dividendus $\surd .|2 - \surd .2 + \surd 2$
$\surd .|2 - \surd .2 + \surd 2$

4
$+2 + \surd 2$

$6 + \surd 2$

$\surd .|6 + \surd 2 - \surd .32 + \surd 512$
$\surd .|2 + \surd 2$

12
$+2$

$14 + \surd 128$

$-\surd .2 + \surd 2$
4 16

$\surd .8 + \surd 32$
4 16

$-\surd .32 + \surd 512$

$2 + \surd 2$
$2 + \surd 2$

4
2

$\surd .6 + \surd 32$

$\surd .|32 + \surd 512|\surd 16$ | 4 | 24
$\surd .|6 + \surd 32|\surd 1$ | 1 | 32

162 | 4 | 56
128 | 32 | 56

320 | 128 | 336
280

3136
32

6272
9408

$\surd 100352$

$\surd .320 + \surd 100352.$

— $\surd .32 + \surd 512$ Multiplicanda sunt cum $\surd .2 + \surd 2$. verum id non negligendū, cum $\surd$.quod toti numero facto $\surd .6 + \surd 2 - \surd .32 + \surd 512$ præfixum est ad hoc posterius membrum quoque suam vim diffundat, ideo affectionem ejus gradu altiore involutam, quam si tantum ita scriptum esset $\surd .32 + \surd 512$. atque eam ob causam $\surd .2 + \surd 2$ ut uterque terminus sit homogeneus ante quadrari debere, tunc enim $2 + \surd 2$ eodem gradu esse, quo illa $\surd .32 + 512$. Atque rursum cum universales numeri quiescente tantisper universalitatis nota multiplicentur, $2 + \surd 2$ quadrandū, ut universalitatis quoque notam adipiscatur, ut hic factum vides in $\surd .6 + \surd 32$.

itaque factus ab utroque dati numeri membro erit
$\surd .14 + \surd 128 - \surd .320 + 100352.$

Vnde

Vnde latus erutum dabit $\surd.4+\surd8.-\surd2+1$.

Formula investigandi latus quadratum in hujuscemodi numeris sequitur leges supra in binomijs expositas, est enim numerus hic $14+\surd128-\surd.320+\surd100352$ *instar binomij tractandus, ut* $14+\surd128$ *membrum prius,* $\surd.320+\surd300352$ *posterius intelligatur.*

Formula investigandi lateris quadrati in huiuscemodi numeris sic habet.

$14+\surd128-\surd.320+\surd100352$

dimid. $7+\surd32$

$7+\surd32$ — $\surd80+\surd6272$ dimid.

$80+\surd6272$ quad:

49

32

dimid. quad. $81+\surd6272$

$80+\surd6272$

1

Differentia á quadratis segmentorum est 1, cujus $\surd$ itidem 1, unitatem hanc addito ad semissem membri maioris videlicet $7+\surd32$ totus erit $8+\surd32$, hujus semissis $4+\surd8$ subductus de eodem $7+\surd32$ relinquet $3+\surd8$, hujus latus (nam prior pars $4+\surd8$ latere caret) dabit pro secundo segmento $\surd2+1$. id priori signo suo coniunctum dabit integrum optati numeri latus $\surd.4+\surd8.-\surd2+1$.

Notabis autem hunc numerum tanquam binomium concipi, cujus prius membrum $\surd.4+\surd8$. *atque distinctionis gratia punctum post 8 affigi: alterum veró membrum esse* $-\surd2+1$, *ut hic signum — totum consequens de toto antecedente neget. Ideoque in analysi ad numeros absolutos 1 addendum á latus numeri 2, atque hanc summam de priore membro universo subducendam, tumque ad extremum hujus reliqui latus erutum exhibere valorem ejus in numeris absolutis.*

Proponatur numerus $\surd.2-\surd.2\frac{1}{2}+\surd1\frac{1}{4}$ dividendus per $\surd.2+\surd.2\frac{1}{2}+\surd1\frac{1}{4}$, dabitur quotus $\surd5+1-\surd.5+\surd20$.

Formula numerationis ista est.

dividendus. $\surd.2-\surd.2\frac{1}{2}+\surd1\frac{1}{4}$ — $\surd.|2+\surd.2\frac{1}{2}+\surd1\frac{1}{4}$ divisor

$\surd.2-\surd.2\frac{1}{2}+\surd1\frac{1}{4}$ — $\surd.|2-\surd.2\frac{1}{2}+\surd1\frac{1}{4}$

ab his factus $\surd.2-\surd.2\frac{1}{2}+\surd1\frac{1}{4}$. — $\surd.1\frac{1}{2}-\surd1\frac{1}{4}$. factus

iterum multip. cum $\surd.1\frac{1}{2}+\surd1\frac{1}{4}$. — $\surd.1\frac{1}{2}+\surd1\frac{1}{4}$

Dabit $\surd.6+\surd20-\surd.5+\surd50$ — fit $\surd1$, sive 1, qui erit divisor.

Operis hujus δοκιμασία talis est. Latus numeri novissimi 20 addito ad penultimum 5, hujus summæ latus seorsim adnotato, deinde primi numeri 5 latus (quod æquale est semissi lateris 20) ad 1 addito, ab hac summa subducito latus suprascriptum é $\sqrt{5}+\sqrt{20}$ erutum, reliquus dati numeri, qui tangens est 9 graduum, valorem in absolutis numeris exhibebit.

Lateris quadrati analysis per numeros absolutos erit hujuscemodi.

200000000000000000000000	94721359549900000000000
4|4|7|2|1|3|5|9|5|4|9|9|	3|0|7|7|6|8|3|5|7|7|1|7|

447213595499
5 addendus

947213595499

447213595499 $\sqrt{20}$
2236067977495 $\sqrt{5}$
1

3236067977495
307768353717 Subducito

1583844032

Erit itaque posita diametro 20000000000 tangens 9 graduum 1583844403. Si cui collibitum erit, hâc eâdem via ad quotcumque myriadum myriadas tangentem istam producere licebit.

Si quæratur tãgẽs 4 gradiũ & 30 scrupulorũ dividito $\sqrt{\ }.2\sqrt{\ }.2+\sqrt{\ }.2\frac{1}{2}+\sqrt{\ }1\frac{1}{4}$ per $\sqrt{\ }.2+\sqrt{\ }.2+\sqrt{\ }.2+\sqrt{\ }1\frac{1}{4}$, quotus erit $\sqrt{\ }4\frac{1}{2}+\sqrt{\ }2\frac{1}{2}+\sqrt{\ }.\frac{1}{2}5\sqrt{\ }+5-\sqrt{\ }5-+1+\sqrt{\ }.5+\sqrt{\ }20$. formula ita est.

Dividend. $\sqrt{\ }.2-\sqrt{\ }.2+\sqrt{\ }.2\frac{1}{2}+\sqrt{\ }1\frac{1}{4}$ $\sqrt{\ }.2+\sqrt{\ }.2+\sqrt{\ }.2\frac{1}{2}+\sqrt{\ }1\frac{1}{4}$ divisor
$\sqrt{\ }.2-\sqrt{\ }.2+\sqrt{\ }.2\frac{1}{2}+\sqrt{\ }1\frac{1}{4}$ $\sqrt{\ }.2-\sqrt{\ }.2+\sqrt{\ }.2\frac{1}{2}+\sqrt{\ }1\frac{1}{4}$

Factus $2-\sqrt{\ }.2+\sqrt{\ }.2\frac{1}{2}+\sqrt{\ }1\frac{1}{4}$ Factus $\sqrt{\ }.2-\sqrt{\ }.2\frac{1}{2}+\sqrt{\ }1\frac{1}{4}$
$\sqrt{\ }.2+\sqrt{\ }.2\frac{1}{2}+\sqrt{\ }1\frac{1}{4}$ $\sqrt{\ }.2+\sqrt{\ }.2\frac{1}{2}+\sqrt{\ }1\frac{1}{4}$

Fa. $\sqrt{\ }.8+\sqrt{\ }.40+\sqrt{\ }320.-2+\sqrt{\ }.2\frac{1}{2}+\sqrt{\ }1\frac{1}{4}$ Factus $\sqrt{\ }.1\frac{1}{2}-\sqrt{\ }1\frac{1}{4}$
$\sqrt{\ }.1\frac{1}{2}+\sqrt{\ }1\frac{1}{2}$ $\sqrt{\ }.1\frac{1}{2}+\sqrt{\ }1\frac{1}{4}$

$\sqrt{\ }.12+\sqrt{\ }80+\sqrt{\ }.200+\sqrt{\ }38720.-\sqrt{\ }5+1.+\sqrt{\ }.5+\sqrt{\ }20.$ | $2\frac{1}{4}$ / $1\frac{1}{4}$

$\sqrt{\ }1$ seu 1.

Si

Si præterea latus eruas é, ſegmento primo quatenus nota univerſalitatis ei præfixa protēditur videlicet é numero √.12+√80+√.200+√38720 invenies latus √4½+√2½ √.5+√5, hinc deducatur numerus √6+1+√.5+√20 vt nota defectus totum hunc numerum ſimul tanquam unum includat, dabitur numerus √4½+√2½+√.5+√5.—√5+1+√.5+√20. tangēs 4 graduum 30 minutorum, qui duplicatus exhibet √18+√10+√.20+√80.—√20+2+√.20+√320 latus polygoni ordinati 40 augulorum circulo circumſcripti ſuppoſita diametro partum 2.

Operis δοκιμασία

Latus numeri 18 erutum dabit 4242640687119. Item latus 10, erit 3162277660168. ex 80 autem erit 8944271909991. huc addantur 20, totius latus erit 5379988095711, atque ad iſtum numerum aggregato latera iuventa ex 18 & 10, totus erit 1278490644299. Deinceps autem porró latus é 320 erutum dabit 17888543819998, huc addito 20 ſummæ latus 6155367074355. De hinc latus é 20 erit 4472135954999, cum quo ſi antecedentia & preterea 2 componas dabitur ſumma 12627503029340, qui numerus de 1278490644299 deductus relinquet 015740341364 latus polygoni 40 angulorum circulo circumſcripti poſita diametro partium 2. idoque aſſumpta diametro 20000000000 tangens 4gr. 30m. dabitur bené accuraté partium 7870170683, cujus periculum facere tibi licebit in canonibus operis Palatim á Valentino Ottone publicatis.

Haud ſecus numero √.2 √.2+√.2+√3 diviſo per √.2+√.2+√.2+√2 dabitur quotus √.16+√192+√216+√200.—√6+2+√3+√2 tangens 3gr. 45m. numerus autē iſte ad abſolutum & explicabilem revocabitur hoc modo. Eruito latus é 192, 216 & 200 (adhibitis in conſilium quotlibet circulis) hiſque omnibus ad 16 additis rurſum latus ex ea ſumma eruito, idque ſeorſim annotato: dehinc iterum latus é ſequentibus 6, 3, 2 erutum ad 2 addito, totamque hanc ſummam de ſuperiore deducito, reliquus 0655434628 dabit tangentem 3gr. 45min. poſita diametro partium 2: atque ideó cum diameter ponetur 20000000000 tangens quoque erit 655434628.

Proportionis, ſive aureæ regulæ exempla quædam.

Quemadmodum 4+√.8—√51⅐ ad √.8+√51⅐, ſic 4 ad quem.

Operis formula ita habet.

$$
\begin{array}{lll}
4+\surd.8-\surd 51\tfrac{1}{5} & \surd.8+\surd 51\tfrac{1}{5} & 4 \\
4-\surd.8-\surd 51\tfrac{1}{5} & 4-\surd.8-\surd 51\tfrac{1}{5} &
\end{array}
$$

$$
\begin{array}{l}
+16 \\
-\ 8-\surd 51\tfrac{1}{5} \\
\hline
8+\surd 51\tfrac{1}{5} \\
8-\surd 51\tfrac{1}{5} \\
\hline
+64 \\
-51\tfrac{1}{5} \\
\hline
12\tfrac{4}{5} \\
4\ \overline{\qquad} \\
\text{divisa}\quad 3\tfrac{1}{5}
\end{array}
$$

Factus dabitur $\surd.128+\surd 13107\frac{1}{5}-\surd 12\frac{4}{5}$, qui per $3\frac{1}{5}$ divisus dabit $\surd.12\frac{1}{2}+\surd 125-\surd 1\frac{1}{4}$, Hic numerus rusum cum $8-\surd 51\frac{1}{5}$ multiplicatus dabit $160-\surd 20480$. hic cum quadratos numerorũ $\surd.12\frac{1}{2}+\surd 125$ & $8-\surd 51\frac{1}{5}$ inter se multiplicabis atque facti latus erues habebis $\surd.160-\surd 20480$; itemque secundum membrũ $-\surd 1\frac{1}{4}$ cum eodem $8-\surd 51\frac{1}{5}$ habebis factum $8-\surd 80$, qui ad antecedentē additus dabit quartum proport. $\surd.160-\surd 20480.+8-\surd 80$.

Et rursũ si datis hisce tribus $4+\surd.8-\surd 51\frac{1}{5}$ $\surd.8+\surd 51\frac{1}{5}$ & $\surd.8-\surd 51\frac{1}{5}$ quæratur quartus proportionalis, ordine dispositis numeris secundum formulam sequentem operator.

$$
\begin{array}{lll}
4+\surd.8-\surd 51\tfrac{1}{5} & \surd.8+\surd 51\tfrac{1}{5} & \surd.8-\surd 51\tfrac{1}{5} \\
4-\surd.8-\surd 51\tfrac{1}{5} & & \surd.8+\surd 51\tfrac{1}{5} \\
\hline
8+\surd 51\tfrac{1}{5} & & \surd 12\tfrac{4}{5}\ \text{factus} \\
8-\surd 51\tfrac{1}{5} & & \\
\hline
\text{Divisor}\quad 12\tfrac{4}{5} & \dfrac{1}{\surd 12\tfrac{4}{5}} &
\end{array}
$$

$\surd 12\frac{4}{5}$ factus per divisorem $12\frac{4}{5}$ divisus dabit in quoto $\frac{1}{\surd 12\frac{4}{5}}$

Hic numerus porró per $4-\surd.8-\surd 51\frac{1}{5}$ multiplicatus, factusque rursũ $8-\surd 51\frac{1}{5}$, numerus qui inde exiſtet per $\surd 12\frac{4}{5}$ ad extremum, divisus, dabit in quoto $\surd 80-8-\surd.136-\surd 28483\frac{1}{5}$ numerum optatum.

Ad extremnm, quemadmodum $\surd.1\frac{1}{4}+\surd\frac{1}{16}+\surd.1\frac{1}{4}+\surd\frac{45}{64}$ ad $\surd.2\frac{1}{4}-\surd\frac{1}{16}-\surd.1\frac{1}{4}+\surd\frac{45}{64}$, ita 2 ad quem? Respondebis ad $\surd.28-\surd 320-\surd.960-\surd 737280$.

Operis forma hæc eſt.

$\surd.1\frac{1}{4}$

$$\frac{\begin{array}{l}\sqrt{.}\big|1\frac{1}{4}+\sqrt{\frac{5}{16}}+\sqrt{.1\frac{7}{8}}+\sqrt{\frac{45}{64}}\\ \sqrt{.}\big|1\frac{1}{4}+\sqrt{\frac{5}{16}}-\sqrt{.1\frac{7}{8}}+\sqrt{\frac{45}{64}}\end{array}}{\begin{array}{l}3\frac{1}{16}\\ +\frac{5}{16}\end{array}}$$

$$\frac{\begin{array}{l}\sqrt{.}\big|2\frac{1}{4}-\sqrt{\frac{5}{16}}-\sqrt{.1\frac{7}{8}}+\sqrt{\frac{45}{64}}\\ \sqrt{.}\big|1\frac{1}{4}+\sqrt{\frac{5}{16}}-\sqrt{.1\frac{7}{8}}+\sqrt{\frac{45}{64}}\end{array}}{\begin{array}{l}3\frac{15}{16}\\ -\frac{5}{16}\end{array}}$$

$$\begin{array}{l}3\frac{3}{8}+\sqrt{\frac{45}{64}}\\ -1\frac{7}{8}+\sqrt{\frac{45}{64}}\\ \hline 1\frac{1}{2}+\sqrt{1\frac{1}{4}}\\ 1\frac{1}{2}-\sqrt{1\frac{1}{4}}\\ \hline 2\frac{1}{4}\\ 1\frac{1}{4}\\ \hline \sqrt{1}\ \text{divisor.}\end{array}
\qquad
\begin{array}{l}1\frac{1}{2}-\sqrt{1\frac{1}{4}}\\ 1\frac{1}{2}-\sqrt{1\frac{1}{4}}\\ \hline \sqrt{.3\frac{1}{2}}-\sqrt{11\frac{1}{4}}\end{array}
\qquad
\begin{array}{l}3\frac{5}{8}+\sqrt{.\frac{5}{64}}\\ +1\frac{7}{8}+\sqrt{\frac{45}{64}}\\ \hline \sqrt{.}\big|5.\frac{1}{2}+\sqrt{1\frac{1}{4}}-\sqrt{.30}+\sqrt{180}\\ \sqrt{.}\big|1\frac{1}{2}-\sqrt{1\frac{1}{4}}\\ \hline 8\frac{1}{4}\\ -1\frac{1}{4}\\ \hline \sqrt{7}-\sqrt{20}\end{array}
\qquad
\begin{array}{l}\sqrt{.}\big|30+\sqrt{180}\\ \sqrt{.}\big|3\frac{1}{2}-\sqrt{11\frac{1}{4}}\\ \hline 105\\ 45\\ \hline \sqrt{.60}-\sqrt{2880}\end{array}$$

Factus $\sqrt{.7}-\sqrt{20}-\sqrt{.60}-\sqrt{2880}$
per tertium 2 multiplicatus

quartus propor. $\sqrt{.28.}-\sqrt{320}-\sqrt{.960}-\sqrt{737280}$
tionalis optatus.

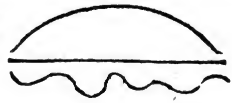
Atque ita hic & irrationalium numerorum & exemplorum finis esto.

LVDOLPHI á CEVLEN HILDESHEIMENSIS Liber ſecundus, in quo Geometrica quædam Fundamenta ὁλοσχερῶς ex *Euclidis* στοιχειώσει ſelecta explicantur.

Definitiones

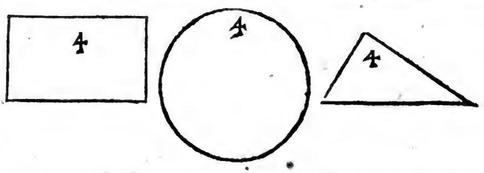
1 PVnctum eſt indiviſibile omnis magnitudinis principium, ſola cogitatione comprehenſibile.

2 Linea eſt longitudo omnis latitudinis expers, cujus terminus, hoc eſt principium atque finis, punctum.



3 Linea recta eſt quæ in directum intra ſuos terminos ſeu puncta porrigitur: curvæ intra ſuos terminos inflexæ jacent. ideoque intra eoſdem terminos curva longior erit quam recta.

4 Figura in ſuperficie eſt, quæ longitudine & latitudine ſola, abſque ulla craſſitie, prædita linearum ambitu concluditur. quæſi locus intra terminātes lineas æquabiliter porrectus jaceat vocatur plana.

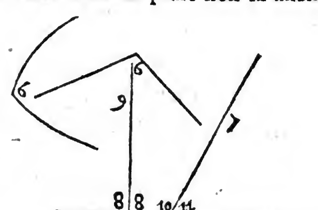


5 Superficierum veró inflexarum, vt ſunt cavæ vel gibbæ, formæ in campanis, columnis, & alibi occurrunt.

6 Angulus planus eſt, quando duæ lineæ in eodem plano non in unum facta porrectione concurrunt, hoc eſt, unam continuam lineam non conſtituunt.

7 Angulus á duabus rectis lineis comprehenſus rectilineus dicitur.

8 Si recta in rectam inſiſtens angulos deinceps faciat æquales, hi anguli recti dicuntur.



9 Recta angulos deinceps faciens rectos alteri perpendicularis insistit.

10 Obtusus est angulus major recto.

11 Acutus est angulus minor recto.

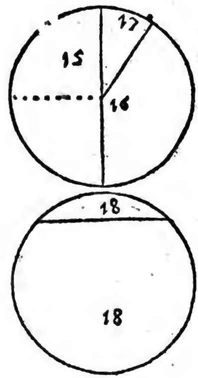
12 Terminus est quo magnitudo terminatur, vel includitur. ut duo extrema puncta in linea; & lineæ quibus superficies comprehenditur.

13 Figura est quæ vel uno tantum termino, vel etiam pluribus concluditur.

14 Circulus est figura plana unica linea, quæ peripheria dicitur comprehensa, in cujus medio punctum, quod vocatur

15 Centrum, á quo omnes lineæ ad ambitum seu peripheriam eductæ æquales sunt.

16 Recta per centrũ educta utrimque in peripheria terminata, ea & peripheriam & circulum bisecat, & dameter circuli dicitur.



17 Ideoque Semicirculus est figura á diametro & semiperipheria comprehensa.

18 Sectio est segmentum circuli á recta inscripta & peripheriæ parte comprehensa, ea semicirculo inæqualis, vel major vel minor erit.

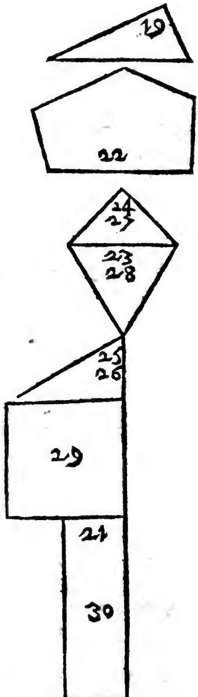
19 Figuræ omnes rectis lineis comprehensæ rectilineæ vocantur.

20 Triangulum tribus rectis lineis comprehenditur.

21 Quadrangulum quatuor.

22 Multangulnm est rectilineum pluribus quam quatuor rectis comprehensum.

23 Trian-



23 Triangulum æqualate-rum quod tria latera habet æqualia.

24 Æquicrurum, quod duo tantum.

25 Scalenum vel varium, quod nullum.

26 Triangulum rectangu-lum, quod habet uni-cum angulum rectum.

27 Obtuſangulum, quod unicum obtuſum.

28 Acutangulum quod om-nes acutos.

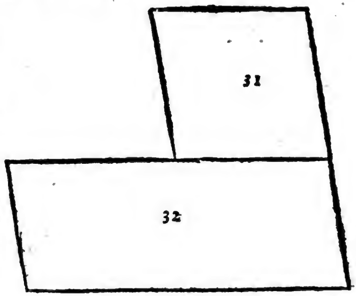
29 Quadratum eſt rectan-gulum æqualaterum, id eſt quatuor lateribus æqualibus & totidem angulis rectis compre-henſum.

30 oblongum eſt parallelogrammum rectangulū inæquilaterum.

31 Rhombus est parallelo g rammum obliquangulum æ quilaterum.

32 Romboides est paral lelogrãmum obliquangulum inæquil aterum.

33 Lineæ parallelæ sunt, quę ubique æqualiter distant: ideoque in eodem plano nusquam concurrunt.



Axiomata, sive communes Notiones.

1 EIdem æqualia inter se sunt æqualia.

2 Si æqualibus æqualia addantur tota sunt æqualia.

3 Si ab æqualibus æqualia subducantur reliqua sunt æqualia.

4 Si ab æqualibus inæqualia subducauntur reliqua sunt inæqualia.

5 Si inæqualibus æqualia addantur tota sunt inæqualia.

6 Ejusdem æquemultiplicia sunt æqualia.

7 Ejusdem eædẽ partes, vt ejusdem duo semisses vel triẽtes, sũt inter se æquales.

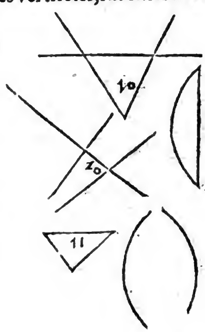
8 Totum est majus parte.

9 Anguli recti á rectis lineis comprehensi sunt æquales.

10 Si rectæ duæ sectæ recta eádem parte faciant angulos duos minores duobus rectis, eodem coutinuatæ tandem concurrent.

11 Duæ rectæ superficiem non comprehendunt.

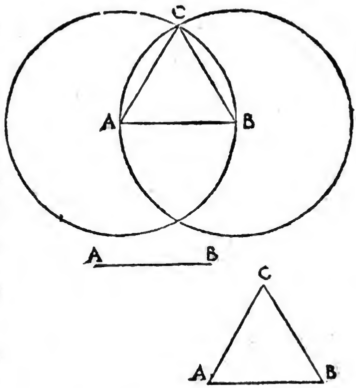
12 Curva una, vel duæ curvæ, aut recta una & duæ curvæ superficiem comprehendere possunt.



PROPOSITIONES QVAEDAM EX EVCLIDE Selectæ OPERI INSTITVTO SERVIENTES.

PROPOSITIO 1. PROBLEMA. 1

Super data recta triangulum ordinatum construere.

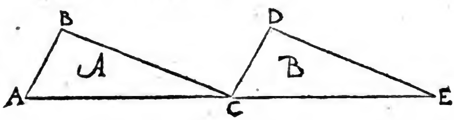


Exponatur linea A B, crusque circini unum statuatur in A, alterum distendatur ad B, eoque intervallo ex termirus A & B tanquam centris duæ æquales peripheriæ mutuo se secent in C, rectæ C A, C B á concursu C ad terminos A & B connexæ, constituent triangulum æquilaterum A B C super data linea A B.

Cujus demonstratio levissimo negotio é definitione 15 primi libri *Euclidis* derivari potest.

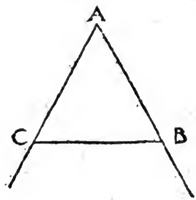
PROPOSITIO. 2.

Si duo triangula æquantur angulis duobus æquicruris, sunt æquilatera & æquiangula.



Est propositio quarta 1 lib. *Euclidis*, cujus sententia hæc est. Duo triangula notata characteribus A & B, habento A B B C, æqualia lateribus C D D E: itemque angulos ab istis lateribus comprehensos A B C C D E inter se æquales, etiam bases A C C D æquales habebunt, & angulos reliquos B A C, B C A, reliquis angulis D C E, D E C æquales: quod, si oculos in consilium adhibeas perspicium est, certé demonstratu propter congruentiam haud difficile. Atque hic istud semel in universum monitum te velim, angulum tribus literis definiri, ejusque mediam vertici deberi, vt in angulo C D E eum indicari qui in D communi sectione crurum C D D E comprehendi intelligitur. Basin autem anguli esse lineam ei subiensam, & cui crura tanquam insistant, quemadmodum hic C E basis est anguli C D E. quod semel hic monuisse sufficiat.

PROPOSITIO. 3.



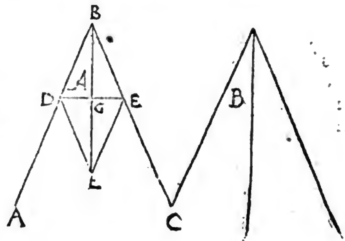
Si triangulum est æquicrurum est in basi æquiangulum: ijsque cruribus prodictis anguli sub basi æquabuntur.

PROPOSITIO. 4.

Si triangulum est in basi æquiangulum est æquicrurum: latera quippe æqualibus angulis subtensa æqualia erunt.

PROPOSITIO 5. PROBLEMA. 2.

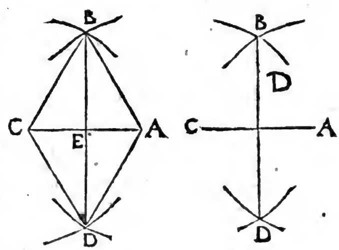
Angulum rectilineum bifariam secare.



Exponatur angulus B comprehensus à lineis A B & B C, centro B intervallo quocunque absumantur æqualia crura B D, B E, atque inde á terminis æqualium crurum D & E tanquam centris duæ æquales peripheriæ concurrant in F. recta B F à concursu ad verticem anguli eum bisecabit. Res perspicua est ex propositione *Euclidis* 4 & nostra 2. Vt hic in figura charactere A insignita videre est.

PROPOSITIO 6. PROBLEMA. 3.

Datam lineam rectam bifariam secare.

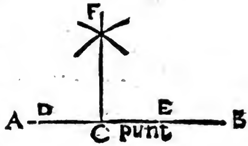


Proponatur A C, super eam constrtuito triangulum æquicrurum quodcunque A B C, ejusque angulum verticalem B bifariam per antecedentem propositionem secato, sitque linea eum bisecans B D, hæc datam A C bisecabit in puncto E, quod é præmissa facile demonstrari potest.

PROPO-

PROPOSITIO 7. PROBLEMA 4.

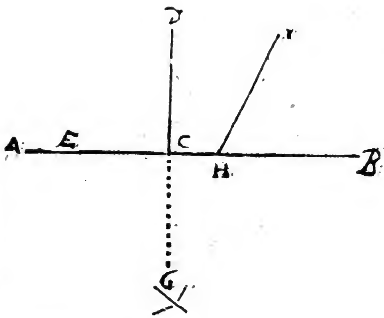
Super datâ rectâ é dato in ipsa puncto perpendicularem erigere.



E dato puncto C tanquam centro in recta linea A B duæ partes utrimque secentur æquales C D C E,& é punctis sectionum D & E duæ æquales peripheriæ concurrant in F, recta C F, a concursu F ad datum punctum C erit perpendicularis super datam A B.

PROPOSITIO 8. PROBLEMA 5.

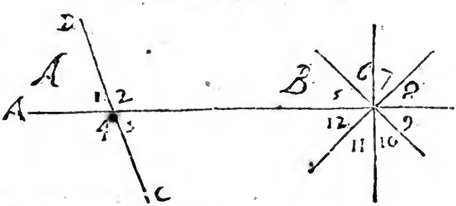
A dato sublimi puncto in rectam subjectam perpendicularem demittere.



Esto recta A B, punctumque extra ipsam infra suprave D. ex D tanquam centro peripheria describatur incidens subjectam A B in E & F, ex quibus tanquam centris duę ęquales peripherię mutuo sese intersecent in G, recta connectens puncta D & G ad angulos rectos intersecabit datam A B in puncto C, eritque D C perpendicularis optata.

PROPOSITIO 9.

Si duæ rectæ lineæ intersecantur anguli ad verticem æquabuntur.



Anguli ad verticem κτ̃ κορυφὴν sũt hic primus & tertius,seu 1 & 3.itemque secundus & quartus ijsdem characteribus insigniti. 2 &4.

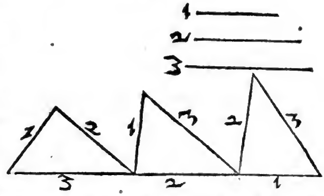
PROPO

PROPOSITIO. 10.

Si quotcunque rectæ in eodem puncto mutuo sese intersecent, omnes in communi sectione quatuor rectis æquabuntur. Quia videlicet cum quatuor rectis ad idem punctum positis æqualem locum occupant. quales sunt omnes his characteribus 5,6,7, 8, 9, 10,11,12 in superiore diagrammate insigniti.

PROPOSITIO 11. PROBLEMA 6.

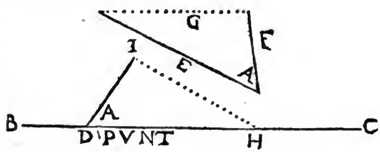
E datis tribus rectis, quarum duæ quælibet, reliqua sint majoris triangulum construere.



Exponantur tres lineæ rectæ 1, 2, 3, ex istis construendum esto triangulum. assumito quamlibet istarum pro basi, deinde ab ejus terminis duæ æquales peripheriæ reliquarum intervallis concurrat, rectæ ab eorum concursu ad subjectæ basis terminos comprehendent triangulum é datis rectis optatum. Fabricam hanc triplici diagrammate expressimus, prout vel hanc vel illam datarum pro basi reliquis supponere libuit.

PROPOSITIO 12. PROBLEMA 7.

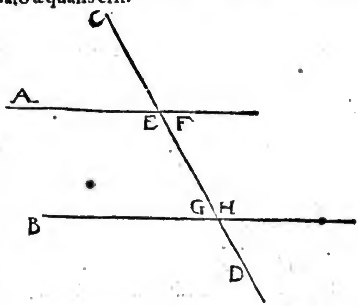
Ad datum datæ rectæ punctum angulum rectilineum dato æqualem construere.



Exponatur recta B C & punctum in eâ D, ad quod construendus sit angulus dato A (qui sub E & F lineis comprehenditur) æqualis. Ponatur DH linea æqualis cruri E, itemque D I, cruri F denique I H basis statuatur basi G æqualis, angulus IDH dato æqualis erit.

PROPOSITIO. 13.

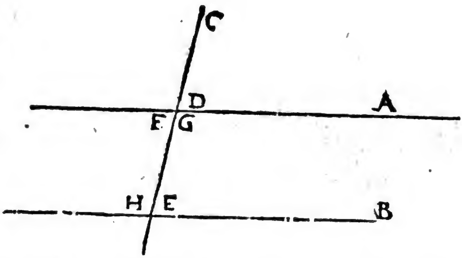
Si rectæ duæ recta sectæ faciunt angulos alternos æquales, istæ sunt parallelæ.



Anguli interiores & alterni sunt E & H, itemque F & G, namque si continuo ordine ita numeres vt E primus F secundus, H tertius, G quartus sit, primus tertio, secundus quarto alternus erit. Si itaque alterni æquales sunt, sequetur quoque lineas ipsas inter se esse parallelas.

PROPOSITIO 14.

Si recta duæ recta secta faciat angulum externū interno & opposito æqualem, vel internos eadem parte duobus rectis æquales erūt parallelæ.



Vt in subjecto diagrammate CE linea utrãque AD & BH incidēs faciat angulum CDA externū interno CEB æqualem, rectæ AD BE propterea erunt parallelæ. Itemque si interni AGE, GEB ab eadem secãtis parte duobus rectis æquales sint, statim sequitur rectas AD, BE pararllelas esse.

PROPOSITIO 15.

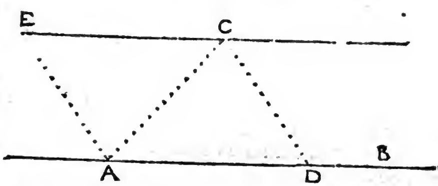
Si recta duas parallelas rectas secat anguli alterni inter se æquabuntur; atque externus interno & opposito; & interni eadem parte duobus rectis æquales erunt. Ita enim triplex angulorum çqualitas parallelismum arguit; & vicissim triplex angulorum æqualitas á parallelismo arguitur.

PROPOSITIO 16.

Lineæ eidem parallelæ inter se sunt parallelæ.

PROPOSITIO 17. PROBLEMA 7.

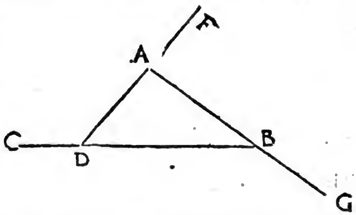
Data recta per datū positione punctum parallelam ducere.



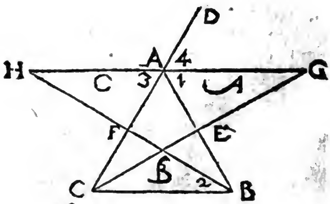
Exponatur recta AB, & pūctum C per quod exigenda sit parallela; sumito quodcunque intervallum á dato puncto C ad subjectam pertingens, deinde esto quęcumque AD, & concipito triangulum ACD, cui aliud statuatur çquilaterum, ut EC ipsi AD, & EA ipsi CD çqualis sit, recta EC infinitè producta datę AB erit parallela.
Complures extant ducendarum parallelarum fabricæ, hæc autem sola toto isto opere maximé frequentatur.

PROPOSITIO 18.

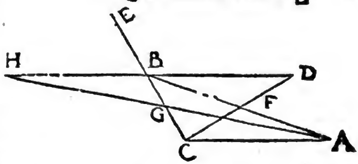
Trianguli latere continuato angulus exterior major est utrolibet interiore opposito.



Si propositi trianguli ABD latus B D cõtinuetur angulus exterior ADC major erit angulo interiore & opposito DAB, vel DBA. itemq; continuato latere AB, angulus exterior D B G major erit utrolibet interiore opposito DAB vel ADB. denique quoque continuato latere A D exterior B A F major utrolibet *B* D A uel A B D.



Namque in hoc secundo diagrãmate latus A C continuetur in D, tum angulus exterior erit B A D, ut cum ostendas minorem utrolibet interiore ACB vel ABC bisecato latus utrumque A C & AB in F & E, & ab angulis oppositis B C per bisectionũ pũcta F & E rectas BF CE cõtinuato in H & G vt B H dupla sit ipsius BF, & CG ipsius CE. recta HG ipsarũ vertices H & G connectens quoque per A punctũ trasibit. Namque per 4 propos. 1 lib. *Eucl.* triangulum H F A triangulo



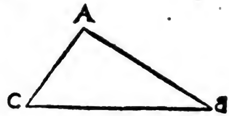
BFC, & AEG ipsi C E B æquilaterum erit, quia HFA CF*B* anguli ad verticem æquales, & crura H F FA cruribus F *B* FC æqualia sunt, ideoque H A ipsi C *B* quoque æquabitur, & anguli quoque HAF & FC*B*, nec non AH F & F*B*C æqualibus lateribus subtensi æquales erunt. Sed cum FAH angulus angulo DAG æqualis sit qui pars duntaxat externi est, totus sanié DA*B* ipso interno AC*B* major erit.

Eodem modo demonstrabitur angulum EAG, angulo C*B*A æquari, unde manifestum est totum angulum DA*B* non tantum alterutro oppositorum majorẽ, sed utrique simul çqualem esse. Atque inde (per nonam propositionem hujus libri) quoque planum est trianguli tres angulos duobus rectis æquari, quod post paulo etiam aliter demonstrabitur.

PROPO-

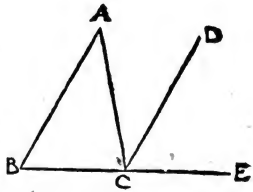
PROPOSITIO 19

Trianguli majus latus ſubtendit majorẽ angulũ.



Id eſt maximo lateri maximus angulus opponitur. In triangulo A*B*C latus maximum eſt *B*C, & propterea angulus *B* A C ampliſſimus: & deinceps *B* A latus ei quantitate proximũ etiã angulũ AC*B* amplitudine ei proximũ ſubtendit, majorẽ ſcilicet quã ſit ABC.

PROPOSITIO 20.

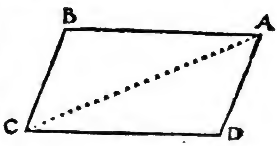


Trianguli latere producto externus angulus duobus internis & oppoſitis æquatur. & tres anguli trianguli duobus rectis æquales ſunt.

Eſto triangulum A B C, cujus latus B C cõtinuetur in E, hic angulus externus ACE plane æqualis erit internis & oppoſitis ad A & B

Ex C agatur C D parallela contra A B, erit itaque angulus BAC angulo ACD alterno (per 15 hujus) æqualis, & per eandem propoſitionem angulus DCE externus æquabitur interno & oppoſito ABC. atque ideo jam totus angulus ACE utrique ABC BAC ęqualis erit. ſed cum per 9 hujus uterque angulus ACB ACE ſimul duobus rectis æquales ſint, quia recta in rectam inſiſtit: tres autem anguli trianguli ſimul ſumpti hiſce duobus æquales demonſtrati ſint. efficitur tres angulos trianguli duobus item rectis æquari.

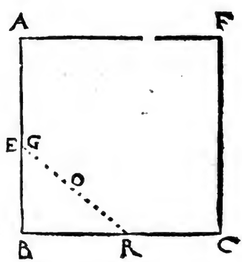
PROPOSITIO 21.



Si rectæ contermінent æquales & parallelas, ſunt æquales & parallelæ.

In expoſito diagrãmate *A*B *C*D æquales ſunto & parallelæ, quas connectant eadem parte *A*D & B*C*, iſtę quoque æquales erunt & parallelæ, quod demonſtratu perfacile erit é 4 primi *Euclidis*, ducta diagonio *AC*.

PROPOSITIO 22. PROBLEMA 8.



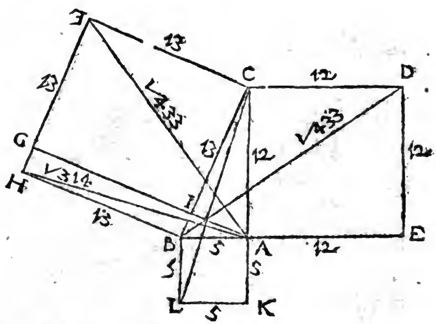
Super data recta linea quadratum conſtruere.

Erigito ſupet data *C* B perpendicularem eidem æqualem, & ex *A* & *C* tanquam centris duæ peripheriæ æquales intervallo datæ B*C* concurrant in FC rectæ *A*FF*C* connexæ comprehendet cum prioribus quadratum *A*B*C*F.

Cæterum A *B* perpendicularem ſuper *B* C conſtitui hoc modo, circini crus unum prolibitu ſupra lineam collocavi in puncto O, atque inde intervallo quolibet O K decircinavi peripheriam R *B* E, hic diameter ab R ſecet peripheriam ductam in E, recta *B* E connexa & pro arbitrio continuata erit optata perpendicularis. demonſtratio dependet é 31 propoſitione 3 libri *Euclidis*.

PROPOSITIO. 23.

Si baſis trianguli ſubtendit rectum quadratum baſis æquatur quadratis crurum.

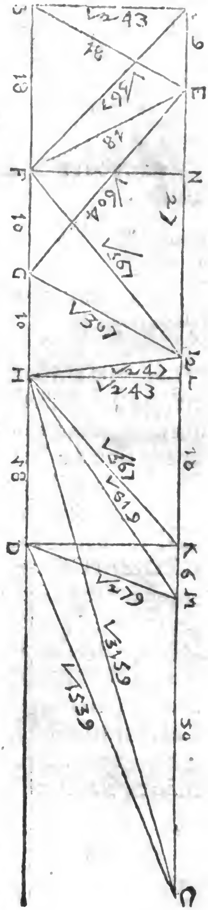


Eſt 27 propoſitio primi *Euclidis*, cujus demonſtratio ita habet. Eſto triangulum ABC, cujus angulus ad A rectus, crura recti AB AC, baſis autē BC. ad latera omnia quadrata deſcribātur, ductiſque rectis reliquis figura compleatur, ut vides.

Hic quia triangulum BCL dimidium eſt quadrati BLKA inter eaſdem parallelas eademque baſi poſiti, per ſequentem 25 propoſitionem, quæ eadem eſt cū 41 primi *Euclidis*; Atque eandem quoque ob cauſam HBA triangulum dimidiū parallelogrammi GHBI: habent enim baſin HB communem. cumq; iſta triangula angulum HBA, angulo CBL æqualem habeant atque æquicrurum, etiam ipſa triangula erunt æquilatera inter ſe: eamque ob cauſam æqualia per 6 tam hujus unde efficitur per 6 axioma quadratum AAKL parallelogrammo HGIB æquari. Eodemque modo quadratum BCDE parallelogrammo GFCI æquale demonſtrabitur. Quare integrum quadratum HFCB utrique quadrato BAKL & CDEA pariter æquabitur. Quod demonſtraſſe oportuit.

PROPOSITIO. 24.

Triangula



Triangula in æquali basi & intra easdem parallelas sunt æqualia.

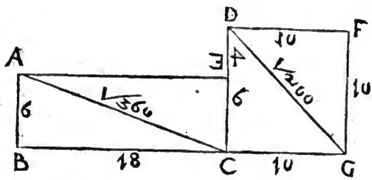
PROPOSITIO. 25.

Parallelogrammum in æquali basi intra easdem parallelas cum triangulo positum hujus erit duplum.

Intuere diagramma: Hic triangula ABF & BEF in eadem basi inter parallelas easdem AC BD consistunt, suntque ideo æqualia: idem iudicium esto de triangulis EFG, FIG & GIH, quæ cum illis in æqualibus basibus & intra easdem parallelas constituta ipsis & inter se æqualia sunt.

Præterea parallelogrammum rectangulū HLKD eandem habet basin eandemque altitudinem cum triangulis HKD, HMD & HCD, atque ideo singulorum duplum, vel illa hujus dimidia erunt.

PROPOSITIO 25.



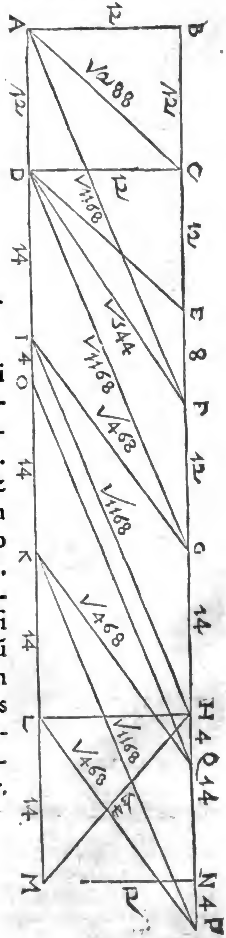
Parallelogrammi anguli oppositi & secta diagonio segmenta æquantur.

Vt hic in utroque parallelogrammo AECB CDFG perspicitur. neque id quicquam ad rem facit rectangula sint an obliquangula semper enim anguli oppositi æquabuntur: & á diagonijs AC, ac DG in duas æquas partes dissecabuntur.

PROPOSITIO 26.

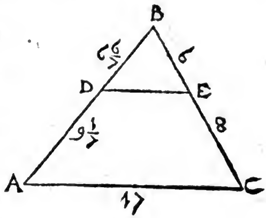
Triangula vel parallelogramma æquealta sunt ut basis.

Si enim sint in basi æquali sunt æqualia, sin bases habebunt inæquales earundem rationem quoque sequentur. Sunto namque lineæ BP & AM parallelæ, hic quadratum ABCD parallelogrammo ACED & AFCD æquale erit. itemque HLMN parallelogrammo DGHI, nec non ipsi OQPK. Parallelogrammũ autem ABDC ad DGHI eam habet rationem quam bases 12 ad 14, seu in minimis terminis quam 6 ad 7. demonstratio ex elementis petatur.



PROPOSITIO 27.

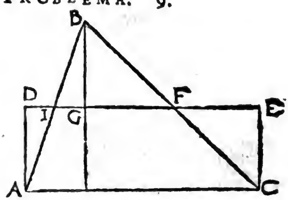
Si recta in triangulo est parallela basi secat crura proportionaliter: & contra, si recta in triangulo crura proportionaliter secat est basi parallela.



Esto in triangulum ABC, cujus crura AB CB recta DE contra basin AC parallela interſecet, ea quoque crura ſecabit proportionaliter, ut ſegmenta crurum eandem habeant rationem, ut B D ad D A ſit quemadmodum BE ad EC. ſtatuamus enim latus AB eſſe partium 16, AC 17, BC 14. cum itaque hic ob parallelarum affectionem angulus BDE externus interno BAC æqualis ſit, atque is qui ad E angulo C; angulus autē tertius ad B utrique triangulo communis, erunt ipſa triangula (per 5 propoſ. 6 lib. *Euclidis*) inter ſe ſimilia & lateribus proportionalia, ut BC 14 ad AB 16, ſic BE 6 ad BD 6$\frac{6}{7}$, hoc ſegmentum de tota B A 16 deductum relinquet 9$\frac{1}{7}$. atque ita crurum ſegmēta in eadem erunt ratione, ut 8 ad 6 ſic 9$\frac{1}{7}$ ad 6 $\frac{6}{7}$. Itemque ut BC 14 ad AC 17, ſic BE 6 ad E D 7$\frac{2}{7}$. Et quemadmodum AB 16 ad DB 6$\frac{6}{7}$, ſic AC ad DE. Ita enim ipſa ſegmenta inter ſe & totis ſunt proportionalia.

PROPOSITIO 28. PROBLEMA. 9.

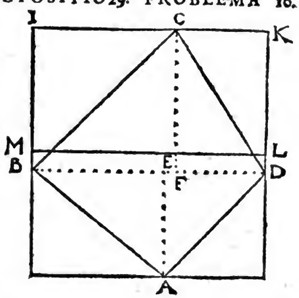
Parallelogrammum dato triangulo æquale conſtruere.



Detur triangulum ABC á cujus vertice B demittatur perpendicularis BH, eamque biſecato in G, parallelogrammum é baſi AC & altitudine GH, ut DACE dato triangulo æquabitur, quod vel inde perſpicitur quia abſumpta ſegmenta IBG A D I, & BGF ipſi CEF æquilatera ſunt & æqualia.

PROPOSITIO 29. PROBLEMA 10.

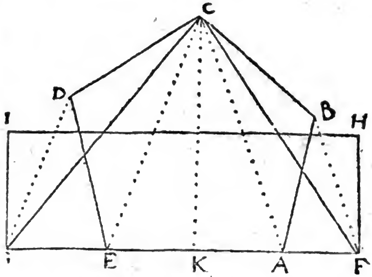
Dato trapezio parallelogrammum rectangulum æquale conſtruere.



Eſto trepezium A B C D mutandum in parallelogrammum rectangulum. agatur diagonius B D, in eamque utrimque ab angulis A & C perpendiculares demittantur CF & AE fit parallelogrāmum rectangulū BIKD é baſi BD & altitudine CF, itemque aliud ſuper eadem baſi B D & perpendilari AE; hoc ex utroque compoſitum biſecetur á linea ML, biſegmentum MIKL dato trapezio ABCD erit æquale. demōſtratio ex analogia antecedentis propoſitionis eſt perſpicua.

PROPOSITIO 30. PROBLEMA. II

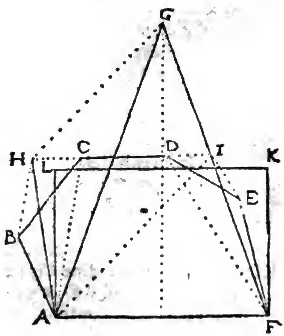
Datum quinquangulum in parallelogrammum rectãgulum transformare.



Datum quinquangulũ trãsformetur in trangulum. ductis enim diagonijs CE, CA, ab D & B contra ipsas parallelæ sunto DG, BF usque ad basin continuatam. rectæ á vertice C ad terminos parallelarum G & F constituent triangulum GCF dato quinquangulo æquale. Huic triangulo fiat per 28 præmissam parallelogrammum GIHF æquale.

PROPOSITIO 31. PROBLEMA 12.

Datum multangulum quodlibet in parallelogrammum rectangulum, cujus basis dato dati multanguli lateri æquale sit transformare.

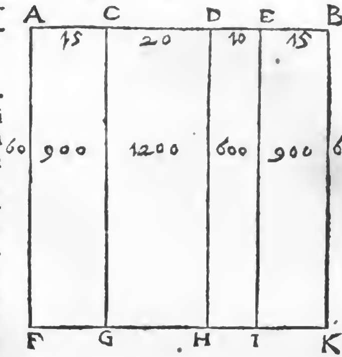


Multangulum propositũ ABCDEF transformetur prius in triangulum AGF cujus basis eadem sit cum base dati multanguli AF. tumque triangulum istud convertatur in parallelogrammum ALKF quemadmodum propositione 28 edoctus es. Multangulum itaque datum ABCDEF eidem parallelogrammo pariter æquabitur.

PROPOSITIO 32.

Si recta

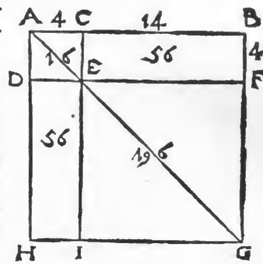
Si recta est secta in quotlibet segmenta rectangulum è tota & segmentis æquatur quadrato totius.



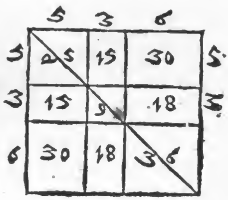
Sit AB linea secta in quascunque quatuor partes, cujusmodi sunt AC,CD,DE,EB, rectangula é segmentis istis & tota AB seu AF, videlicet AFGC, CGHD, DHIE, EIKB, æquabuntur quadrato á tota AFKB. totum enim æquatur suis partibus. sed idē numeris quoque comprobari potest: ponatur enim tota AB partium 60, segmenta autem AC 15, CD 20, DE 10, EB 15, quæ sigillatim per 60 multiplicatæ, dabunt areas segmentorum ACGF 900, CDHG 1200, DEIH 600, EBKI 900, omnium autem summa 3600 æquatur quadrato á tota AB 60.

PROPOSITIO 33.

Si recta est secta in duo segmenta, quadratum totius æquatur quadratis segmentorum & duplici rectangulo utriusque.



Recta A B pro arbitrio secetur in C, ajo qnadrata segmentorum ACED, E FGI, cum duplici rectangulo segmentorum, quæ sunt CBFE & DEIH æquari quadrato totius AB, utpote ABGH. Nam vel oculis patet totum partibus æquari, & numeris in consilium adhibitis demonstrationem habet perquam facilem.



In secundo hoc diagrammate linea recta secta est in tria segmēta inæqualia, in qua numerorum subsidio videre est qnadrata segmentornm cum eorundem duplicibus rectangulis æquari quadrato totius.

oblongum CIHD, reliquũ oblongũ AIHB, reliquo quadrato HDFG æqule erit.

In numeris ita conſtabit, ſit data CD partium 12, ſemiſsis CE 6. quadrata ex CE 36 & CA 144 addita conflabunt quadratũ ex AE 180, hujus latus $\sqrt{180}$ quantitas lineæ AE ſive EF, unde deducta ED 6, reliqua erit DF $\sqrt{180} - 6$, ſeu DH majus datæ ſegmentũ: id rursũ de tota BD 12 deductũ relinquet ſegmentũ minus HB $18 - \sqrt{180}$. Quamobrẽ ſi AB 12 multiplicetur per HB $18 - \sqrt{180}$, oblongũ hoc quadrato majoris ſegmenti HD $\sqrt{180} - 6$ æquale erit. multiplicationis typũ hic habes·

HB $18 - \sqrt{180}$ | 6 1 | $\sqrt{180} - 6$ DH ſeu DF

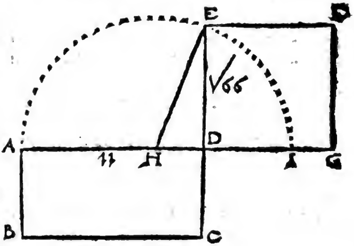
AB $12 \quad \sqrt{144}$ | 6 1 | $\sqrt{180} - 6$

720
36 72
18 18
216 — $\sqrt{25920}$
area oblongi ABHI

12
12
24 | 180
12 | 36
$\sqrt{144}$ 216
$\sqrt{180}$
11520
144 216 — $\sqrt{25920}$
$\sqrt{25920}$

PROPOSITIO 37. PROBLEMA 14

Dato rectilineo æquale quadratum conſtruere.



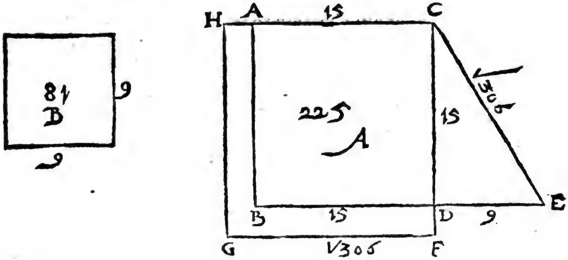
Detur primo rectangulũ ADCB, ejus longitudini AD continuetur DG æqualis ejuſdẽ latitudini DC, & continuata ſit diameter circuli, perpendicularis é puncto continuationis D eidẽ occurrens in E erit latus quadrati propoſito rectangulo æqualis. Nam cum tota biſecta ſit in H, ducatur radius HE, tum rectangulum ADCB ab inæqualibus ſegmentis AD & DI comprehenſum cum quadrato interſegmenti HD, æquatur quadrato biſegmenti HI ſeu HE. ſubducto utrimque quadrato communi HD, reliquum quadratum ex DE, ſeu DEFG, reliquo oblongo ADCB æquale erit.

Quod ſi aliud quodvis rectilineum proponatur, illud ante ad oblongum, ſive parallelogrammum rectangulum erit revocandum: ut ita totum negotium ad hanc fabricam devolvatur.

PROPOSITIO 38. PROBLEMA 15.

 Datis

Datis duobus quadratis æquale quadratum construere

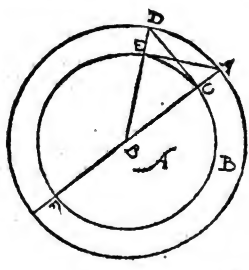


Hoc est quadratum quadrato addere, vt figura ex utraque composita quadrata sit. Atque hac via iterata factione quotlibet quadrata in unum quadratum ipsis æquale contrahetur, quod idem de quibuslibet similibus figuris verum est, & eadem ratione præstari potest. Operis ratio é 15 propositione lib. 2. *Euclidis* dependet. Quomodo autem quælibet rectilinea in unicum triangulum quadratumue, aut aliam quamlibet rectilineam figuram contrahantur infra dicetur.

Proponantur quadrata A & B addenda, D E ęqualis lateri quadrati B continuetur in directum cum latere B D. atque angulo recto ad D subtendatur C E, quadratum ad eam descriptū datorum utrique æquabitur; quale hic vides CHGF ad rectam C F ipsi C E æqualem descriptum. Cujus viritas é 47 primi *Euclidis*, vel é nostra 22 propositione repetenda est. atque jam haud erit dificile 2, 3, 4 aut quotlibet denique quadrata in unum continuato opere contrahere, vel etiam dato quadrato aliud duplum triplumve, aut quamlibet multiplum construere.

PROPOSITIO 39. PROBLEMA 16.

Rectam à dato puncto educere qua datum circulum contingat.

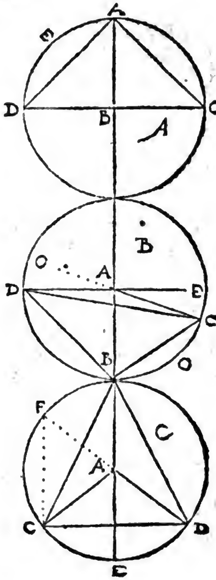


Detur circulus A punctumque D, á dato puncto ad circuli centrum connectatur B D, eoque intervallo centro B decircinetur peripheria D A, huic ab E termino minoris radij B E, perpendicularis E A secundæ peripheriæ occurrat in A, & ab A agatur radius A B, recta communē sectionem C cum dato puncto D connectens tanget peripheriam datam. Namque cum trian-

gula

gula BAE B DC anglum ad B communem & ęquicrurum habeant erunt ęquiangula: cumque angulus ad E rectus sit, qui ad C quoque rectus erit, & D C perpendicularis extremæ diametro BC in C circulum dictum continget.

PROPOSITIO. 40.

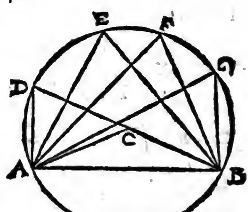


Angulus in centro duplus est anguli in peripheria in eandem peripheriam insistentis.

Vt in primo hoc diagrammate insistit angulus A C D in peripheriam AED, & ABD in centro illius duplus. Namq; ABC triangulum æquicrurum propter radios AB & BC habet angulos ad A & C ęquales; externus autem DBA utrique interno æqualis alterius erit duplus, per 32 propos. primi lib. *Euclidis.*

Et in secundo diagrammate angulus C D B anguli in centro CAB quoque dimidius est. sit enim ab illius vertice diameter DE, constat igitur angulum BAE æquari utrique ADB & ABD, continuataque CA in O, erit angulus B A O æqualis simulutrique C A B & ABC. Et D A O utrisque A D C & A C D simul, per 32 lib. 1. *Eucl.* vel 18 hujus. Est autem ADC triãgulum æquicrurum, & ea propter anguli ADC ACD æquales, itemque ADB & ABD: quare si de toto angulo BAE angulus E A C, & de ejus semisse A D B angulus A D C detrahatur, reliquus C DB reliqui B A C erit dumdus. Namque si de toto duplo tantum deducatur quantum á dimidio, reliquus é toto duplus quoque erit reliqui é dimidio. atque eodem argumento DAB dupus quoque fuerit anguli DC B. Quod demonstrasse opor-

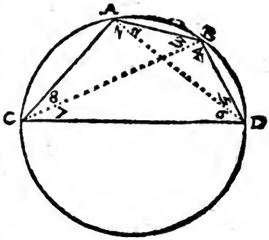
PROPOSITIO 41.



Anguli in eadem sectione sunt æquales.

Consectarium est ex antecedente propositione protinus manifestum. angulus in sectione, angulus in peripheria eandem notionem habent. ut hic ADB angulus in peripheria vertice D terminatus, insistens in peripheriam AB omnibus angulis in eandem peripheriam insistentibus AEB AFB AGB ęqualis est, quia sunt dimidij ejusdem anguli in centro ACB. Affectio peripheriæ pulcherrima & elegantissima, per quam plurima problemata geometrica cũ concinnitate quadã expediuntur, quemadmodũ in sequentibus patebit.

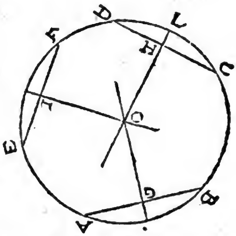
PROPOSITIO 42.



Anguli in oppositis sectionibus æquantur duobus rectis.

Seu quod idem sit, omnis quadranguli circulo inscripti oppositi anguli duobus rectis æquales sunt. ita hic A & D duobus rectis æquantur, denique C & B. unde efficitur angulos A & D simul, ipsis *C* & *B* æquales esse. Ductis enim diagonijs AD & C*B*, anguli *C*AD *CB*D in eadem sectione, vel in eandem peripheriam insistentes æquantur per præmissam: atque eandem ob causam A B *C* & *A* D C. quare duo anguli trianguli *C* A D & A D *C* simul uni A B D æquales erunt. atque ideo A *C*D & A*B*D tribus angulis trianguli æquales cum sint, duobus rectis quoque æquabuntur quod demonstrasse oportuit.

PROPOSITIO 43.



Anguli in centro peripheriave circulorum æqualium sunt æquales.

Veritas hujus ex 40 propositione præmissa perspicitur.

PROPOSITIO 44.

Si inscripta inscriptam perpendiculariter bisecet est diameter circuli.

Sic in exposita figura *A* B bisecatur á perpendiculari G O, & D C ab H O, F E ab I O, quarum concursus est ipsum circuli centrum. Atque ideo hac via dati circuli centrum inveniri potest: vel data duntaxat aliqua peripheriæ parte tota absolvi.

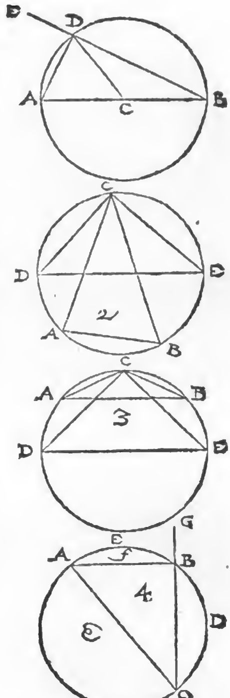
PROPOSITIO 45. PROBLEMA 17.

Datam peripheriam bisecare.

Datos datæ peripheriæ terminos recta connectito, eamque perpendiculari bisecato, hæc eadem continuata ipsam peripheriam quoque bisecabit. in antecedente diagrammate perpendicularis HL & rectam DC in H, & peripheriam DLC bisecat in L.

PROPOSITIO. 46.

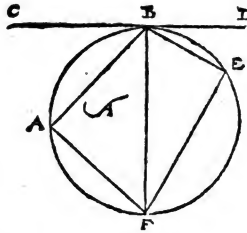
Angulus in ſmicirculo rectus eſt, in majore ſectione minor recto, in minore major : angulus autem majoris ſectionis eſt major recto, minoris minor.



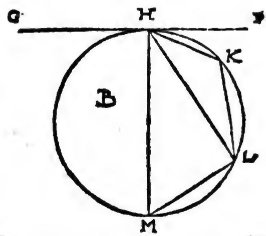
E 31 propoſitione libri 3. *Euclidis*, quam non ignorare multum intereſt, ideoque eam demonſtratione muniemus. Angulum B D A in ſemicirculo rectum eſſe demonſtrabitur hoc modo, ducto enim radio DC fient duo triangula æquicrura A *DC* & C D *B*, quare angulus D trianguli A D *B* reliquis ad A & B ſimul æqualis erit, & propterea rectus, cum ſit trium angulorum A, D, *B*, ejuſdem trianguli, hoc eſt duorum rectorum dimidius. Angulum porro ſecundi diagrammatis *A* C *B* in majore ſectione recto minorem vel inde patet, quod duntaxat recti pars ſit. Et angulũ 3 diagrammatis in minore ſectione A C B recto majorem demonſtratione ſimillima convinces. Contra veró in quarto diagrammate angulus E *B* A á peripheria EB & inſcripta A*B* comprehẽſus minor recto, quia pars ſit recti rectilinei *A* B G. Denique angulus majoris ſectionis A*B*D, ab A B & peripheria B D comprehenſus major eſt quam rectus rectilineus *A B C*.

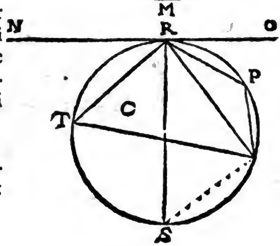
PROPOSITIO. 47.

Angulus secantis & contigua aquatur angulo in opposita sectione.



Recta CD contingat peripheriam ABE in B, ex hoc contactus puncto agatur diameter BF comprehendens cum contingente angulum DBF, in opposita autem sectione inscribatur angulus BAF, hic rectus erit per antecedentem, ille veró per 39 hujus, atque ideo inter se æquales secundó GI circulum contingat in H, inde quælibet linea circulo inscribatur HK, quæ cum contingente comprehēdat angulum IHK, is angulo KLH æqualis futurus est, & LHI ipsi HML; & GHL angulo HKL. Nam cum angulus HLM in semicirculo rectus sit; anguli HML & LHM uni recto pariter æquabuntur, rectus autem quoque est MHI; Itaque subducto utrimq; communi angulo MHL, reliquus LHI, reliquo HML æquabitur: porró autem cum per 42 hujus anguli ad M & K in oppositis sectionibus duobus rectis æquales sint, itemque GHL & LHI anguli deinceps, sublatis utrimq; æqualibus, hinc HML, illinc LHI, reliqui GHL & HKL æquales erunt.



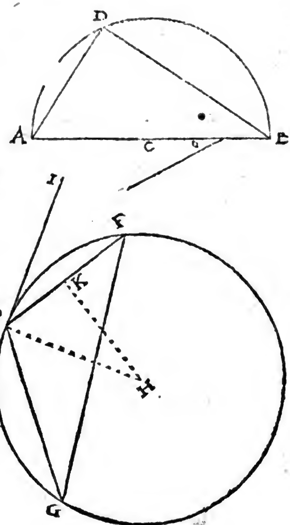


In tertio diagrammate angulus PRO angulo RTQ æquatur, cujus demonstratio ex præmissis faccillima est.

Super

PROPOSITIO 48. PROBLEMA 18.

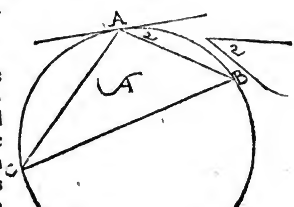
Super data recta linea circuli sectionem dati anguli capacem describere.



Si angulus qui dabitur rectus sit semicirculo super data recta descripto rem factam habes. angulus enim omnis in semicirculo rectus est, per 31 prop. 3 lib. *Euclidis*: sin veró rectus non sit, ut hic O angulus datus, dataque recta EF, tum ad terminum datæ rectæ angulum IEF dato O æqualem construito, & ab æquati vertice E rectam EH cruri EI perpendicularem erigito, et á K medio datæ perpendicularis altera KH priorem secet in H centro circuli per EF datæ terminos describendi, in cujus sectione ECF angulo IEF opposita inscribetur angulus EGF ipsi IEF, seu dato O æqualis. Cujus veritas ex antecedente perspicitur.

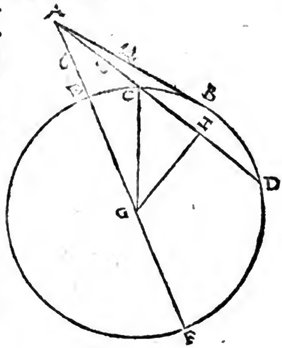
PROPOSITIO 49. PROBLEMA. 19

A dato circulo sectionem dati anguli capacem absumere.



Detur angulus Z & circulus A, unde sectio absumenda sit capax anguli dicti, ducatur recta circulũ contingens in A, ad punctum contactus fiat angulus á secante AB & ipsa contingente datæ Z æqualis, tum sectio opposita ACB erit optata circuli pars dati anguli capax, ut ACB angulus eidem inscriptus dato æqualis sit.

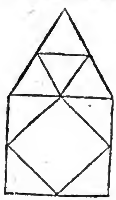
Quod ſi linea ab externo puncto *A* educta ſeſe in centrum ipſum non induat, vt in ſecunda figura A*D*, veritas quidem eadem, ſed demonſtratio talis erit. quadratũ *A*G excedit (per 47 pr. primi) quadratum AG, ipſo G H: Et quadrata á G H & HC æquantur quadrato radij GC. Itẽque rectangulum D A C, cum quadratis C H & H G, ſeu unius C G, æquatur quadrato AG; cui perinde quoque æquatur rectangulnm *F*A E cum quadrato E G. quamborem & illinc & hinc ſubducto communi quadrato radij GC vel G E, reliquum rectangulũ D A C reliquo F A E pariter æquabitur, & ea propter quoque quadrato lineæ A*B* ab eodem puncto circulum contingentis.



PROPOSITIO 52.

Rectangula é qualibet ex eodẽ puncto ſecante & ſecãtis exteriore ſegmẽto æquãtur interſe.

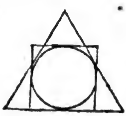
Confectarium eſt é premiſſa protinus manifeſtum : ſed idem per numeros quoque ſuam habet demonſtrationem.



Definitiones.

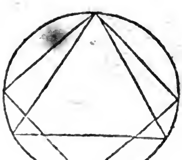
Rectilineum rectilineo inſcribi dicitur cum ſinguli illius anguli ſingulis hujus lateribus terminantur.

Rectilineum rectilineo circumſcribi dicitur cum ſingula figuræ ambientis latera per ſingulos inſcriptæ figuræ angulos educuntur.



Hæc utraque definitio de rectilineis homo geneis, ſive æquali laterum numero terminatis intelligenda eſt.

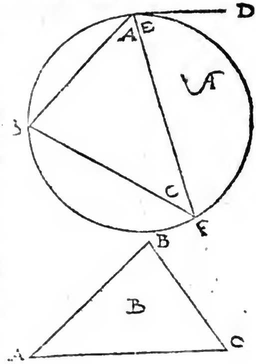
Rectilineum circulo circumſcribi dicitur, cujus omnia latera peripheriam tangunt.



Rectilineum circulo inſcribi dicitur, cujus omnes anguli in peripheria terminantur.

Recta circulo inſcripta dicitur, cujus termini in perepheria terminantur.

PROPOSITIO 53. PROBLEMA 20.

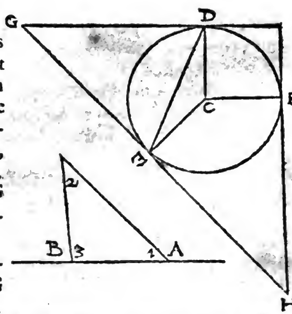


Triangulum dato triangulo æquiangulum in datum circulum inscribere.

Recta quælibet E D datum positione circulum contingat, & angulus F E D à secante & contigua comprehensus angulo C dati trianguli B C A ponatur æqualis; deinde ad idem latus FE angulus alter construatur æqualis dati trianguli angulo A, recta B F utriusque inscriptę terminos connectens inscribet triangulum B A *F* dato triangulo B A C ęquiangulum. Cujus veritas ex ipsa fabrica perspicitur, cum enim bini anguli binis per fabricam æquales sint, ideo per 32 pro. 1. lib. *Euclidis* consequens est etiam reliquos ad B æquales esse.

PROPOSITIO. 54. PROBLEMA 21.

Dato circulo triangulum dato triangulo æquiangulum circumscribere.

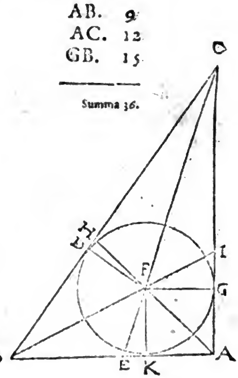


*F*abrica talis est. Dati trianguli latus quodcunque utrimque continuato; vt hic AB, & in centro circuli C, ad radium C D tanquam commune crus utrimque angulos DC*F* D *C* E externis dati trianguli angulis A & B æquato, rectæ in D, E & *F* circulum contingentes K G, G H, H K comprehendent triangulum K H G dato triangulo æqniangulum. Cujus demonstratio hęc est.

Cum in quadrangulo E G D C anguli ad E & D recti sint, reliqui ad C & G duobus rectis pariter ęquabuntur: sed in triangulo anguli deinceps ad A duobus rectis itidem ęquantur: quare (cum externus ad A angulo in centro per constructionem æqualis sit) internus ad A angulo E G D quoque æqualis erit. Eodem plane modo angulus dati trianguli internus ad B, angulo D K *F* æqualis evincetur: quamobrem & tertius ad H, angulo dati trianguli tertio quoque æquabitur, totumque triangulum toti æquiangulum erit & simile.

PROPOSITIO 55. PROBLEMA 22

In datum triangulum circulum inscribere.



Dati triaguli ABC duos quoscunque angulos A & B bisecato, rectarum AH *BI* bisecantium concursus F erit centrum circuli in datũ triangulũ inscribendi: perpendiculares enim ex *F* puncto in latera demissæ æquales erunt. ideoque centro F intervallo FK circulus KDG descriptus latera trianguli ibidem continget. Nam cum anguli triangulorum FAG, FAK, ad A æquales sunt, & qui ad G & K recti, & A*F* latus utrique commune, triangula ipsa erunt æquilatera per 20 propos. 1 lib. *Euclidis*, & propterea *F*G *F*K latera æqualia, simili via constabit *F*D ipsis quoque æquari; & quia subjectis lateribus perpendiculares sunt circulus in ijs latera dati trianguli continget.

Hinc & aliud efficitur in omni triangulo G*C* & KB á contactu ad angulos æquari lateri contermino BC.

Ideoque data trianguli laterum summa, radio circuli eidem inscripti, datur basis anguli recti. duplicato enim radio, eoque de laterum summa subducto reliqui dimidium erit basis quæsita. exemplo res erit illustrior. summa laterum trianguli A*B*C esto 36, radius FK 3, duplum 6 id de 36 deductum relinquet 30, dimidium 15 pro recta BC angulo recto subtensa. est enim FK ipsi GA, & FG ipsi KA æqualis, quamobrem duplo radij, hoc est KAG de summa laterum subducta, relinquitur BK & GC, atque BC, quarum hanc solam illis pariter æquari paulo ante ostendimus.

Iisdem datis dabitur area trianguli.

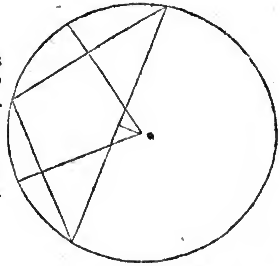
Radius enim FK 3 cum dimidia omnium laterum summa 18 multiplicatus comprehendet aream trianguli.

Data trianguli cujuscunque laterum summa & area datur radius circuli eidẽ inscripti.

Divisa enim area per dimidiam laterum summam, dat radij mensuram in quoto.

PROPOSITIO 56. PROBLEMA 23.

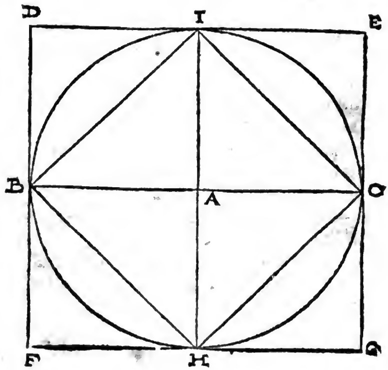
Dato triangulo circulum circumſcribere.



Dati trianguli duo quælibet latera lineis normalibus biſecentur, earum concurſus O erit centrum circuli, cujus veritas ex 45 propoſitione manifeſta eſt.

PROPOSITIO 57. PROBLEMA 24

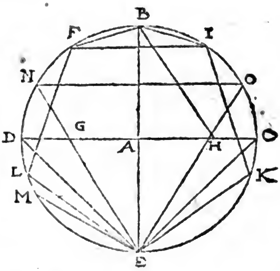
Dato circulo quadratum circumſcribere.



In datum circulum inſcribatur diameter BC; quam altera diameter HI normaliter interſecet, rectæ harum terminos connectentes inſcribent circulo quadratũ BICH, rectæ in his punctis circulum contingentes DE EG GF & FD comprehendent quadratum circulo circumſcriptum.

Atq; hinc facile fuerit circulum dato quadrato vel inſcribere vel circumſcribere.

PROPOSITIO 58. PROBLEMA. 25



Dato circulo quinquangulum ordinatum inſcribere.

Cum multæ & variæ ad hanc fabricã ſint viæ, unicam eamque expeditiſsimam & parabiliſsimã dumtaxat proponam. ſunto BE DC normales diametri, & AC radius biſecetur in H, & connectatur HB, cui æqualis ſtatuatur HG, denique connectatur GB, ea erit optatum inſcripti quinquanguli latus. ſegmentum autem AG latus decanguli eidem circulo inſcripti.

In cæteris autem ordinatis aliquot figuris inſcribendis, cujuſmodi ſunt eæ quæ continentur lateribus 6, 12, 20, 30, 60, 15 hæc via tenenda eſt, & primo quidem in ſexangulo notandum eſt ejus latus æquari radio circuli, ut hic ME hujus peripheria, videlicet $\frac{1}{6}$ totius circuli ſubducta de LE $\frac{1}{5}$, relinquet ML $\frac{1}{30}$, a duplicata dabit $\frac{1}{15}$, eademqae biſecta erit $\frac{1}{60}$ totius, ideoque rectæ illis latera ſubtẽſæ ſunt ordinatorum polygonorum inſcriptorum totidem lateribns & angulis comprehenſorum:

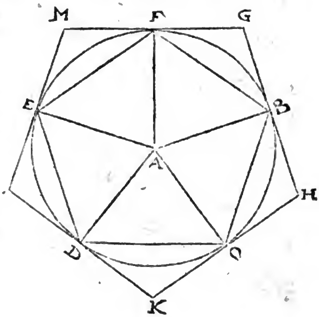
Sit EN latus trianguli æquilateri ſubtendens totius peripheriæ $\frac{1}{3}$, ea de ELF $\frac{2}{5}$ ejuſdẽ ſubducta reliquam faciet peripheriã NF $\frac{1}{15}$, eique ſubtẽſa latus erit quindecanguli.

Peripheria ED & EC ſunt quadrantes totius, hinc ſi ſubducas EL vel EK $\frac{1}{5}$ reliquæ DL & CK erunt $\frac{1}{20}$ totus.

Hoc fundamento multorum polygonorum latera tam in lineis quam per numeros in aſſumpta menſura explicari poſſunt, quemadmodum in ſequentibus videre eſt.

PROPOSITIO 59. PROBLEMA 26.

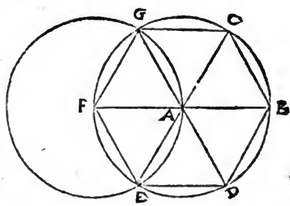
Dato circulo quinquangulum ordinatum circumscribere.



Primum dato circulo per antecedentem propositionem quinquangulum inscribito, ad cujus angulos radij agantur, rectæ circulum in FEDCB punctis contingentes comprehendent quinquangulum ordinatũ MGHKL circulo circumscriptum. Cujus demonstratio hæc est: cum enim radij AB AC AD AE AF quinquangulum inscriptum in quinq; triangula ęquilatera & æquiangula dispescant, quorum anguli in centro æquales, & anguli quadrãguli AFGB ad F & B contingentias recti sint. atque eodem modo in quadrangulo BACH se res habeat, anguli ad A & G, & A & H duobus rectis æqules iuter se quoque æquales erunt: atque cum urrobique anguli in centro A in æquales peripherias insistentes æquales sint, reliqui an G & H quoque æquales erunt. Simili plane modo omnium angulorum GHKCM æqualitas evincitur. Sed & latera MG & GH ęquari ita demõstratur, cum enim triangula FGB BHC bases FB BC ęquales habeant & angulos ad basin æquales, latera FG GB, lateribus BH HC & inter se quoque æqualia erunt per 5 prop. 1. libri *Euclidis*.

PROPOSITIO 60. PROBLEMA 27.

Dato circulo sexangulum rodinatum inscribere.

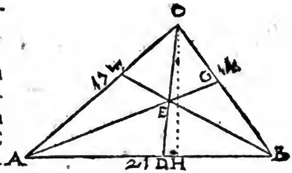


Duę æquales peripheriæ per mutua centra transeant, seque secent in punctis G & E, diametri GD, FB, EC ex his sectionum punctis & centro F eductę dispescent totam perpheriam in partes sex æquales, quibus subtensæ rectæ comprehendẽt sexangulum ordinatũ FGCBDE. Sunt enim hic sex triangula æquilatera, quorum latera radio circuli æquantur, namque cum AGF, AEF sint per fabricam æquilatera, & triangula ADB ACB ipsis ad verticem posita quoque æquilatera erunt. angulus autem trianguli æquilateri valet $\frac{2}{3}$ recti; & duo FAG FAE simul $\frac{4}{3}$, hi de duobus rectis subducti relin-

relinquent $\frac{1}{3}$ pro angulo GAC, cui æqualis eſt verticalis EAD. Quare anguli in centro æquales & æquicruri baſes baſibus æquales habebunt, & ſexangulum FGCBDE ab ijs cōprehenſum erit æquilaterum.

PROPOSITIO. 61.

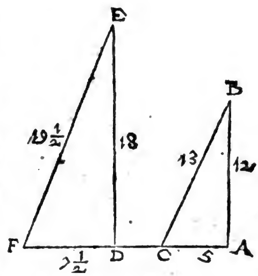
Si recta in triangulo biſecat angulum ſecat baſin ratione crurum.



Angulus trianguli ACB biſecetur á recta CD, eademque diſpeſcet baſin AB in ſegmenta AD & DB, horum ratio erit eadem quæ crurum AC CB angulum biſectum C comprehendentium. Res eadem erit ſi angulum A biſecet recta AG, erit enim & hic, ut AB ad AC, ſic BG ad GC. vel ſi BF biſecet angulum B: erit enim hic quoque, ut BA ad BC, ſic AF ad FC.

PROPOSITIO 62.

Triangula æquiangula ſunt proportionalia cruribus & baſibus æqualium angulorum: & contra, *Triangula lateribus proportionalia ſunt æquiangula.*



Sunto EFD BCA triangula æquiangula, ſintque E & B, F & C, D & A anguli æquales, tum crura BA 12 · AC 5 proportionalia erunt cruribus ED 18 DF $7\frac{1}{2}$ æqualium angulorum A & D. vel alternis quoque ut AB 12 ad DE 18, ſic AC 5 ad DF $7\frac{1}{2}$. Et ut AC 5 ad CB 13 ſic DF $7\frac{1}{2}$ ad FE $19\frac{1}{2}$: nam & iſta angulos æquales comprehendunt ad C & F. Denique crura angulorum B & E, AB 12 BC 13, & DE 18 EF $19\frac{1}{2}$ erunt inter ſe proportionalia in hoc noviſſimo exemplo ratio eſt utrobiqui ſubſequi tredecupla vel ut 12 ad 13.

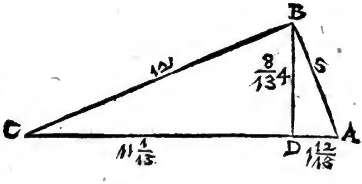
Propoſitio hæc ſummam utilitatem habet in gæodæſia altitudinis, latitudinis profunditatis: hujus uſus in geometricorum problematum ſolutionibus quoq; perilluſtris eſt, quemadmodum infra plurimis locis patebit. Quin ut ſemel abſolvam, omnis Gæodeſia quibuſcunque tandem organis inſtituatur, duorum triangulorum ſimilitudine perficitur, quorum alterum vel in ipſo organo, vel ſaltem organi ſubſidio cognitum datur, alterum veró in re menſuranda concipitur. Itaque propter ſimilitudinem, dato hujus trianguli latere uno, reliqua pro quæſita diſtantia per proportionem nullo negotio concludentur.

PROPOSITIO 63.

Si recta in triangulo est perpendicularis ab angulo recto in basin, secat triangula similia toti & inter se.

Et, Perpendicularis est proportionalis inter segmenta basis.

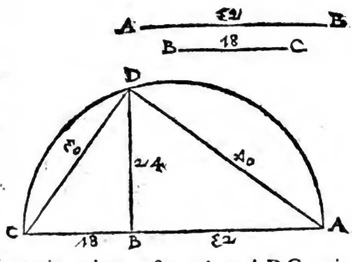
Et, Crus anguli recti utrumlibet est proportionale inter basin recti & basis segmentum eidem cruri conterminum.



Exponatur rectangulum triangulum ABC, cujus angulus ad B sit rectus, unde perpendicularis sit B D in A C basin recti, ea dispescet totum triangulum in duo triangula ABD & DBC, quæ & inter se & toti triangulo A B C sunt æquiangula. Nam cum angulus A D B rectus æqualis sit angulo recto A B C, angulusque A utrique & toti ABC & particulari A B C communus sit, sequitur per 32 propos. 1. libri *Euclid.* reliquum DBA reliquo ACB æquari. eodem modo demonstrabitur BDC toti ABC equiangulum: quare & ipsa ABD BCD inter se erunt æquiangula. Vnde per antecedentem latera circa æquales angulos erunt proportionalia. videlicet vt C D ad *D* B, sic D B ad D A, & quemadmodum C *A* ad A *B*, sic AB ad A D, vel vt AC ad *C* B, sic *C* B ad C D. In numeris unius trianguli base AB supposita partium 5, & perpendiculari BD $4\frac{8}{13}$, erit A D $1\frac{12}{13}$, & D C $11\frac{1}{13}$, B C 12 earundem partium.

PROPOSITIO 64. PROBLEMA 27.

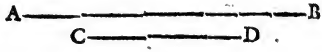
Inter datas duas rectas mediam proportionalem invenire.



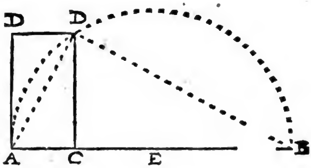
Exponantur duæ rectę A B 32 BC 18, eæ in directum continuatæ, vt in *C A*, fiant diameter semicirculi; & ex puncto cotinuationis B erigatur perpendicularis B D usque ad peripheriam, ista erit inter datas *AB* & B *C* proportione media. Connexis enim lineis á terminis diametri ad perpendicularis verticem *A* D, D *C* comprehendent triangulum rectangulum A D C, cujus angulus ad D rectus propter semicirculum, atque ideo per antecedentem erit vt A B 32 ad *B* D 24, sic B D 24 ad B C 18. Idem aliter quoque demonstrari, & numeris comprobari protest.

PROPOSITIO 65. PROBLEMA 28.

Inter data rectæ segmenta, rectam datam quæ illius dimidio non sit major proportione mediam collocare.

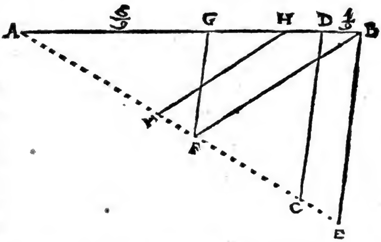


Data A B bisecetur in E, quo tanquam centro intervallo A E describatur semicirculus A D B. & á diametri termino A perpendicularis sit A D, per cujus verticem D parallela contra diametrum sit peripheriam secans in D, inde perpendicularis in subjectam diametrum sit D C, ea dividet diametrum A B in duo segmenta A C *C*B, inter quę ipsa sit media proportionalis, hoc est vt A C ad C D, sic *C*D ad C *B*. cujus veritas ex antecedente propositione plana est.



PROPOSITIO 66. PROBLEMA 29.

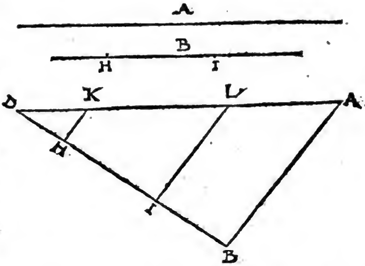
A data recta partem partesve datas desecare.



Seu quod idem est, á data recta optatam partem auferre. Proponatur recta A B secanda ratione qualibet data: recta quęlibet faciat angulum cum data ad A, eademque in optatas partes tribuatur, vt hic in A E ad libitum assumptæ sunt partes 9 ęquales. itaque hujus adminiculo $\frac{1}{5}$ $\frac{1}{3}$ $\frac{1}{2}$ $\frac{1}{9}$ é linea A *B* desecari poterit. Postaletur enim $\frac{1}{9}$, á fine nonæ partis in assumpta A E connectatur linea E B, contra quam á C ejusdem initio parallela sit C D, earum intersegmentum BD quoque nona pars erit lineæ A B. Rursum si postuletur $\frac{1}{5}$ lineæ A *B*, tum in assumpta A *F* á terminis *F* & I quintæ partis A F sunto parallelæ, ab F in finem datæ B, ab I in H usque contingens punctum, eæ intercipient H B $\frac{1}{5}$ datæ A *B*. atque ita in cæteris omnibus observata construxionis analogia.

PROPOSITIO 67. PROBLEMA. 30.

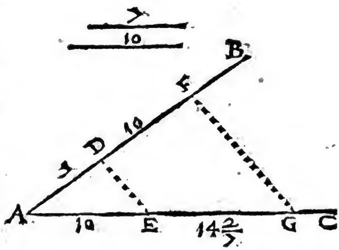
Datam rectam ſecare ratione ſegmentorum alterius datæ.



Fabrica antecedēti germana eſt. Proponatur recta A ſecanda ſecundum rationem ſegmentorum lineę B, quę in pūctis H & I interſecta eſt. datæ rectæ A & B faciant angulum, & AD æquetur datę A, & AB ipſi B; horum extrema A & B conjungat recta AB contra hanc ex punctis ſectionum H & I parallelæ ſint I L & HK, ab his AD in K & L interſecabitur ratione ſegmentorum DH. HI IB.

PROPOSITIO 68. PROBLEMA. 31.

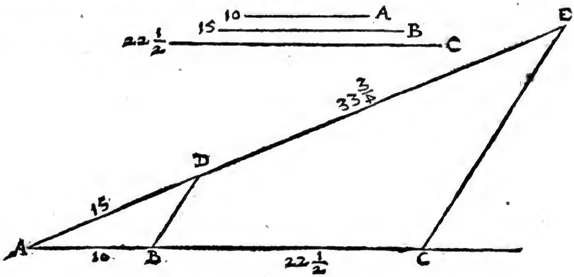
Datis duabus rectis tertiam proportionalem invenire.



Hoc eſt, ut ſit quemadmodum prima ad ſecundam, ita ſecunda ad tertiam vel inverſé ut quęſita tertia ad ſecundam, ſic ſecunda ad primā. Datæ duæ lineæ 7 & 10 faciant angulum quemcunque ad A, ut AD prima ſit & AE ſecunda, tum prima AD7 continuetur intervallo ſecūdæ in F, ut DF ſit æqualis ſecundæ AE 10, terminosque primæ & ſecundæ connectat DE, cui parallela FG interſecabit GE $14\frac{2}{7}$ tertiā proportionalem. demonſtratio petenda é 2 propoſ. 6. lib. *Euclidis*.

PROPOSITIO. 69. PROBLEMA 32.

Datis tribus rectis continue proportionalibus invenire quartam.

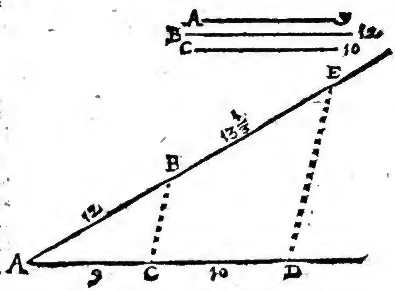


Expositæ ſunto rectę tres A B C, quibus quarta proportionalis poſtulatur. Igitur prima & ſecunda, A & C, ſeu AB & AD faciant angulum quemcunque BAD, quarum terminos jungat DB, & prima AB continuetur in C, ut BC tertiæ datarum C æqualis ſit, CE parallela contra BD interſecabit DE quartam proportionalem. iſto exemplo datas tres in continua analogia propoſuimus, ut numeri 10 15 22½ iudicio ſunt, itaque & quarta D E 33¾ in eadem ratione porró continuabitur.

PROPOSITIO 70. PROBLEMA 33.

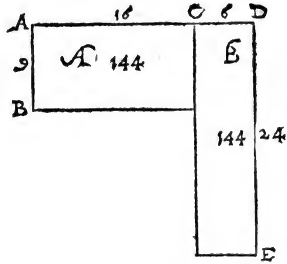
Datis tribus rectis quartam in disjuncta analogia proportionalem invenire.

Quamvis in expoſito caſu termini continuè proportionales non ſint tamen fabricæ forma & ratio eadem eſt cum antecedente, ut in appoſito diagrammate perſpicitur. Sit prima A partiũ 9, B 12, C 10, quare quarta proportionalis erit 13⅓.

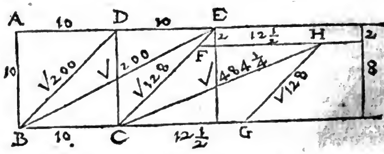


PROPOSITIO 71.

Rectangula æqualia sunt lateribus reciproca: Et, *Rectangula lateribus reciproca sunt æqualia.* Et generalius, *Parallelogramma æquiangula æqualia sunt lateribus reciproca: & contra.*



In primo diagrammate rectangulũ BC, habeat latus BA partium 9, CD partium 6, DE partium 24, utriusque area sub istis lateribus comprehẽsa erit 144. quare ut latitudo CD 6 ad latitudinem AC 16, ita vicissim longitudo AB 9 ad longitudinẽ DE 24. est enim utrobique ratio subdupla subsuperbitertia, seu $\frac{3}{8}$. Atque ita quod alterius longitudini desit, id latitudine rursum compensatur, & quod secundæ figuræ latitudini deest, id longitudini accedit.

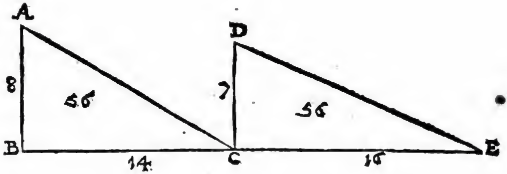


Atque hinc causa patet ob quam in proportione reciproca operis formula invertatur, hoc est primus multiplicetur per secundũ & factus dividatur per tertiũ. Exemplum. esto pannũ cujus longitudo ulnarum 4, latitudo $2\frac{1}{2}$, cui aliud subtexendum sit, cujus latitudo ulnæ $1\frac{1}{2}$: quæritur quotnam ulnas longum esse debeat 2 respondeo ulnas $6\frac{2}{3}$. Hic si directam sequaris proportionẽ ita esset, ut $2\frac{1}{2}$ ad 42 sic $1\frac{1}{2}$ ad quem, & cõcluderes $2\frac{2}{5}$. quod á veritate longissimé dissidere vel inde patet, quod latitudo secundi panni minor sit latitudine primi. quamobrem hoc modo collocandi erunt termini, ut $1\frac{1}{2}$ latitudo ad latitudinem $2\frac{1}{2}$, sic vicissim lõgitudo 4 ulnarum ad ulnas $6\frac{2}{3}$. Atque ita vides panni dati longitudinem cum latitudine multiplicatam parallelogrammum rectangulum comprehendere: tantã autem debere alterum panni genus habere aream: ideoque factum é longitudine & latitudine primi per latitudinem secundi divisum in quoto nobis exhibere optatam secundi longitudinem. Vnde planum est in proportione rociproca primum per secundum multiplicari factumque per tertium dividi debere. res eadẽ erit in obliquangulis, quorum anguli erunt æquales, ut in secundo diagrammate figuræ DBCE & FCGH quorum anguli ad B & C æqueacuti sint, & latera proportionalia: quemadmodum GC $12\frac{1}{2}$ ad CB 10, ita DB $\sqrt{200}$ ad CF $\sqrt{128}$: quia parallelogramma ista sunt æquiangula & lateribus reciprocé proportionalia erũt inter se æqualia, quod numeris comprobare erit in proclivi.

PRO-

PROPOSITIO. 72.

Triangula & angulum unum & aream æqualem habentia reciprocātur cruribus æqualis anguli. Et contra; *Triangula æqua angulo & cruribus æqualis anguli reciproca ſunt æqualia.*

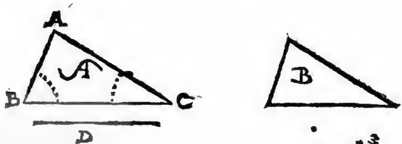


Sunto duo triangula *A*BC DCE æqualia angulis B & *C*, & crurum proportio ſit reciproca, ut nempe A *B* altitudo primi ad D*C* altitudinem ſecūdi eam habeat rationem, quam CE longitudo ſeu baſis ſecundi ad CB longitudinem ſive baſin primi. Ajo quoque triangulum primum ſecundo æquari.

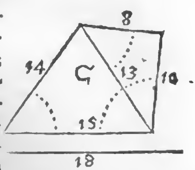
Sed & contra quoque, ſi duo triangula BDE CFH æqualia (in ſecundo antecedentis propoſitionis diagrammate) & angulū ad D angulo ad *F* habeant æqualē: horum ideo crura erunt reciproce proportionalia. propoſitio hæc utilis eſt in quorundam haud levis momenti problematum ſolutione, cujuſmodi infra ſumus exhibituri.

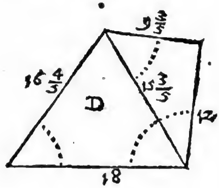
PROPOSITIO 73. PROBLEMA 34.

Super data recta figuram datæ figuræ ſimilem conſtruere.



Sit primo datum triangulum A *B*C cui ſuper linea D ſit conſtruēdum aliud ſimile, ut BC ſit latus lateri D homologum. æquato itaq; angulos ad terminos lineæ D, angulis in baſi B & *C*, crura æquatorū angulorum ad mutuum occurſum producta comprehendent triangulum B dato ſimile.





operis ratio pendet ex 12 & 20 hujus libri propoſitione. Si vero data figura quadrangula ſit aut multangula, ea primum reſolvatur in ſua triangula, & triangula ſigillatim conſtruantur ſimilia ſimili partium ſitu, ac tota figura toti

toti similis erit, ut in his quadrangulis vides: namque super linea 18 constructum est triangulum D simile triangulo C figuræ datæ, deinde ad latus $15\frac{3}{5}$ triangulum simile triangulo super latere 13 constituto, idque simili omnium partium situ, quæ in hujusmodi figurarum resolutione accurate erit obseruanda: quare tota figura toti similis est constructa, quod factum oportuit.

PROPOSITIO 47.

Figuræ similes habent inter se eam habent rationem quam homologorum laterum quadrata.

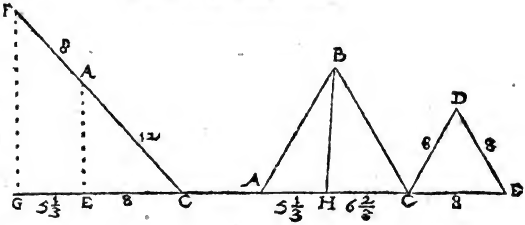
Vt numeris hujus theorematis veritatem comprobemus sunto nobis exempli loco illa duo quadrangula similia, quorum aream investigato, & primum quidem trianguli ABC á lateribus 13, 14, 15 comprehensi, ea erit 84 tum trianguli BG*F* in secundo diagrammate á lateribus $16\frac{4}{5}$ 18 $15\frac{3}{5}$ area erit $120\frac{24}{25}$, horũ ratio est ut 25 ad 36. reliquorum porró duorum triangulorũ areæ BD*C* & GHF erunt $\sqrt{1598\frac{7}{16}}$ & $\sqrt{3314\frac{13}{25}}$, quarum ratio itidem est ut $\sqrt{\frac{25}{16}}$ ad $\sqrt{\frac{81}{25}}$ hoc est ut $\frac{5}{4}$ ad $\frac{9}{5}$, vel ut 25 ad 36, quemadmodum illa priora eandem ergo rationẽ quoque habebunt quadrata ab homologis lateribus AC & BF videlicet 225 ad 324, qui numeri ad minores terminos revocati dabunt rationem 25 ad 36, quam supra inter areas triangulorum *ABC* & EGF intercedere comprobavimus.

Quod de quibusvis homologis lateribus verum esse intelligatur sic quadrata laterum BC 13 & GF $15\frac{3}{5}$, videlicet 169 & $\frac{84}{25}$ habent rationem inter se quam 4225 ad 6384, seu 25 ad 36, quanta inter triangulum BDE & GHF supra inventa fuit. Denique etiam ipsa quadrangula ABD*C* $84+\sqrt{1591\frac{7}{16}}$ & EGHF $120\frac{24}{25}+\sqrt{3314\frac{13}{25}}$ eandem quoque rationẽ habent quã 25 ad 36. Hujus periculũ expeditissime facies, si primum ABDC $84+\sqrt{1598\frac{7}{16}}$ per ultimum 36, & secundũ EGHF $120\frac{24}{25}+\sqrt{3314\frac{13}{25}}$ per 25 multiplicaveris facti nãque ob analogiã terminorũ æquabuntur, utrobique enim fit $3024+\sqrt{2071575}$. Theorematis hujus veritas nõ rectangulas solum, sed curvilineas quaslibet figuras spectat, sive istæ sint meræ ut circuli ellipses, seu mixtæ, cuius generis figuræ infinite variari possunt. Hoc argumentum tractat *Euclides* 2 propositione libri 12 Circulos inter se eam habere rationem, quam á diametris quadrata. Præterea omnes figuræ similes circulis inscriptæ sunt quoque ut quadrata á diametris circulorum quibus inscribuntur: quin & sectiones circulares similes eandem legem subeũt. In summa omnes figuras similes earumque latera homologa affectio hæc tangit.

PRO-

PROPOSITIO. 75.

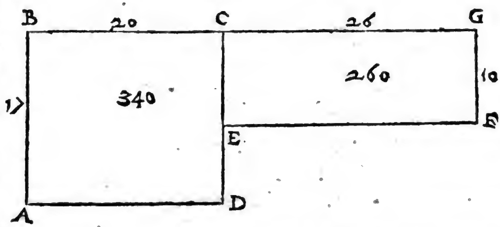
Si tres rectæ continuæ proportionales sint erit vt prima recta ad ultimam sic figura superficiaria è linea prima ad figuram ei similem è linea secunda.



Sunto duo triangula similia ABC CDE super basibus AC & CE, quibus per per 68 propositionem hujus inveniatur tertia proportionalis G E, cui æqualis sit HC, & connectatur BH quæ absumat triangulum BCH. cum itaque angulus BCH angulo DEC æqualis sit, & æqualium angulorum latera reciprocé proportionalia. namque ut DE ad B C, sic EC ad CA, & ex fabrica vt E C ad C A, sic HC ad C E, quare ex æquo vt D E ad BC, sic vicissim H C ad C E; ideo per 72 propositionem triangula BHC & CDE erunt æqualia. sed cum sit, quemadmodum basis AC ad basin HC (per 26 hujus) sic triangulum ABC ad BHC, seu eidem æqualem D E C. quare cum AC CE & CH continué proportionales sint, vt prima AC ad ultimam HC, sic triangulum A BC ad sibi simile CDE. Si libeat per numeros quoque hoc idem comprobare tibi licebit, inventa etenim area trianguli ABC $\sqrt{3888}$ & HBC seu CDE $\sqrt{768}$, datur ratio 9 ad quatuor sive dupla sesquiquarta, atque eadem est primi 12 ad tertium $5\frac{1}{3}$, videlicet 36 ad 16, seu 9 ad 4, quemadmodum oportuit.

PROPOSITIO 76.

Prallelogramma æquiangula rationem habent é lateribus compositam.

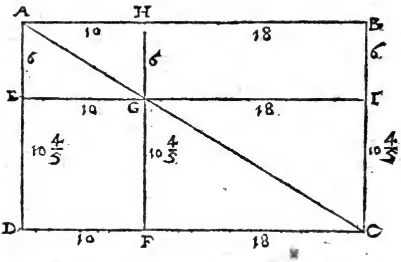


Sunto duo parallelogrāma rectāgula, primi latitudo AB sit partium 17, longitudo BC 20; secundi latitudo GE 10' longitudo GC 26, areæ ab his lateribus comprehensæ sunt 340 & 260: quare ipsarum inter se ratio erit, vt 340 ad 260, vel 17 ad 13, videlicet super quadripartiensdecimas tertias $1\frac{4}{13}$. Eadem ratio existet cum rationes laterum addes. ratio lateris BC ad CG est 20 ad 26 sive 10 ad 13, hoc

modo Et ratio lateris A B ad G F est vt 17 ad 10, hoc modo 10 17 quamobrem ratio ab illis composita erit $1\frac{4}{13}$, ut supra. 10 13

PROPOSITIO 77.

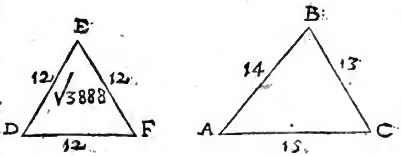
Diagonalia sunt toti parallelogrammo & inter se similia.



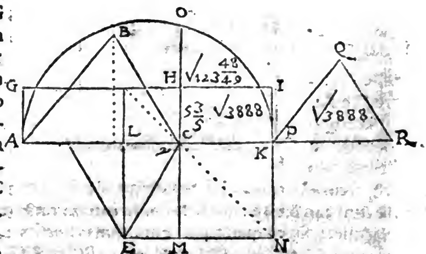
Sit AC diagonius parallelogrammi ABCD, in ea per idem punctum G rectę duæ EI HF contra ejus latera parallelæ intersecantes cõprehendũt diagonalia duo AHGE & GICF inter se & toti A B C D similia. Namque triangula AEG, GFC ADC ob laterum parallelismum æquiangula & similia sunt, ideoque ipsa parallelogramma AHGE, GIC F & ABCD ijsdem lateribus comprehensa similia erunt. quod numeris experiri haud difficile est.

PROPOSITIO 78. PROBLEMA. 35.

Dato rectilineo simile & alteri dato æquale constituere.



Detur magnitudine triangulum E D F, cui æquale quidem aliud, sed dato ABC simile postulatur. dato triangulo ABC cui simile postulatur constuatur æquale parallelogrammum ACHG ad basin AC; & sub eadem altitudine HC construatur HCKI parallelogrammum æquale parallelogrammo LEMG hoc est ipsi triangulo E D F. Parallelogrãma itaque hæc per fabricam æquealta sunt vt bases AC & C K per 1 proposit. 6. lib. *Euclidis*. inter has igitur A C & C K media



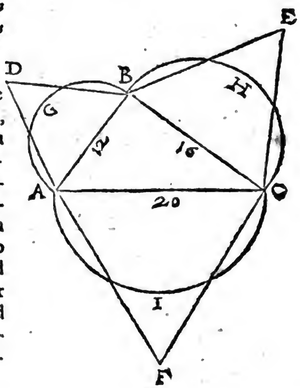
propor-

proportionalis sit C O. Ajo istam esse basin trianguli dato quidem A B C æqualis & alteri E D F similis. Namque vt G A C H, ad H C K I, sic basis A C ad C K: atque vt A C ad C K, sic figura ex A C ad figuram ex C O similem, qualis sit P Q R. Itaque vt A G H C ad H C K I, sic triangulum A B C ad triangulum P Q R. Sed cum primum tertio æquetur, etiam secundum H C K I parallelogrammum quarto P Q R triangulo erit æquale: quamobrem triangulum P Q R dato triangulo E D F æquale, & alteri dato ABC simile est constructum, quod fecisse oportuit.

Istuc per numeros ita experiri licebit. trianguli A B C latus A B sit 14, A C 15, BC 13. & E D F latera singula 12. ergo C K erit $\sqrt{}\,123\frac{4}{4}\frac{8}{9}$ media proportionalis inter A C 15 & C K $\sqrt{}\,123\frac{4}{4}\frac{8}{9}$ est C O $\sqrt{}\sqrt{}\,27895\frac{2}{4}\frac{4}{9}$. cui æqualis posita est P R. Vnde proportio, vt AC 15 ad AB 14, sic P R $\sqrt{}\sqrt{}\,27895\frac{2}{4}\frac{4}{9}$ ad P Q $\sqrt{}\sqrt{}\,21168$. Eadem via deprehendes pro latere RQ $\sqrt{}\sqrt{}\,15737\frac{3}{4}\frac{4}{4}$, & pro perpendiculari ex Q demissa in basin P R $\sqrt{}\sqrt{}\,8670\frac{2}{6}\frac{5}{2}\frac{8}{7}$. Hujus semissis $\sqrt{}\sqrt{}\,541\frac{1}{6}\frac{6}{2}\frac{3}{7}$ multiplicatus per basin P R $\sqrt{}\sqrt{}\,27895\frac{2}{4}\frac{4}{9}$ dabit aream trianguli P Q R $\sqrt{}\,3888$ æqualem dato triangulo E D F.

PROPOSITIO 79.

In triangulo rectangulo figura ad basin descripta æquatur figuris ad crura similibus similiterque sitis.



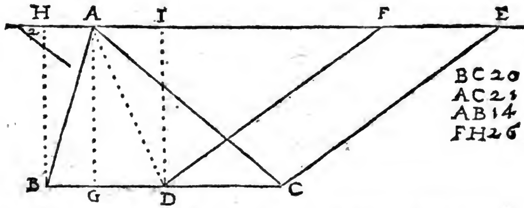
Figuræ rectilineæ, an curvilineæ an mistilineæ sint nihil interest, modo tria rectanguli trianguli latera pro istarum figurarum homologis lateribus assumantur. Vt in diagrammate subjecto A B C triangulum habeat angulum ad B rectum, triangula super ejus lateribus descripta sunto æquilatera. Ajo triangulum A C F ad basin recti A C descriptum æquarier triangulis A D B & B E C descriptis ad crura A B & B C. Idem in semicirculis hic depictis locum habere intelligatur.

Idem numerorum subsidio ita comprobabitur. In exemplo proposito. rea trianguli A F C est $\sqrt{}\,30000$, area trianguli A D B $\sqrt{}\,3888$, B E C $\sqrt{}\,12288$, duo ista triangula in unam summam composita conflabunt $\sqrt{}\,30000$, ideoque triangulo

AFC æquabũtur. Idẽ in semicirculis numeri evincẽt, & duos AGB BHC uni AIC æquari demonstrabunt. Namque si semissem AB multiplices per $3\frac{}{1}\frac{1}{0}\frac{4}{0}\frac{1}{0}\frac{5}{0}\frac{9}{0}\frac{2}{0}\frac{6}{0}\frac{5}{0}$ dabitur semipepheria AGB $18\frac{}{1}\frac{8}{0}\frac{4}{0}\frac{9}{0}\frac{5}{0}\frac{5}{0}\frac{5}{0}\frac{9}{0}$, hujus dimidium multiplicatum per semissem AB dabit aream semicirculi AGB $56\frac{}{1}\frac{5}{0}\frac{4}{0}\frac{8}{0}\frac{6}{0}\frac{6}{0}\frac{7}{0}\frac{7}{0}$. simili ratione deprehendes aream semicirculi BEC $100\frac{}{1}\frac{5}{0}\frac{3}{0}\frac{0}{0}\frac{9}{0}\frac{6}{0}\frac{4}{0}\frac{8}{0}$. Hęc utraque summa composita dabit $157\frac{}{1}\frac{7}{1}\frac{9}{0}\frac{6}{0}\frac{3}{0}\frac{2}{0}\frac{5}{0}$. atque adeó huic summæ æqualis plane deprehendetur area semicirculi AIC. nam hæc semiperipheria est $31\frac{}{1}\frac{4}{0}\frac{1}{0}\frac{5}{0}\frac{9}{0}\frac{2}{0}\frac{6}{0}\frac{5}{0}$, cujus dimidium per semissem AC multiplicatum dabit aream semicirculi AIC $57\frac{7}{0}\frac{9}{0}\frac{6}{0}\frac{3}{0}\frac{2}{0}\frac{5}{0}$. ut prius: neque adeó hęc geodesia $\frac{}{0}\frac{}{0}\frac{}{0}\frac{}{0}\frac{1}{0}\frac{}{0}\frac{}{0}$ unius unitatis á vero desit aut redundat.

PROPOSITIO 80. PROBLEMA 36.

Dato triangulo æquale parallelogrammum in dato angulo construere.

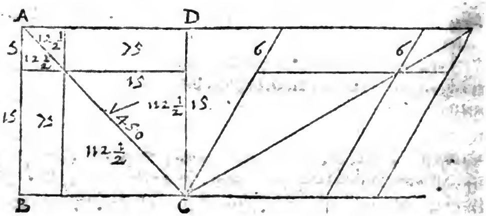


Detur triangulum ABC & angulus 2, huic æqualis cõstruatur in basis medio ut hic FDC, & per A trianguli verticẽ contra basin BC parallela sit AE, inque ea ponatur FE ipsi CD æqualis parallelogrammum FDCE in optato angulo triangulo ABC quoque æqnabitur.

PROPOSITIO. 81.

Complementa sunt æqualia.

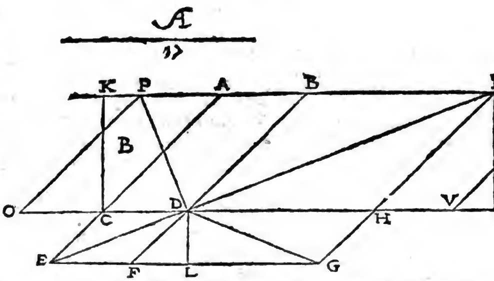
Ducta enim diagonio propter eam consistunt duo diagonalia, á quorum lateribus cõternis duo complementa comprehenduntur.



henduntur. ista inter se æqualia esse ajo. quod facillime utroque modo & per lineas & per numeros demonstrari potest.

PROPOSITIO 82.

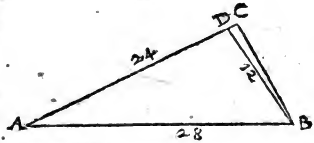
Ad datam rectam in dato angulo datum rectilineum applicare.



Detur recta A angulus V, & triangulum B. Hujus basis bisecetur in C & per verticem trianguli contra ejus basin agatur parallela, & DH datæ A ponatur æqualis, atq; ãgulus adC dato angulo V ponatur æqualis, cujus crus occurrat parallelę AE & compleatur parallelogrammum ABDC, & rectæ DH sit æqualis BI, sitque diagonale BDHI; cujus diagonius ID continuata concurrat cum AC in E, & cõpleatur parallelogrammum AEGI. itaque per antecedentem complementum FDHG complemento ABDC, hoc est triangulo OPD æquale erit: quare FDHG ad datam lineam, A datumque angulum dato triangulo B æquale construximus: Hujus generis alia & multó etiam difficiliora libro tertio longe expeditiore & faciliore via confecta dabo.

PROPOSITIO 83.

In triangulis obtusangulis basis plus potest cruribus duplici rectangulo ex altero crure & ejus continuatione ad verticis perpendicularem.

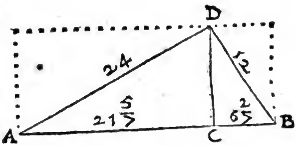


Triangulum propositum ABD habeat angulum ad D obtusum; eo casu quadratum basis AB majus erit quadratis crurum AD DB duplici rectangulo á latere AD & ejus continuatione DC ab angulo D ad perpendicularem BC. Est 12 propositio 2 libri *Euclidis*, quam intellexisse multum interest Geodętarũ omnium; idem de sequẽte, & supra proposita 62 propositione dictum intelligi quoque volo: nam horum subsidio triangulorum accurata geodesia expediri solet.

Demonstratio hujus theorematis ita habet. quadratum ex AB æquatur per 47 propos. 1. lib. *Euclid.* quadratis ex *BC* & AC, est enim angulus ad C rectus. Et per 4 propos. lib. 2 quadratum AB æquatur quadratis DC CB (quæ ambo ipsius DB quadrato æqualia sunt) & AD cum duplici rectangulo *A*D in DC: quare AD DB quadrata simul mino ra sunt quadrato AB duplici rectangulo sub *A*D in DC cõprehenso. In numeris istæc ita expediuntur. quadrata ab AD & DB 576 & 144 in unam summam addita subducantur de quadrato AB 784, reliqui 64 dimidũ 32 erit rectangulum sub A D & D*C* comprehensũ, id per crus AD 24 diuisũ in quoto dabit D*C* $1\frac{1}{3}$. hujus quadratum $1\frac{7}{9}$ subductum de quadrato BD 144. relinquet quadratum BC $142\frac{2}{9}$ cujus latus $\sqrt{142\frac{2}{9}}$, dabit perpendicularem quæsitam *B*C. Vnde trianguli area inventu est facilis: multiplicato enim basis AC dimidio 12 per perpendicularem BC $\sqrt{142\frac{2}{9}}$, dabitur area trianguli $\sqrt{20580}$, hoc est in numeris absolutis $123\frac{1}{1}\frac{0}{0}\frac{8}{0}\frac{3}{0}$ minus vero, vel $143\frac{1}{1}\frac{0}{0}\frac{8}{0}\frac{4}{0}$ majus vero.

PROPOSITIO. 84.

In triangulo basis acuti minus potest cruribus duplici rectangulo ex altero crure & ejus segmento à dicto angulo ad verticis perpendicularem.



Theorema generale est ad investigationem perpendicularis intra triangulum cadentis data trium laterum quantitate. Exponatur itaque triangulum ADB cujus anguli ad A & *B* acuti, atque á vertice D perpẽdicularis sit DC in basin AB. Ajo quadratum AD basis anguli acuti ad B minus esse quadratis crurum ejusdem AB *B*D duplici rectangulo sub AB & BC comprehenso.

Est enim quadratum AD duobus A*C* CD, item quadratum DB duobus B*C* CD æquale, subducto communi manebit illic quadratum AC hic *C*B. sed quadratum totius AB & BC æquatur quadrato A*C* & duplici rectangulo AB in B*C*, subducto itaque utrimque A*C* quadrato relinquentur duo rectangula *A*B in B*C*, quibus AD quadratum á quadratis crurum DB *BA* exceditur.

Quadrata crurum anguli acuti B, ut AB & AD 784 576, addita conflabũt 1360, inde quadratum ab BD 144, quę dicti acuti basis est, subductũ relinquet 1216, cujus dimidium 608 erit rectangulum sub A B & segmento A*C* comprehensum divisis itaque 608 per AB 28 quotus erit A*C* $21\frac{5}{7}$. Et secundo addantur quadrata D*B* & AB existet summa 928, unde quadratum AD 576 subductum, relinquent 352 hujus dimidum 176 per A*B* divisum dabit in quoto segmentum *C*B $6\frac{2}{7}$. quod priori A*C* $21\frac{5}{7}$ additum restituet totam AB partium 28. Porró autem si quadratum lineæ *C*B subduxeris de quadrato DB reliqui latus dabit quantitatem perpendicularis D*C* $\sqrt{104\frac{24}{49}}$. hęc per dimidum lineæ AB 14 multiplicata dabit aream trianguli ADB $\sqrt{20480}$, ut supra in propositione antecedente.

Illud

Illud hic ad extremum obiter notandum, si tota perpendicularis per totam basin in quam cadit multiplicetur, fieri parallelogrammum rectangulum ista base & altitudine comprehensum; idque dati trianguli esse duplum, quemadmodum subjectam figuram contemplanti perspicuum est. Atque inde causa in promptu est eccurnam trianguli basis in altitudinē multiplicata, faciat dati trianguli duplum: verum hoc tyronibus tantum dictum esto.

Atque ita altera hujus libri pars finē habeat, in qua ὁλοσχερῶς & pingui quod ajunt Minerva rudioribus, non autem eruditis Geometrica hæc fundamenta descripsimus.

Atque hic secundi libri finis esto.

LVDOLPHI A CEVLEN,

Variorum Problematum Libri 4.

A

WILLEBRORDO SNELLIO R.F.

é vernaculo in latinum translati, ac varijs
locis demonstrationibus aucti
& illustrati.

Anno 1615.

Amplisſimo Conſultisſimoque viro,

D. ÆMILIO ROSENDALIO I.V.D.

et in Curia Hollandica ſenatori prudentisſimo.

VIR AMPLISSIME,

SI apud alium verba facerem, qui harum artium & ſcientiarum delicias vel ignoraret vel non deguſtaſſet, jam mihi earum utilitas undique eſſet depromenda, ex Armamentario, arma; é Navali, naves; ex Architectonica, machinæ: é Tactica, numerorum ſcientia; graphice, pictura, viſus præſtiga et miracula ex Opticis; denique é cælo ipſo Sol & Luna cum reliquorum ſiderũ errantium & inerrantium choro eſſent deducendi: & præterea é ſummorum Philoſophorum auctoritate quoq; purpureus aliquis pannus, qui late ſplendeat ornatus gratia eſſet aſſuendus. Tanta enim & tam præclara elogia paſſim per eorum libros ſparſa extant, ut leviculos quoſdam homunciones, qui tanquam novi Epicuri, harum artium ſcientiam vel averſantur vel ſugillant, ſuo pondere facile opprimant. Namque præter uſum, quem per omnes vitæ partes habent ſingularem longé lateq; diffuſum, mentem quoque & cogitationem á ſenſibus avocant & convertunt ἐπὶ τὴν τοῦ ὄντος θέαν, *his enim animus humanus barbarico cæno occæcatus et infoſſus* ἐκκαθαίρεται καὶ ἀναζωπυρεῖται *repurgatur & reſuſcitatur. Ideoque non Philoſopum ſolum, ſed maximé quoque* τοὺς μέλλοντας τῶν μεγίστων ἐν τῇ πόλει μεθέξειν *qui in ampliſſimo dignitatis & honoris gradu apud ſuos cives futuri ſunt, & optimum quemque his artibus erudiendos in politeia ſua Plato ſciſſit. Neque verò id leviter aut perſunctorié tantum, ſed* ὡς οἷόν τε μάλιστα προστακτέον, ὅπως οἱ ἐν τῇ καλλίστῃ πόλει σοι μηδενὶ τρόπῳ τούτων ἀφέξονται, καὶ γὰρ τὰ πάρεργα αὐτοῦ οὐ σμικρά *quam diligentiſſime cavendum ne in Republica pulcherrime á te conſtituta ullo modo ab Arithmetica & Geometrica cognitione abſtineant, utilitas enim haud exigua hinc emanat. Cum iſtæc toties & tot locis iterantem audio, videor mihi videre ſollicito vultu divinum illum ſenem anxie & ſollicité has ipſas artes ſuis auditoribus, diſcipulis, civibus commendantem, & Academiæ ſuæ elogium interpretantem* οὐδεὶς ἀγεωμέτρητος εἰσίτω. *Namq; ut Philoſophos mittam harum cognitione deſtitutos (quos tanquam homines profanos ipſa rerum Natura é ſuo ſacrario longe ſubmovet, & veluti nimium protervos ac libidinoſos procos á caſtiſſimo ſuo corpore longe aſpellit, neque unquam abſque hac dote admiſſura eſt) ut, inquam, hos miſſos faciam, cedo mihi Prætorum edicta, Imperatorũ reſcripta, juriſconſultorũ reſponſa de herciſcunda familia, de fructu arboris in confinio, de inſula in flumine nata, de alluvionibus, atque alia ſexcenta in quibus judex harum ignarus plane cæcus ſit. certe Tiberiadis autor hoc lumine orbatus non ſecus ac Polyphemus aliqui in ſpelunca ab uliſſe occæcatus errabunda veſtigia circumfert: quin etiam eidem ſubtiliſſimo licet juris interpreti ad legem Papiniani de divortio earundem ignorantia vel invito vocem illam expreſſit,* Nulla eſt, *inquit in* toto libro hac gloſſa difficilior, cujus computationem nec ſcolaſtici, nec Doctores intelligunt. *Et Aphricanus quoque ubi legem faliciam tractat. Qui ducenta in bonis relinquebat, nonne ab Accurſio malé acceptus, á reliquis minus feliciter eſt explicatus? ut non immeritó divinus ille Philoſophus exclamet has artes longe potiores eſſe* μυρίων ὀμμάτων *decem oculorum corporeorum millibus. Verum ut initio dixi hæc pluribus diſputare apud te nequaquam mihi eſt opus, qui iſta jam diu pervideris, & ingenij acie, qua plurimum polles, aſſideris. Atque eam ob cauſam tu unus maxime occurrebas (ut jam arctioris neceſſitudinis vinculum*

lum

lum huc non arcessam) cujus nomini quicquid in hoc opere nostrum esset inscribendum judicarem, qui cum penitiore juris scientia accuratissimam Mathematum cognitionem conjunxeris. En itaq; tibi vir Amplissime hosce problematum variorum libros, in quibus quorundam Geometricorum problematum tractationem ita exhibemus, ut quandoque numeros quoque in hujus subtilitatis societatem admiserimus. Est enim numerus omnis commensus, rationis & proportionis accuratus interpres. Et, si quid Aristoteli credimus 2 cap. 1 Metaph. Arithmetica est ἀκριβεστέρα τῆς γεωμετρίας; cui, si id dicat numeris ob infinitam sectionem, quam Geometra actu non assequatur, cujuslibet magnitudinis partes & partium particulas accuratius exprimere, plane assentior. Eam ob causam numerorum, maxime irrationalium & surdorum usum istius libris illustratum philomatis invidere non debuimus: idque adeo tanto magis, ut clarum cuilibet sit, quantopere ad usum inutilis sit Pythagorea illa ἀλογίας in tredecim species distributio, in qua Euclides, *totum 10 Elementorum librum occupavit, cum generales istæ numerationis leges nihil pensi habeant ad quamnam speciem hic vel ille numerus sit referendus. Una enim & catholica hujus numerationis regula est. Verum* Euclides, *Pythagoreæ sectæ philosophus, potissimum se ad Pythagoream solidorum corporum adscriptionem composuit, inquit* Proclus, *tanquam illud esset Geometriæ summum bonum & finis extremus. ideoque de utiliate libri decimi minus sollicitum mirari desino: nullum enim inter eas species elementum extat quod usquam in Archimede, Apollonio, Sereno, Theodosio, Menelao, Ptolomæo Theone, Eutocio, Diophanto, ipsoque adeo* Euclide *extra elementa vel citetur vel usum ullum habeat: crux igitur quædam istic tantum defixa est, quæ solo calculo in abaco facillime tollatur: & quamvis ista tanquam subtilia in Mathematica bibliotheca conservari possint: attamen ut minus utilia à στοιχειώσει segregari debent. nam si ista usum habeant, totum hoc genus, cujus ille liber particulam duntaxat aliquam explicandam sibi sumit haud dubie plus longe reconditæ eruditionis & scientiæ complectetur. atqui cum certum sit* Euclidem *istarum contemplationem nudis lineis & magnitudinibus astrinxisse (ejus enim rei documenta in omni antiquitate exstant certissima, apud Archimedem Eutocium, Ptolomæum, alios) non mirum est purioris judicij logicos sublata illa sterili ἀκριβολογία earum tractationem ad surdorum & irrationalium logisticam cum celeberrimis hujus ævi Mathematicis rejecisse. Nam & illi ipsi qui hujus numerationis sunt studiosissimi, quique omnem ætatem in ea sola triverunt, cum numeri ultra modum excrescunt, & quasi mole sua ruunt, ecquid manum de tabula tollunt? introspice sodes librum hunc quintum, in quo istas lacunas à nobis suppletas videbis. Quinimò ubi ipsi se ipsos ob multiplicem numerorum nexum vix expediunt, nunquid facta analysi ad explicabiles delabuntur? videlicet ne tritum illud occinatur, τὸν μαθηματικόν. Est itaq; numerorum ista tractatio eatenus probanda, quatenus ad alia etiam aliqua utilitas inde redundet. Hæc ideo liberius apud te, vir Consultissime, disputavi, ut, quoniam harum non es ignarus, ipse quoque videas jure meritissimo partem hanc ab elementis segregatam. Quamobrē si ista quæ hic afferimus à te ea fronte accipiuntur, qua me excipere es solitus, tum demum labores nostros nullubi melius collocari potuisse intelligam.*

LVDOLPHI

LVDOLPHI à CEVLEN HILDESHEIMENSIS

Liber tertius, DE

Figurarum transmutatione & sectione.

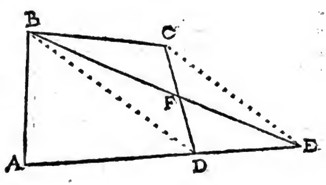
FIgurarū in alias varias transformationem, optatæ partis desectionem, earundemque cum inter se mutuo additionem tum subductionem, & magnitudinum numerationem Arithmeticæ simplici analogam, quas species vocant hoc tertio libro tractare constituimus.

Harum tam diversarum rerum miscellam cum minus commode unico titulo complecti possemus à potiore parte insignivimus, singulis tamen locis pressiorem inscriptionem & argumento congruam præfixuri.

Parte hujus libri prima omnes illę propositiones continentur, quæ figurarum reductionem docent, vt quadranguli in triangulum, aut multanguli in quadrangulum, vel etiam in quadrangulum dati lateris datique anguli, ac denique quotcunque rectilineorum in unum optatum, aliaque his affinia quamobrem primum problema hic esto.

PROPOSITIO. I.

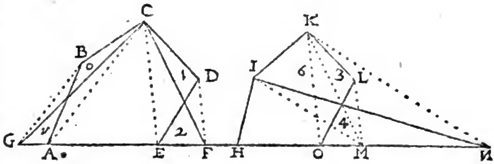
Dato triangulato æquale triangulū construere.



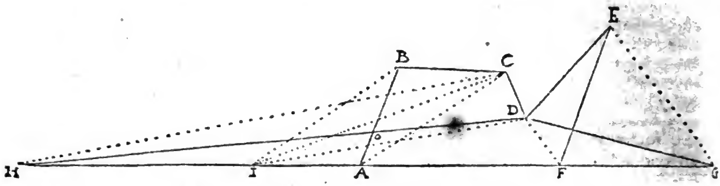
Propositum esto quadrangulum ABCD per quā agatur diagonius BD, & huic parallela a vertice anguli C occurrat basi AD continuatæ in E, recta BE constituet triangulum ABE dato quadrangulo ABCD æquale: cum enim rectę BD CE per fabricam parallelæ sint, triangula BED BCD vertice in ipsis terminata in eandem basin BD insistentia erunt per 27 propos. lib. 1 ęqualia, itaque subducto utrimque communi segmento BFD reliquum BFC reliquo DFE per 3 nostrum axioma libri 2 æquabitur. Itaque si ad ABFD illic addas triangulum FBC hic FDE: totum quadrangulum ABD toti triangulo ABE pariter æquabitur.

PROBLEMA 2.

Dato quinquãgulo æquale triangulum construere.



Dato quinquãguloABCDE æquale est constructum triangulum GCF: nãque triangulum GCA triangulo ABC, & FCE triangulo ECD æquale est, addito utrimque communi ACE totum quinquangulum ABCDE toti triangulo GCF æquabitur. *In* secundo diagrammate triangulum OKM triangulo OKL æquale est, ideoque quadrangulum HIKM quinquangulo HIKLO æquabitur. hinc rursum triangulo IKM æquale est triangulum INM ob parallelismum IM & KN, quare totum triangulum HIN toti quadrangulo HIKM, hoc est dato quinquangulo HIKLO pariter æquabitur. cujus veritas ex demonstratione problematis primi liquido constat.

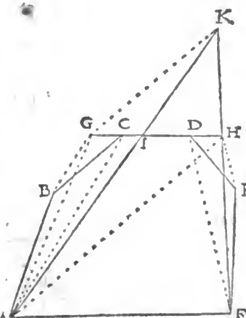


Cæterorum polygonorum rectangulorum reductio & demonstratio huic est simillima: semper enim hac reductione sigillatim unum latus detrahitur. Ita propositum sexangulum ABCDEF primum in quinquangulum ICDEF, hoc in quadrangulum HDEF, idque ad extremum in triangulum HDG transformatur. ut hic vides.

PRO-

PROBLEMA 3.

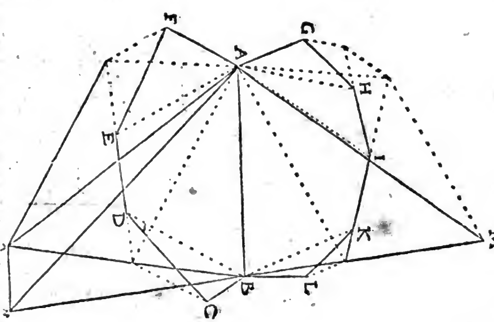
Dato rectilineo ad datum ejus latus triangulum æquale construere.



Datur triangulatum (qua voce generali quadrangula & multangula omnia tanquam á triangulis composita comprehendimus) *A*BCDEF, cui triangulum æquale postulatur super base A F. continuetur latus C D & contra F D A C diagonios parallelę E H B G lateri CD occurrant in punctis H & G, rectæ F H A G connexæ constituent quadrangulum A G H *F* æquale dato sexangulo *A*BCDEF: denique in hoc quadrangulo ducatur diagonius A H contra quam per angulum G parallela G K occurrat continuato lateri *F*H: rectæ A K F K cum basi A *F* comprehendent triangulum A K *F* æquale sexangulo dato A B *C* D E F. cujus veritas ex primi problematis demonstratione manifecta est, vel é 37 propos. 1 lib. *Euclidis* & 27 propositione nostræ stœchiosis evinci est.

Sed idem quoque præstari potest ex analogia fabricæ secundi diagrammatis problematis antecedentis, si latus alterutrum ipsi A F conterminum, vt F E assumatur pro basi & triangulatum continenter triangulis mulctetur donec ad angulum A deventum sit, quemadmodum illic ab autore expressum vidisti.

Fabricæ huic affine est & hoc paradigma undecanguli, cui æquale construximus triangulum A N M. namque id in duas partes á diagonio *A* B dissectum, & segmento AGHIKLB triangulum AMB æquale constructum est super

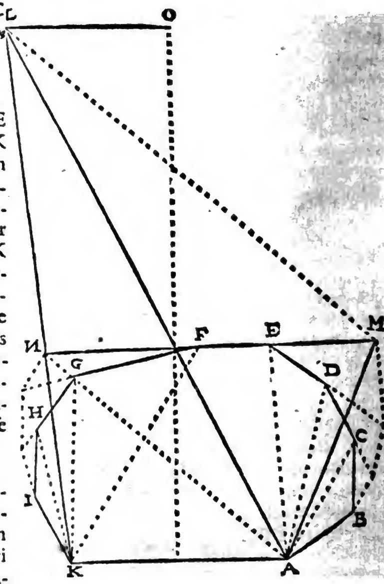


basi

basi AB. Itemque altrinsecus segmento *A*B*C*DE*F* triangulum ABO. Denique duo triangula AMB AOB contraxi in unicum ANM. ducta enim ON parallela contra AB & continuato latere MB; triangulum AN*B* triangulo AOB æquatur. ideoque totum triangulum AMN dato undecangulo construximus æquale.

PROBLEMA 4.

Dato triangulato ad datum ejusdem latus triangulum æquicrurum & æquale construere.



Datur decangulum ABCDE FGHIK, super cujus basi AK construendum sit triangulum æquicrurum eidem æquale. exposita figura primum per antecedens problema 3 reducatur ad triangulum sibi æquale ALK super data basi AK; à cujus vertice sit LO parallela contra basin, recta PO basin AK recte seu perpendiculariter bisecans eidem occurrat in O, unde cõnexæ rectę OA OK comprehendent triangulum AOK dato triangulato super data base æquale.

Datam autẽ figuram quamlibet dehinc in parallelogrammum in dato angulo ad datam lineam ex antecedentis libri propositione 82, vel in quadratum æquale transformare ex 37 ejusdem haud erit operosum.

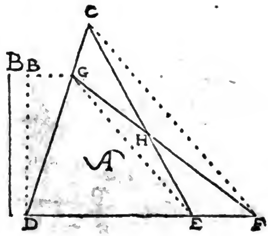
At quomodo rectilineum quodlibet in partes optatas datave ratione secari posit infra suis locis dicetur.

PRO-

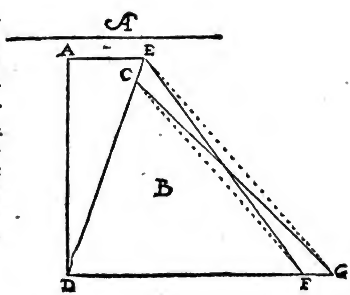
PROBLEMA 5.

Dato triangulato ad datam altitudinem triangulum æquale construere.

Exponatur triangulum A, altitudo lineaB,huic statuatur æqnalisBD, perque ejus verticem BG sit parallela contra basin D F hinc connectatur GE, cui perC triãguli verticem parallela sit CF, recta GF connexa comprehendet triangulum G D F dato CDE æquale.

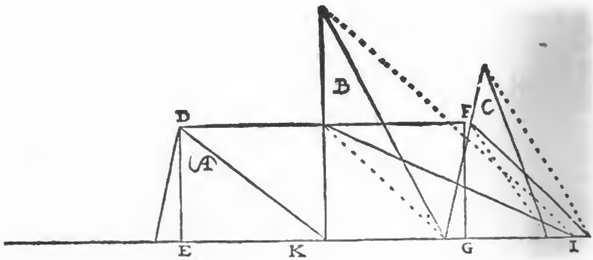


In estoexemplo altitudo data minor erat altitudine dati trianguli, nũc detur altitudo trianguli C D G minor data A, cui æqualis sit perpendicularis AD, & per ejus verticem expressa AE parallela contra DG. hinc agatur ab E ad terminũ trianguli G recta EG,eique á C trianguli vertice parallela CF, inde connexa EF constituet triangulum EDF sub data altitudine æquale dato CDG.



PROBLEMA 6.

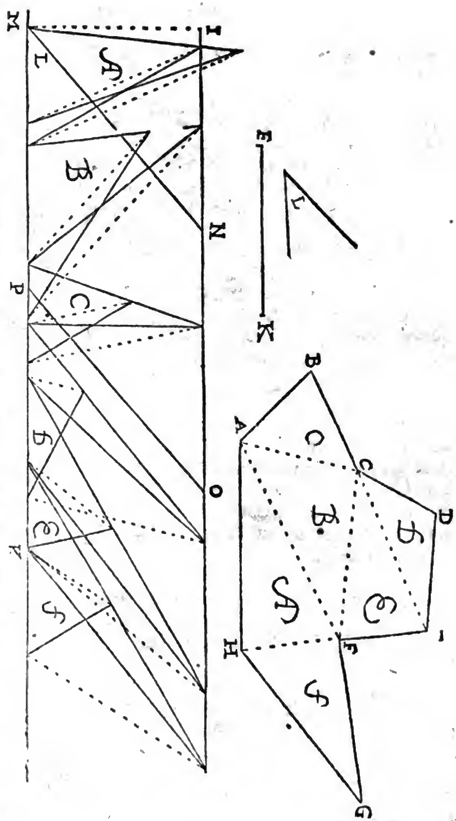
Triangula quotlibet in unicum parallelogrammum rectangulum datæ altitudinis contrahere.



Data ſunto triangula tria A B C, quibus unicum poſtuletur æquale rectangulum parallelogrammum, cujus altitudo æquetur altitudini trianguli A. Revocato itaque duo reliqua triangula ad eandem altitudinem trianguli A, ſintq; omnium trium baſes continuatæ in HI, cujus ſemiſsi KI æquetur EG. rectangulum igitur DEGF omnibus iſtis triangulis pariter æquabitur.

PROBLEMA 7.

Datum triangulatum ad datam altitudinem in dato angulo applicare.



Seu quod idem ſit dato triangulato parallelogrammum in dato angulo ad datã altitudinem ęqnale conſtruere. Sit octãgulum ABCDEFGH cui ad altitudinẽ EK cõparandum ſit parallelogrammum æquale in angulo L. reſolvito datum octangulum in ſua triangula atque ſingulis conſtruantur triangula alia æqualia ad datam altitudinem FK vel IM, earumque omnium baſibus æqualis recta MK biſecetur in P, anguloque L dato æqualis ſtatuatur N M P, & compleatur denique parallelogrammum NMPO, hoc ipſum erit dato octangulo æquale.

Si cui videbitur nimis operoſa hac ratio revocet primò triangulatum datum in unicum triangulum, per proble-

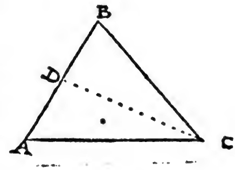
 ma

ma primum & secundum, tumque illi unicum in data altitudine datoque angulo æquale ut ante construatur.

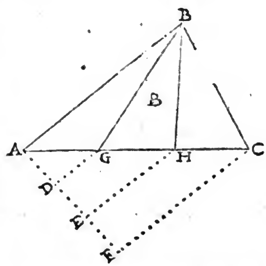
De Figurarum Sectione.

Problema. 8.

Datum triangulum recta ex angulo ducta secare ratione data.



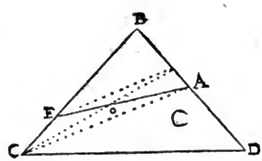
Bisecundum esto triangulum ABC recta ex angulo C educta. dati anguli basis AB bisecetur in D recta á vertice ad punctum in basi medium D bisecabit triangulum, quod é 38 propositione lib. 1. & prop. 1 lib. 6 *Euc.* demonstratu perfacile est.



Haud alia ratio est si in tres quatuorve, aut etiam plures partes æquales secandum sit triangulum; ut si in tres tantum, secetur basis AC in totidem partes æquales per prop. 10. lib. 6. *Euc.* nam si cũ AC basi recta quælibet faciat angulum, inque ea tres lineæ ęquales deinceps continuæ statuantur AD DE EF, & contra FC terminos connectens parallelæ sint EH DG, eæ intersecabunt datam AC in tres partes æquales. hinc connexæ ad angulum B rectę BG BH desecabunt tria triangula BAG, BGH, BHC inter se æqualia. Haud alia ratio est si triangulum propositum data qualibet ratione intersecandum sit.

Problema 9.

Datum triangulum è dato in latere puncto secare ratione data.

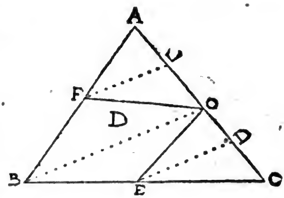


Ex puncto A in latere BD educenda est linea absumens partem trianguli tertiam. secetur itaque latus BD in tres partes æquales, á vertice C ad datum punctum connectatur recta AC contra quam ab F sit parallela FE, recta ab E ad A connexa absumet EBA segmentum optatum. agatur enim CF: erit itaque

e*C*BF totius trianguli CBD pars tertia: & cum triangula *CEF* EAF in ea-m basi intra easdẽque parallelas consistant erunt æqualia. segmentum autem)F est commune eorum utrique: quare reliquum EO*C* reliquo *F*OA ıale erit. quamobrem si ad quadrãgulum EOAB addantur ista æqualia tri-gula, triangulum EAB, ipsi C*FB* pariter æquabitur. ideoq; totius BCD pars t tertia. atque ita in cæteris quibuslibet optata ratione secandis consimili pla-modo agendum.

Licebit itaque hinc

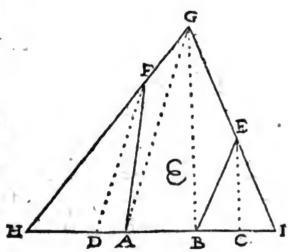
Datum triangulum A B C duabus rectis ex o in latere puncto O ita secare, vt spatium rmedium ab eductis comprehensum reli-rum utriusque sit duplum.



Secetur enim A*C* quadrifariam, & ovissimis sectionum punctis L & D ıtra rectam B O á vertice B ad medi-) eductam, agantur utrimque paral-:L*F* DE, rectæ OF OE compre-ıdent intersegmentum ıOFBE duplum alterutrius extremorum OF*A*, OEC. Cujus demonstratio petatur ex antecedentis analogia. atque simili ıe via conficies vt medium intersegmentum habeat quamlibet rationem ım.

Et licet hinc

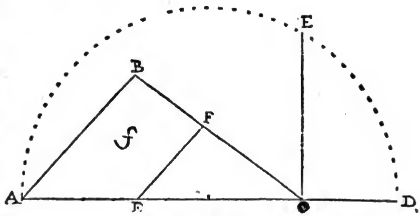
Datum triangulum HGI é datis duobus ctis A & B ita secare, vt recta inde edu-reliquis lateribus occurrentes totius trian-$\frac{4}{7}$ comprehendant.



Ad puncta data *A* & *B* á vertice G nectantur rectæ AG BG, tum basis ı partes 7, quot videlicet nomen par-n habet, dividatur, sitque HD $\frac{1}{7}$, & ', atque ab his sectionum punctis cõ-priores GA GB parallelæ sunto & C E, quarum vertices cum datis ctis connectant AF BE, spatium ab nterceptum AFGEB comprehendent $\frac{4}{7}$ dati trianguli, & FAH $\frac{1}{7}$ EBI $\frac{1}{7}$ ıs Demõstratio é superiore problemate perspicua est. Atque hujus analogia ɔ varia & multiplex é dato puncto datisve punctis in latere poterit institui.

PROBLEMA 10.

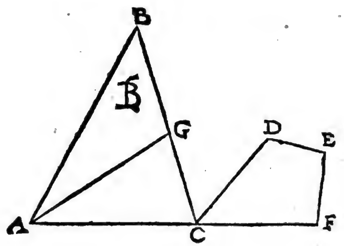
Recta contra datum dati trianguli latus parallela imperatam partem absumere.



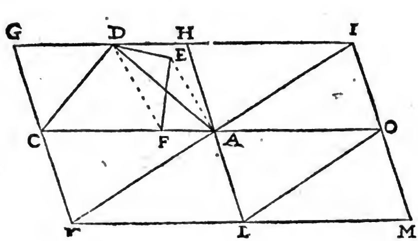
Proponatur triangulum A B C unde $\frac{2}{3}$ absumendæ sint inter latus A B & parallelam E F, continuato basin AC sui parte tertia in D (namq; deductis $\frac{2}{3}$ de integro super est $\frac{1}{3}$) media proportionalis inter A C basin & C D continuationem esto C E, eique æqualis *C* E, recta E F parallela contra A B absumet A B *F* E segmentum trianguli imperatum. Sunt enim tres rectæ A G C E C D continué proportionales: ideoque vt *A* C prima ad C D tertiam (est autem earum ratio ut 3 ad 1) sic A B C triangulum á prima descriptum ad E F C triangulum á secunda: quare E *F* C erit totius ABC $\frac{1}{3}$, & reliqua pars A B E F reliquæ $\frac{2}{3}$. atque ita in cæteris datis partibus quibuscunque.

PROBLEMA 11.

A dato triangulo recta a dato ejus angulo educta absumere segmentum dato rectilineo æquale.



Datum esto triangulum A B C, é cujus angulo A educenda sit recta AG, absumens segmentũ æquale dato rectilineo *C*DEF. Reducito primo datum rectilineum (si id triangulum non sit) ad triangulum C*D*A, & angulo A C *B* æqualis statuatur H A *C*, & describatur parallelogrãmum AH G*C* duplum trianguli A *C*D: hinc continuetur HI æqualis ipsi A C, rectaque IA occurrat GC continuatæ in K, & compleatur parallelogrã-

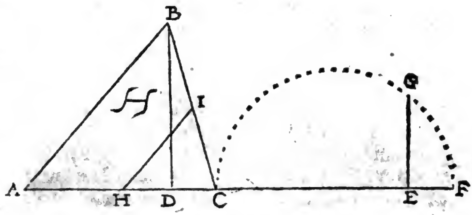


mum

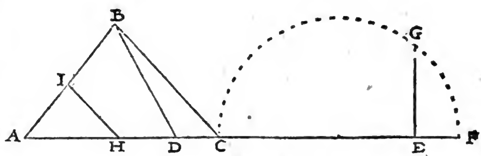
ium GIMK, erit ALMO complementum æquale parallelogrammo GCAH; eoque LAO æquabitur dato triangulo CDA seu quadrilatero CDEF idque dato angulo ACG: quare si recta CG ipsi AL æqualis statuatur & connectatur G erit AGC triangulum dato rectilineo æquale, & relinquetur ACB excessus iti trianguli supra datum rectilineum.

PROBLEMA 12.

A dato triangulo cta contra datum la s parallela triangum dato rectilineo quale absumere.

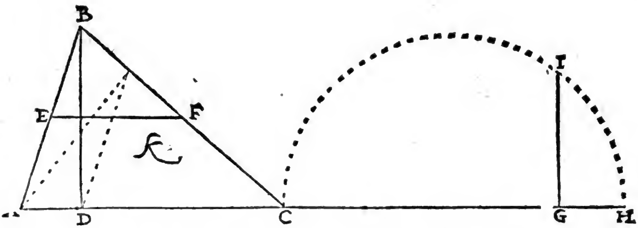


Et primo quidẽ tur triangulum quo recta con- a AB parallela ab- mendum sit tri- ngulũ æquale dato triangulo BDC. recta CF continuetur é duabũs rectis AC D, inter quas proportione media sit EG, cui æqualis sit CH, unde HI parallela ontra AB descabit triangulum IHC æquale dato BCD. demonstratio decimo oblemati affinis est.



Res eadem erit iamsi á triãgulo BC postuletur angulum BCD secari linea cõ- BC parallela. m ut ante in- AC CD seu ip- æquales CE EF edia inveniatur proportionalis EG, cui æqualis sit AH recta HI parallela ntra datum latus BE descabit triangulum A I H æquale dato BCD. Sunt im triangula A B C AIH similia. ideoq; AC AH, BC HI latera erũt propor- nalia; atque ex fabrica ut AC ad AH sic AH ad DC: quare ex æquo ut AH ad C sic BC ad IH: quamobrem propter æqualis anguli crurũ reciprocationem II BCD triangula per 15 prop. lib. 6 *Euclidis* erunt æqualia.

Neque

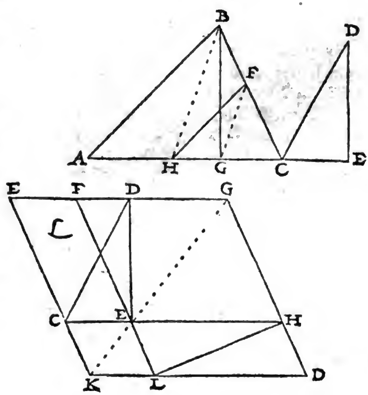


Neque longe alia ratio est si é triangulo ABC sit absumendū segmentum æquale triangulo AB D linea contra basin AC parallela. triangulo enim ABD æquale construatur B AK ut K incidat in crus alterutrum, ut hic in BC, tumque inter CB & BK seu CG & GH ipsis æquales statuatur GI linea proportione media, eidemque æqualis sit BF, recta FE parallela contra A C, absumet triangulum BEF æquale dato. demonstratio ex antecedentibus est perspicua.

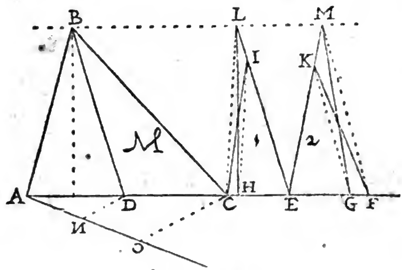
PROBLEMA 13.

A dato triangulo è dato in latere puncto partem dato rectilineo æqualem, vel pro datorum rectilineum ratione secare.

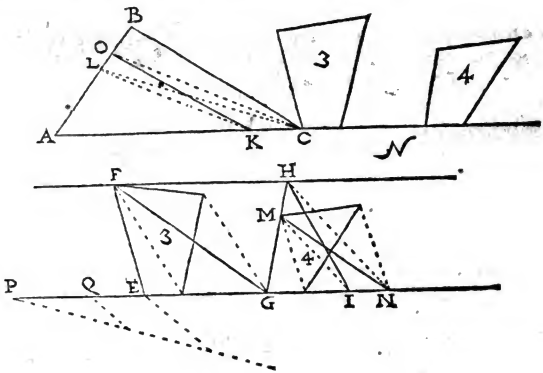
Sit A B C triangulum unde recta ex *F* puncto educta absumendum sit triangulum æquale dato C D E. construatur ad latus B C datumque angulum B C A triangulum E H L seu B*C*G ipsi DEC æquale, sitque rectæ F G parallela á vertice *B* linea BH, recta *F*H eas connectens absumet H F *C* dato triangulo æquale.



Esto



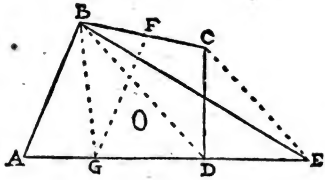
Esto Iam AB C triangulū secandim ratione triangulorum ICE KEF. Vtrumque ad candem altitudinem revocetur sintque LHE MEG æquelata ipsis æqualia, & secundum rationem basium HE EG secato basin dati trianguli AC in D. erunt igitur triangula ABD DBC, ut AD ad DC, hoc est in ratione data, quod é 37 propos. lib. 1, & 1 ac 10 propos. lib. 6. *Euclidis* demonstrari facile potest.



Iam si datum triangulū ABC ex dato in latere puncto O sit dividendum secūdum datam rationem quadrangalorum 3 & 4. revocato ea primum ad triangula æquealta EFG & GHI, quamobrē ipsa erunt ut bases: eadem ratione secato latus AB in K, unde á vertice C ad O sit OC eique prrallela LK recta OK connexa secabit datum triangulum ratione data.

PROBLEMA. 14.

Recta à dato dati quadranguli angulo educta imperatam partem auferre.



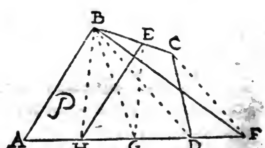
Vt si ex angulo B recta sit educenda, quæ ex dato quadrangulo ABCD quadrantem absumat. reducito datum quadrangulum ad triangulum ABE, & basis AE secetur in partes optatas, sitque pars quarta AG recta BG connexa absumet ABG triangulum æquale parti quartæ.

At si ex A quoque ¼ auferre postules, é puncto G sit parallela GF contra AB & connectatur AF, ea quoque absumet ABF triangulum ¼ æquale. ex fabrica demonstrationem colligere haud est difficile.

Quod si vero G cadat inter D & E, vel F ultra B (eum casum haud operosum erit manente eodem partium situ invare, quem ideo quia diagrammate destituimur explicare nunc non possum.

PROBLEMA. 15.

Puncto in dati quadranguli latere dato absumere partem datam

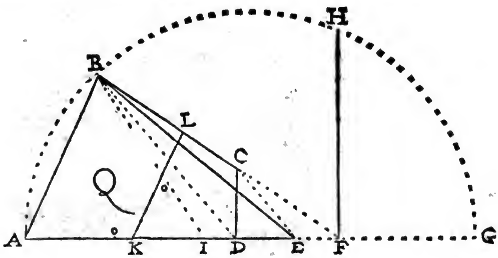


Exponatur quadrangulum ABCD, & in latere BC punctum E, unde recta ipsum bisecans educenda sit. datum quadrangulum reducatur ad triangulum ABF, basisque AF bisecetur in G & connectatur EG, cui á B parallela sit BH, recta connectens EH bisecabit datum quadrangulum.

PRO-

PROBLEMA 16.

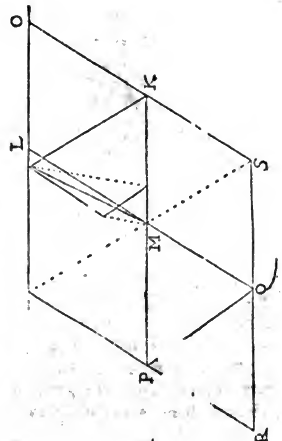
A dato quadrangulo recta dato lateri parallela partem imperatam absumere.



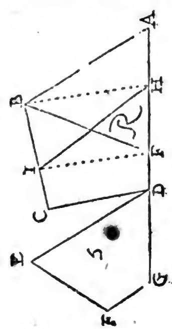
Propositum esto quadrangulū A*B*CD unde absumenda sit pars tertia , idque linea contra datum latus A B parallela. datum quadrangulum revocetur ad triangulum ipsi æquale A*B*E, hinc basin trianguli æqualis A E trifariam secato in I & O, & connectatur I B, erit itaque B I E pars totius quadranguli tertia : hinc *A*E & *B*C latera continuata concurrant in F, Ideo triangulo B I F, quod & $\frac{1}{3}$ dati quadrāguli & præterea ipsi EBF æquale est subducto, relinquētur $\frac{2}{3}$ *AB*E seu quadranguli *A*B C D. itaque continuato *A*F æqualiter ipsi *F* I in G, atque inter *A F* & *F*G proportione media sit *F*H, cui æqualis statuatur F K, inde ex K parallela contra *AB* educta absumet inter parallelas *A*B L K $\frac{2}{3}$ quadranguli dati, & reliquū segmentum K L *C* D reliquum trientem continebti. Nam cum totum triangulum *AB* F ad triangulum I BF habeat rationem quam A F ad I *F*, per propos. 1. lib. 6. *Eucl.* atque ita quoque sit A F ad F G, & triangula similia AB*F* & K L F eandem quoque rationem habeant, quia illud ad lineam primam hoc ad secundam trium continue proportionalium est comparatum: sequitur cū triangulum A B C ad I B F & K L F eandem habeat rationem, ipsa quoque per 75 propositionem nostrorum elementorum inter se æquari. Triangula autem B I *F* K L *F* æquari per 15 propos. lib. 6. *Euclidis*, & per 72 nostrum quoque comprobari potest.

PROBLEMA 17.

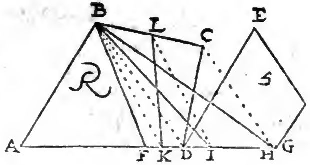
A Dato triangulato recta è puncto in ejus latere dato educta partem imperatam absumere.



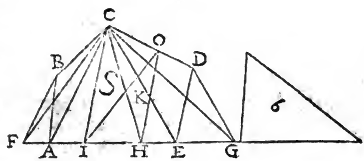
Detur quadrangulum A B C D, punctumque H in latere A D inde recta educenda sit absumens segmentum çquale dato spatio DEFG. Itaque revocato datum hoc spatium ad triangulum æquale in dato latere AB datoq; āgulo BAD, sitque QMP vel A F B. & à vertice B cum dato puncto H connectatur B H, cui ab F parallela sit FI recta H I absumet spatium A B I H æquale dato E FGD.



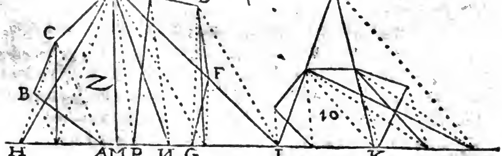
Secundo, é dato puncto K in hoc diagrammate sit absumendum spatium æquale dato D E *F* G & præterea illius pars quarta. Res expedita erit per exẽpli primi analogiã. namque datum quadrangulum D E F G revocetur ut ante ad æquale triangulum ABF in dato angulo A datoque latere A B: revocetur quoque prius quadrangulum A B C D in triangulum ipsi æquale A B H, cujus basis A H pars quarta sit H I, cui æqualis ponatur *F* I. quare triangulum A B I dato quadrangulo DEFG & præterea $\frac{1}{4}$ ipsius ABCD æqualis erit: jam á dato puncto K ad B connectito B K, sitque I L contra eam parallela, recta K L absumet optatum spatium A B L K triangulo A B I æquale.



Tertio quoque detur quinquangulum ABCDE, & punctum in latere O, unde recta educenda sit absumens spatium dato triangnlo 6 æquale. datum quinquangulum revocetur ad triangulum FCG ipsi æquale. hic igitur (quia O punctum datur in latere CD) revocetur datum triangulum 6 ad triangulum æquale CGH in dato angulo CG *F* datoque latere *C* G. porro hoc ipsum quoque triangulum *C*GH quadrangulo *C*DEH æquari haud obscurum est. connectatur jam H O eidemque ab angulo *C* parallela agatur *CI*, recta OI connexa á dato puncto O ad terminum parallelę absumet segmentum IODE optatum. demonstratio é superiorum analogia satis obvia est. Namque datum triangulum 6 per fabricam triangulo C G H, seu quod idem sit quadrangulo HC DE æquale est, ab hoc tollatur triangulum CKO, & pro eo reponatur ei æquale IKH (ob parallelismum linearum H O & C I) totum spatium IODE toti H*C*D*E*, sive dato triangulo 6 æquale erit quemadmodũ postulabatur.



Quarto detur septangulum ABCDE*F*G & punctum O in latere D E, unde recta educenda sit absumens spatium sexanguo 10 & præterea $\frac{1}{7}$ ipsius septanguli

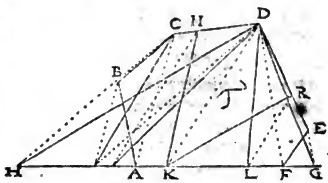


 æquale

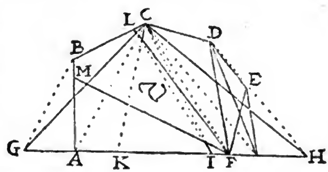
æquale. cõſtruatur triangulũ H D I dato ſeptãgulo æquale, & ſexangulo triangulum itidem æquale & antecedenti æquealtum I L K, cujus baſi æqualis ſtatuatur HM, hinc continuetur MN $\frac{4}{7}$ totius HI: quare triãgulum HMD dato ſexangulo, & M N D $\frac{4}{7}$ ſeptanguli dati ęquale erit. itaque ab N ad datum punctum O connectatur N O, & contra eam ab angulo verticis D parallela DP occurrat baſi AG in P, recta OP, á dato puncto ad terminum parallelæ P connexa abſumet ſpatium optatum OPABCD æquale dato ſexangulo 10 & pręterea quintæ parti ſeptanguli ABCDEFG. quod factum oportuit.

PROBLEMA 18.

A dato triangulato rectis duabus è puncto in ejus latere dato imperatum ſpatium intercipere.



Detur ſexangulum ABCDEF, & punctum K in baſi AF, unde duæ ſunt rectę educẽdæ NK KR quibus interſecetur ſpatium NK RD $\frac{4}{9}$ totius ſexanguli. revocato primum ſexangulum datum ad triangulum æquale HDG, tumque baſin trianguli HG in partes 9 ſecato quarũ quinque ita ſtatuantur, ut earum aliqua pars ab H & reliqua á G in baſi ponatur; hic jam HI aſſumatur earundem 3, & GL 2: tumque ab angulo verticis D ad datum punctum K connectatur recta DK, cui ab L parallela ſit LR uſque in latus DE. & ex I linea IN in latus DC, rectæ inde ad datum punctum K connexę NK KR intercipient ſpatium NKRD optatum. Nam cum NKABC ſit $\frac{3}{9}$ ex fabrica, & RKFE $\frac{2}{9}$ conſequitur dictum ſpatium NKRD reliquas $\frac{4}{9}$ continere.



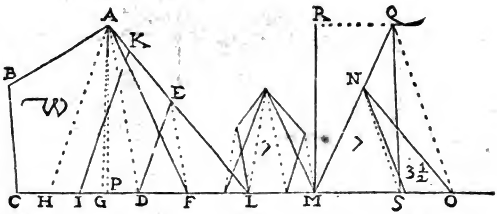
Si punctum datum in aliquo angulorum ſtatuatur factio antecedenti eſt ſimillima. Detur angulus F in ſexangulo iſto ABCD EF, unde itidem inter duas eductas lineas $\frac{1}{3}$ intercipienda ſit, ſed ea lege ut ſegmenta item reliqua inter ſe æquentur, hoc eſt $\frac{1}{3}$ totius quoque contineant.

*D*atum ſexangulum primo revocavi ad triangulum GCH, cujus baſis pars tertia GK, inde (quia B*AF* triangulum majus eſſe deprehendi quam GCK pars tertia) triãgulo CGK triangulũ æquale conſtruxi MAF ad datum latus *A*F datoque angulo BA*F*. hinc triangulo GCI, quæ da æ figurę $\frac{1}{3}$ æquat, æquale conſtruxi ſpatium LFAB. ducta enim *CF* & ab I puncto parallela IL uſque in latus *BC*, recta ab F ad L connexa deſecabit ſpatium L*F*A*B* æqualę $\frac{1}{3}$ totius: quare & reliquũ *F*LCDE $\frac{1}{3}$ quoque æquabitur.

PRO-

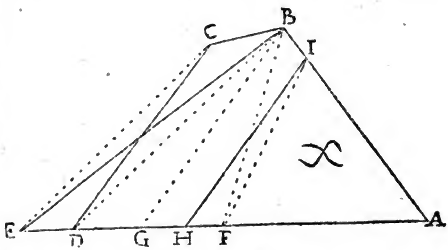
PROBLEMA 19.

Datum triangulatũ recta contra datum latus parallela data ratione secare.



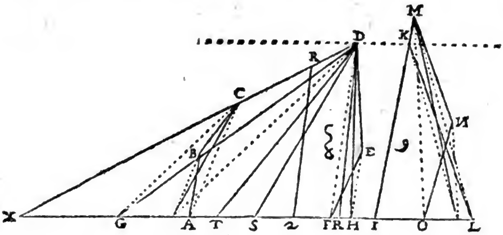
A dato quinquangnlo ABCDE absumẽdum est spatiũ inter DE & lineam parallelam, ut hujus segmenti ratio ad datũ quinquangulum 7, rationem habeat datã, quam 3 ad 2. Agatur ex angulo A recta AH parallela contra datum latus DE, & fit triangulum HAF æquale trapezio AHDE, continuataq; latera CD AE concurrant in L. hinc fiat ut 2 ad 3 sic quinquangulum 7 ad triangulum MNO. id autem ita fit: mutetur datum quinquangulum 7 in triangulum 7, tumq; fiat ut 3 ad 2, sic MS basis trianguli 7 ad basin MO, erit itaque MNO sesquialterũ dati quinquanguli, hoc revocetur ad altitudinem trianguli HAF, quale hic est QMS; hujus basi MS statuatur ab F recta FG æqualis, eritque triangulum FAG triangulo MQS seu MON propterea æquale. Ad extremum media proportionalis inter HL & LG sit LI, parallela IK ab I puncto contra AH vel DE educta absumet spatium KIL æquale triangulo AGL quamobrem IKED triangulo GAF æquale erit, & propterea ad datum quinquangulum 7 rationem habebit datam.

Esto trapezium ABCD linea contra latus BC parallela bisecandum. Revocetur ipsum in triangulum æquale ABE, ejusque basin AE bisecato in F, unde ad angulnm verticis recta FB educta datum triangulũ, ipsumque adeo trapeziũ bisecabit: deinde



ab eodem angulo B acta BG parallela ostendit hoc casu BG triangulum majus esse semisse; quare inter GA & AF media proportionalis sit AH, recta HL ab eo puncto contra GB parallela absumet triangulum HIA æquale triangulo FBA. quare HI parallela contra DC datum quadrangulum in duas partes æquales dispescui, & triangnlum IHA reliquo quinquangulo IHDCB erit æquale.

Et rur-



A ſexangulo ABCDEF abſumẽdum eſto ſpatium IMNO & præterea ipſius ſexanguli $\frac{1}{6}$, idque linea contra datum latus AB parallela. datum ſpatium IMNO in triangulum æquale & dato ſexangulo æque altum convertito, id eſto IKL: & ſexangulum ABCDEF in triangulum GDH eadem altitudine: hinc baſi IL ſtatuatur GT æqualis, cui continuetur TS $\frac{1}{6}$ baſis GH: itaque triangulum GES æquatur dato ſpatio & $\frac{1}{6}$ dati ſexanguli. ſit præterea DR é vertice D prrallela contra datum latus AB, & CD producta occurat baſi FA continuatæ in X, & media proportionalis inter XR & XS ſit XZ, atque ex Z parallela ZR contra datũ latus AB incidat DC in R, ea abſumet ſegmentum ABCRZ æquale $\frac{1}{6}$ dati ſexanguli & inſuper dato ſpatio IMNO.

Cæterum quodnam latus continuandum tibi ſit quemadmodum hic DC continuavimus, hoc modo (ſi forſan ambigas) deprehendes. Per angulum aliquem ducito parallelam, ut puta per C contra datum latus AB, quod ſi ea abſumat partem optato ſpatio majorem, atque parallela cadat intra AR fuiſſet BC latus tibi continuandũ; ſi vero minus hoc ſit & ſpatium á parallela per D ducta majus ſit ut ante, tum DC ut hic factum vides, porro continuari debuit, quin imó ſi neque is caſus ſufficiat angulus E porro eadem ratione tentandus. vel ſi iſta ratio operoſa aut intricatior fuerit, ut non raró evenit, cum datum ſpatium ſemiſſe majus erit, aut neſcio quæ impedimenta occurrent quæ hic nunc enumerare nimis longum eſſet, ſumito dati ſubducendi ſpatij differentiam á dato triangulato, & eo quod reliquum erit ſimili via inſiſtito certé his præceptis, uſu & induſtria confirmatis ſaltem aliqua ratione tete expedies. Et ſi neutra harum ſuccedat (quod aliquando fieri poſſe damus) ut ſi velim ſpatium é dato ſexangulo abſumere quod majus ſit quam ABCDF linea contra AB parallela, ut pars optata intra parallelã, includatur tum prius á dato ſpatio ſubducas ſpatium ABCDF. & quod reliquum erit auferes é triangulo DEF inter DF latus & lineam contra AB æquidiſtanter educendam. Atque ita in cæteris porró. res facilis eſt; experiunti modo paululum animum advertas, & manum promptam habeas.

Sequuntur deinceps magnitudinum quædam affectiones, quas autor vocat ſpecies ad analogiam & ſimilitudinem Arithmeticæ numerationis ſimplicis, videlicet Additio ſubductio multiplicatio & diviſio ita enim has quatuor ſpecies vocant vulgó, quia autem ipſi eas ad hunc numerum aggregare libuit autoris ordinem ſecuti eas tertio hoc libro quoque complexi ſumus.

PROBLEMA. 20.

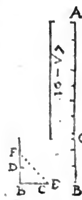
Datam rectam datæ addere.

Exponatur linea A quæ continuanda ſit ęqualiter ipſi B. congruentiâ ſive epharmoſi problema abſolves: diviricatis enim circini cruribus intervallo datæ B, eoque á termino lineæ A in directum continuato linea AB utrique æqualis erit, quemadmodum hic vides.

PROBLEMA 21.

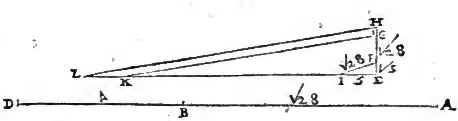
E duabus rectis quarum ſaltem altera ſemoſa menſura ſit aſymmetra, utriuſque quidem quantitate in numeris, alterius verò etiam longitudine data, reliquam quoque ſecundum eãdem menſuram magnitudine exhibere, & priori addere.

Exponatur AB partium 24 (menſuram autem illam vel aliunde huc allatam intelligas, vel ſi libet per præcepta jam tradita in tot particulas eam ſecato, certe quoquo modo datam cognitamque ex theſi eſſe oportet) ad eam addenda ſit inſuper linea potentia ei ſymmetra $\sqrt{13}$. hujus quantitatem inveſtigabis ſubſidio trianguli rectanguli, ſi enim duas lineas ad angulum rectum commiſeris, quarum una ſit earundem partium 2 altera 3, recta angulò recto ſubtenſa earũ terminos connectens dabit optatę quãtitatis lineã $\sqrt{13}$, quia 2 & 3 tantũdem poſſunt per 47 propoſ. 1. libri *Euc.* & 22. noſtrorũ elementorum: quamobrem AB tanto intervallo continuata æquabitur datæ menſuræ $24 + \sqrt{13}$.



Eadem quoque linea $\sqrt{13}$ dabitur ſi inter datæ menſuræ partes 1 & 13 mediam proportionalem invenias, hæc enim erit $\sqrt{13}$, ut ante. aut ſi numerus ſit compoſitus, ut $\sqrt{28}$ inter factores 2 & 14, vel 4 & 7 inventa media proportionalis erit quæſita.

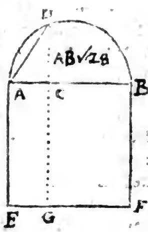
Et rurſum, exponatur linea AB $\sqrt{28}$, huic continuanda eſt alia ejuſdem menſuræ partium 4. hic cũ data mẽſura unde $\sqrt{28}$ deſumpta eſt non detur per proportionẽ ea erit indaganda. conſtruatur itaq; angulus rectus cujus crura ex menſura pro libitu aſſumpta ſint IE 5 & reliquum EF $\sqrt{3}$, quare baſis IF in eadem menſura erit $\sqrt{28}$ cui ponatur æqualis EG, eaq; continuetur in H ut GH ſit unius unitatis ſecundum eandem menſuram: hinc datæ AB ſit æqualis EK & connectatur GK, inde HI contra GK parallela interci-



piat

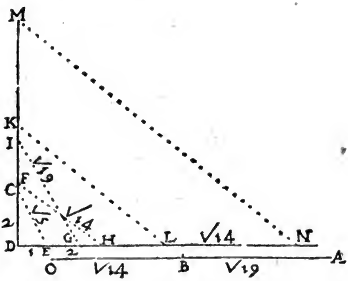
piat segmentum IK: itaque erit ut EG ad GH, sic EK ad KI: quare KI una particula est ad cujus mensuram data AB constructa intelligitur partium $\sqrt{28}$. ideoq; quadrupulum IK fit BD: unde tota AD composita erit $\sqrt{28}+4$, quemadmodū postulabatur.

Potuit vero etiam mensura ad quam data AB est $\sqrt{28}$ inveniri hoc modo quam expeditissime, si inter basin AB totam ejusdemque $\frac{1}{28}$ mediam proportionalem inquiras, ea æquabitur uni particulæ illius mensuræ cujus AB $\sqrt{28}$. Namq; si ex AB quadratum construas, rectangulum ex $\frac{1}{28}$ & ipsa tota linea erit $\frac{1}{28}$ totius quadrati, hoc est erit quadratum unum qualium totum erit 28. atque ideo latus ejus explic abile numero itidem erit, 1 seu una particula optata. hoc ipsum per numeros ita constare potest sit data AB $\sqrt{28}$, hujus $\frac{1}{28}$. erit $\sqrt{\frac{1}{28}}$, horum parallelogrammum, numerus planus ab ipsis factus 1, cujus latus itidē 1. consimilis ratio esset si assumas ex his partes quaslibet, quarum indices sint numeri quadrati ut 4, 9, 16. namque mediæ proportionales inter totam & hac segmenta dabūt mensuram partium datarum: nempe si inter datæ AB $\sqrt{28}$, & ejus $\frac{4}{28}$ mediam proportionalem invenias ea dabit partes debitæ mensuræ duas: si inter eandem & $\frac{9}{28}$ habebis particulas tres: si inter eandem totam & ejusdem $\frac{16}{28}$ habebis particulas 4: atque ita porro ordine per quadratos numeratores infinite continuato. Exemplo unorem juvabo, cum modus iste elegantissimus juxta ac parabilissimus sit.



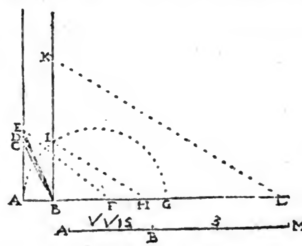
In exposito diagrammate sit AB $\sqrt{28}$ & postulentur in eadē partes 3. intersecato AB in C ut BA ad AC rationem habeat quam quadrata datorum numerorum $\sqrt{28}$ & 3, hoc est ut 28 ad 9, recta AD à vertice perpendicularis CD cum diametri termino A connexa satisfaciet quæsito. eritque talium trium, qualium AB $\sqrt{28}$. sunt enim BA AD AC continuè proportionales quadratūq; DA rectangulo ACGE æqualis erit: est autem ABFE quadratum ad ACGE ut 28 ad 9, sunt enim ut bases: quare quadratum AD erit 9 qualium AB FE 28. ideoque ipsum latus DA 3. ut petebatur. Sed numeris quoque rem comprobabo. sit enim ut ante AB $\sqrt{28}$, AC autem $\frac{9}{28}$ ipsius AB: ajo mediam proportionalem inter $\sqrt{28}$ & ejus $\frac{9}{28}$ exhibere partes in mensura ea secundum quam exposita AB constituta sit $\sqrt{28}$. Novem vigesimæ octavæ de $\sqrt{28}$ sunt $\sqrt{\frac{252}{784}}$ seu $\sqrt{\frac{9}{28}}$, media proportionalis inter totam $\sqrt{28}$ & $\sqrt{\frac{9}{28}}$ sunt $\sqrt{\sqrt{81}}$ hoc est $\sqrt{9}$, pro latere quadrati cujus area habet quadrata novem qualia quadratum ab AB haberet 28, quare latus ejus 3 dabit mensuras ejus generis tres. atque ita porro in cæteris omnibus demonstratio in geometrica etiam facilior. u- ante patuit. atque hac mechanica istum autoris locum Iuvare libuit.

Tertio datæ AB $\sqrt{19}$ continuanda sit linea $\sqrt{14}$, ut composita sit $\sqrt{19} + \sqrt{14}$. sequere modum præscriptum: namque ex assumpta mensura qualibet & secundum eam investigato lineam $\sqrt{19}$, itemque $\sqrt{14}$, ut hic factum vides: gradatim enim eo tandem deventū est hoc modo, rectæ duæ infinitæ angulum rectum comprehendant in his CD crus anguli recti sit partium primo 2, DE 1. tum basis CE erit $\sqrt{5}$, huic DE jam statuatur æqualę crus DF, & reliquum crus DH partium 3. tū basis FH erit $\sqrt{14}$. denique huic æquale sit rursum crus DI, & ipsi CE $\sqrt{5}$ statuatur æquale crus DG, basis IG erit altera optata $\sqrt{19}$: Hinc DK ponatur æqualis ipsi $\sqrt{19}$ & KM lineæ $\sqrt{14}$, datæque AB sit æqualis DL, recta ab M contra KL expositam parallela, intercipiet rectam LN quæsitam $\sqrt{14}$ secundum positam mensuram lineę AB $\sqrt{19}$, quamobrem tota DN sive illi æqualis AO erit $\sqrt{19} + \sqrt{13}$: quemadmodum postulabatur.



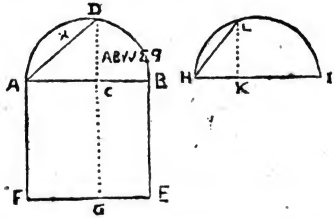
Potuit verò per antecedens nostrum theoremation tam operosæ factionis περιεργεια levissimo negotio declinari. Data enim AB $\sqrt{19}$ ita secetur, ut tota ad suum segmentum se habeat ut 19 ad 14. linea inter datam AB $\sqrt{19}$ & absumptum segmentum proportione media erit optata $\sqrt{14}$. demonstrationem ex proxima annotatiuncula repetas, est enim plane eadem.

Denique etiam in medialibus ad datam AB quæ sit $\sqrt{\sqrt{15}}$ addēdæ sunto partes ejusdem mensuræ 3. Duas rectas infinitas ad angulum rectum inter se committito & mēsuram pro libitu quamlibet assumito taliumque AC 2 statuatur qualiū AB 1: quare subtensa BC erit eorundem $\sqrt{5}$; huic ponatur æquale crus AD, ideoque subtensa DB erit $\sqrt{6}$, huic vero BD etiam æqualis ponatur AE, & ponatur AF partium 3 unde efficitur subtensa EF $\sqrt{15}$, tandem ipsi EF æqualis sit BG, atq; inter AB 1 & BG $\sqrt{15}$; media inveniatur proportionalis BI, hæc erit $\sqrt{\sqrt{15}}$ qualium AB unius. denique ipsi BI continuetur IK partium 3, & datæ AB æqualis sit BH, recta per K contra IH parallela absumet HL lineam partium 3, qualiū BH sit $\sqrt{\sqrt{15}}$. atque ita AB continuata in M dabit AM optatam.



Vt theorematis nostri usum amplificemus, etiam hic atque in omnibus gradibus superioribus locum habere ostēdam: sed haud scio an vulgata illa autoris via in his medialibus $\sqrt{\sqrt{15}}$ cæterisque

cæterisque omnibus altiore gradu affectis expeditior sit, cujus tamen arbitrium penes lectorem relinquo. Exemplum tale esto, postuletur linea partium 3 qualium data AB sit $\surd\surd 192$. Secetur itaque AB in C, ut BA ad AC sit quemadmodum quadrata numerorum $\surd 192$ ad 9; id autem fiet secundum modum præmissum, si ipsi AB statuas æqualem HI eamque seces in K, ut IH ad HK se habeat quemadmodū 192 ad 81, tū enim HL erit talium partium 9



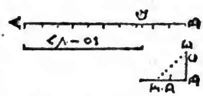
qualium IH $\surd 192$, ipsique HL sit æqualis AC. itaque AB ad AC erit ut $\surd 192$ ad 9: sed cū BA AD DC sint continue proportionales, erit ut prima AB ad ultimam AC, sic quadratum ABEF ad quadratum è secunda AD: Et quia prima linea BA ad AC per fabricam sit ut $\surd 192$ ad 9: & quadratum ex AB sit itidem $\surd 192$, consequens est quadratum ex AD itidē esse 9. quare latere utrobique eruto illic redibit AB $\surd\surd 192$, hic autem AD partium 3, quemadmodum postulabatur. In altioribus gradibus pro numero potestatis labor multiplicādus, ut si in $\surd\surd\surd 600$ postuletur mensura partium duarum. Primum qualium data 6000 talium alia sit 256, inter quas proportione media erit talium 16 qualium data $\surd 6000$. Secūdo huic inventæ alia 16 æqualis statuatur atque inter eam & datam mediā proportionalis erit talium 4, qualium data $\surd\surd 6000$. Denique inter datam & hanc media erit talium 2 partiū qualiū data $\surd\surd\surd 6000$, cujus demonstratio ex antecedente facile derivari potest. Atq; ita porro in reliquis. Sed ut quilibet surdi sive simplici sive multiplici potestatis gradu involuti ut $\surd 2$ $\surd 3$, & mediales $\surd\surd 15$ $\surd\surd 3$, & cæteriquicunq; $\surd\surd\surd 7$ $\surd\surd\surd\surd 19$ aliíque facillimè & expeditissime nullo negotio nulla περιεργεία accuratissime inveniantur organū excogitavimus, cujus descriptionē facillimā ante oculos depictā exhiberē nisi operæ typographicæ versionē istā, dū cū belgica editione pari a facere conātur, nimis importunè urgerent.

A data recta datam subducere. PROBLEMA 22.

Proposita esto recta linea A, atq; inde subducenda sit B, quæritur differētia. res tota epharmosi constat: diductis enim circini cruribus ad intervallum ejus crus unum statuatur in A reliquum in eadem recta terminetur in O, tum AO erit æqualis datæ B, & reliqua erit differentia quæsita, cui posita sit æqualis C. Quantumvis hæc res levicula videatur, eā tamen nescio quo structuræ ambitu 3. prop. 1 libri *Euc.* demonstravit.

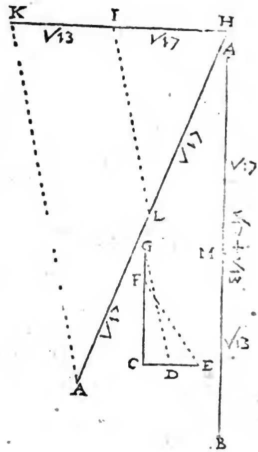
PROBLEMA 23.

E duabus rectis quarū saltē altera famosæ mensuræ sita symmetra utrinsq; quidē quantitate in numeris, alterutrius vero etiam longitudine data, reliquam quoque secundum eandem mensuram magnitudine exhibere & à priori subducere.

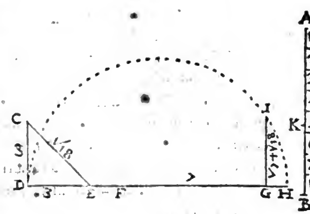


Exponatur linea recta AB partiū 10, quæritur quanta linea superfutura sit, cum inde subduxeris $\surd 6$ secundum ejusdē famosę mēsuræ ęstimationē: duæ rectæ FB BE cōprehendant angulū rectū, cujus crura DB BC sint decima pars lineæ AB, quare subtēsa DC, cui æqualis ponatur BE, erit $\surd 2$, & BF partium duarum mēsuræ expositæ, itaque subtensa FE erit $\surd 6$, cui in data AB statuatur æqualis BG quare reliqua AG erit optata differentia $10 - \surd 6$.

 Sit:



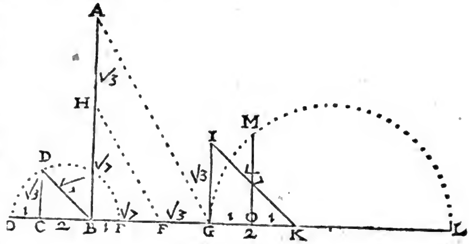
Sit data AB linea composita é duobus nominibus $\sqrt{17} + \sqrt{13}$, unde alterum nomẽ $\sqrt{17}$ sit subducendum. hoc est secare datam rectam in ea nomina é quibus cōponitur. Assumito pro libitu mẽsuram quamlibet, & secundum eam ordine indagato magnitudinem lineæ $\sqrt{17}$, itemque alterius $\sqrt{13}$: istæ sunto HI IK: & cum ea linea HA datæ AB ęqualis angulum faciat, atque secũdum rationes segmentorum HI & IK intersecetur in L: quare HL, seu AM ipsi æqualis erit $\sqrt{17}$, atque reliqua MB $\sqrt{13}$.



Tertio sit AB famosæ mensuræ seu ex hypothesi datæ partium 8, unde subducenda sit linea irrationalis aliqua, cujus quadratum sit $7 + \sqrt{18}$, hoc est quę sit ejusdem mensuræ $\sqrt{.7 + \sqrt{18}}$. Primum invenias secundum constructionem toties jam iteratam lineam $\sqrt{18}$, posito enim utroque crure anguli recti CD & DE partium 3, subtensa recto CE erit $\sqrt{18}$: cui æqualis statuatur DF, & FG continuetur partibus 7: quare tota DG erit $7 + \sqrt{18}$. hinc inter DG & GH 1 sit media proportionalis GI, hujus igitur magnitudo definitur hoc numero $\sqrt{.7 + \sqrt{18}}$. ei æqualis BK subducta de AB relinquet AK $8 - \sqrt{.7 + \sqrt{18}}$.

Quarto

Quarto insuper datur AB $\sqrt{7} + \sqrt{3}$, unde auferenda sit linea æqualis lateri ipsius AB.



Zetema istud cũ alijs compluribus mihi ab harum artiũ callentissimo Simone Stevino olim fuit propositum ãnno 1583, cujus solutionem hanc quæ sequitur tunc attuli.

Primum inquisivi secundum assumptam pro libitu mensuram duas lineas BF $\sqrt{7}$, FG $\sqrt{3}$, atque secundum harum rationem intersecui datam AB in H, ut hic in diagrammate vides. quare BH talium erit $\sqrt{7}$, qualium HA $\sqrt{3}$. Inde GI perpendicularem constitui æqualem ipsi HA, & basin recti IK ipsi BH, quare reliquum crus recti GK erit partium exactè 2 qualium GI est $\sqrt{3}$. eadem bisecta in O exhibet GO ejusdem mensurę unam, tum OL datę AB posita æquali, & OM inter GO & OL proportione media, ea erit latus dati numeri $\sqrt{7} + \sqrt{3}$, hoc est $\sqrt{.\sqrt{7} + \sqrt{3}}$. hæc de AB subducta relinquet AN $7 + \sqrt{3} - \sqrt{.\sqrt{7} + \sqrt{3}}$.

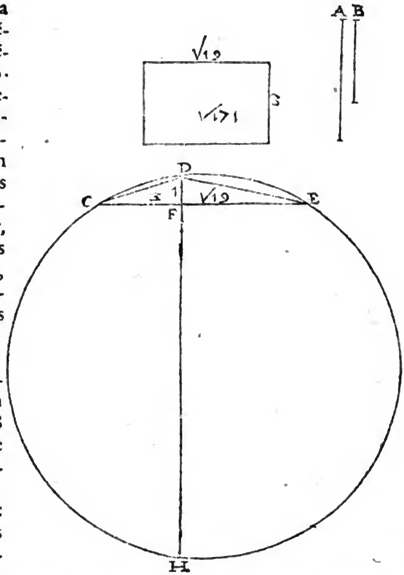
In autoris verbis κατάχρησις est, namque in ipso zetemate postulat latus lineæ AB, cum non hoc vellet, sed latus quadrati æqualis rectangulo comprehenso sub longitudine AB $\sqrt{7} + \sqrt{3}$, & latitudine æquante unam unitatem ejusdem mensuræ. itaque interpretatione nobis juvandus fuit. Problemati sequenti titulum multiplicationis autor præfixerat.

PRO-

PROBLEMA 24.

Rectangulum à duabus lineis in assumpta mensura longitudine definitis comprehensum ad ejusdem unitatem applicare.

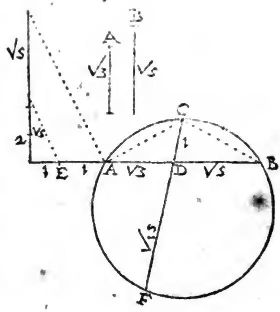
Problema ipſum ab autore, ut reliqua omnia tantum ad ſubjectũ exẽplum concipitur hoc ſẽſu: Dantur duæ rectæ lõgitudine A $\sqrt{19}$, & B 3 ſecundum ejuſdem menſuræ famoſæ æſtimationem, quæritur quantum ſit rectangulum ab ipſis comprehenſum, & ſi inter ſe multiplicentur, quanta ſit numeri ab ijs facti longitudo, hoc eſt, geometrice lineam invenire, cujus numerus facto datarum æquetur.



Conſtruito rectangulum parallelogrammum á datis lateribus A & B tumque latera inter ſe multiplicato, dabitur figuræ magnitudo $\sqrt{171}$. Vt autem lineam huic facto æqualem invenias datas in directum continuato, ut CE, huic á puncto continuationis F erigito FD æqualem tertiæ parti lineæ *B*, & per tria puncta CDE peripheriam deſcribito, ipſamque DF continuato donec eidem occurrat in H. ajo *F*H eſſe optatam, cujus demonſtratio plana eſt é 51 propoſ noſtra, & 35 prop. 3 libri *Euc.* quiá CE & DH rectæ eidem circulo inſcriptæ mutuo ſeſe interſecant in F, ideo rectãgulum ſub *CF* & FE æquatur rectãgulo DF & FH: atqui CF in FE comprehendit $\sqrt{171}$, quare D*F* in FH tantundem quoque valebit. cum igitur $\sqrt{171}$ dividentur per D*F* ſeu 1 dabitur in quoto *F*H linea $\sqrt{171}$.

Secuudo

Sunto datæ duæ lineæ A $\sqrt{3}$ & B $\sqrt{5}$ geometrice multiplicādæ quæritur earum factus. Ante tibi investiganda est mensura assumpta sive famosa, ut jam sæpiculé á nobis factitatum,& hic similis factionis typum exhibemus, invētaque EA sit ejusdem unitas hinc AB continuetur é datis duabus A & B, & ab D continuationis puncto statuatur DC 1 angulo quocunque,tum per tria puncta ACB descripta peripheria DC continuatam interfecet in F,fegmentum DF erit linea optata.



Idem aliter. Inventa famosa mēsura, inquirito more folito lineam cujus quātitas fit $\sqrt{15}$,ea erit optata.

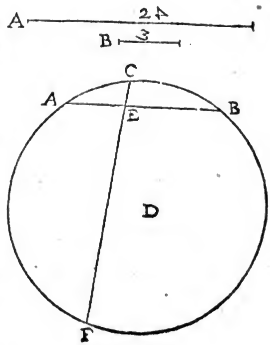
Istud problema, si verum fateri velimus & multiplicationis & divisionis analogiam, exhibuit. Namque Geometrica comprehensio Aritsmeticæ multiplicationi suo quodam modo respondet,hinc enim numeri figurati tanquam talium figurarum indices existunt:contra vero datum parallelogrammum ad datam lineam applicare divisionis instar est. & cum utrumque hoc problemate præstetur,utrāque & multiplicationem& divisionem hoc uno contineri manifestum est.ut sequente jam omnino tanquam novo opus non sit,nisi insignis ἀκυρολογία & illic & hic incauto aut minus versato lectori importere potuisset. Namque quod hic autor postulat duarum linearum multiplicatione Geometrica lineam fieri, tam ἄκυρον est, quam id quod sequitur mutua duarum linearum divisione lineam existere. Neutrum geometra agnoscet,neque per naturam esse potest· sunt ista problemata quibus tyronibus illudi solet.neque enim ea mens est proponentis.& cum Zetema joculare sit, solutionis tamen via legittima est. Ideoque autoris Zetema, qui omnia tantum ὑποδειγματικῶς proposuit, Problemate ita enuntiavi ut scrupulum illum planè sustulerim: quamobrem ut exempla quoque legittimè enuntientur ita concipi debuere.

1. *Dantur duæ rectæ secundum ejusdem mensuræ æstimationem A $\sqrt{19}$,B 3, quæritur si rectangulum ab ipsis comprehensum ad ejusdem mensuræ unitatem applicetur,quæ nam sit longitudo inde existens. Vel ut 1 mensura ad B 3, sic A $\sqrt{19}$ ad quem.*

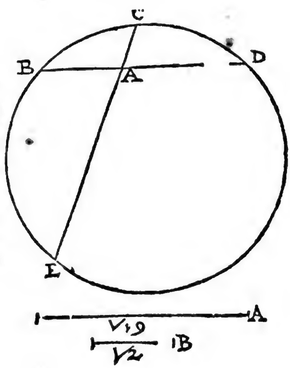
Hic quia unitas est primus proportionis terminus necesse est quartum proportionalem numerum eodem numero definiri,quo numerus è mediorum multiplicatione existens:veruntamen aliud in hac figurarum comprehensione est area ab A & B comprehensa $\sqrt{171}$, aliud verò linea cujus longitudo secundum famosæ mensuræ æstimationem sit $\sqrt{171}$: Hoc numero non surdo facilius intelligetur,aliud inquam est quadratum 16 pedum,aliud linea 16 pedum.quadratum enim 16 pedum habet 16 pedes quadratos,linea vero 16 pedum tot pedibus in longitudinem patet:atqui duarum linearum comprehensio sive multiplicatio parallegrammum facit:quare huic facto nulla linea æqualis dari potest, quoniam inter superficiem & lineam nulla ratio,aut mathematica comparatio intercedit: similem catechresin ultimo exemplo superioris problematis quoque notavimus.

PROBLEMA 25.

Datis duabus lineis in assumpta mensura, rectangulum sub earum altera & ejusdem mensuræ unitate comprehensum ad reliquam applicare.



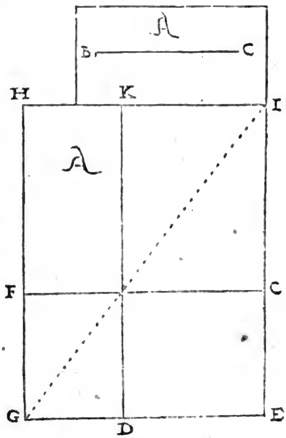
Dividenda esto exposita linea A 24 per B 3, idque Geometrice. Datæ duę A & B comprehendant angulum quemcunque AEF, & EF (æqualem dividendæ A) continuato iu C, ut EC æquetur unitati in assnmpta mensura, quæ est $\frac{1}{3}$ ipsius B, vel $\frac{1}{24}$ lineæ A. & per tria puncta CAF peripheria descripta continuatam AE intersecet in B. hic EB erit quotus quęsitus, vel potius EB erit latitudo existens ex applicatione CE in EF, ad lineam AE: quare si AE sit 3, CE 1, EF 24, erit quæsita EB earundem partium 8.



Aliud. Data A $\sqrt{19}$ dividenda esto per B $\sqrt{2}$. Inquirito primum earum famosam mensuram AC unius unitatis ejusdẽ: quare antecedentis exempli factionem secutus deprehendes A D $\sqrt{9\frac{1}{2}}$. Non raró evenit ut datum parallelogrammum ad datã lineam applicandum sit (zetema est superiori omnino idem, factione tantum mutata) in quo modum mihi usitatum subjecto diagrammate explicabo.

Datum

Datum quadrangulum BFHK ad datã lineam BC applicandum esto, seu quod idem sit, ad datam BC parallelogrammũ construendum dato æquale, quæritur latus reliquum.

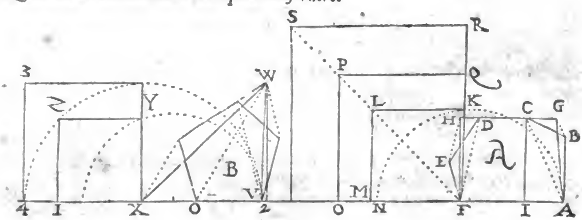


Continuato basin dati parallelogrammi FB in C ut BC continuatio datæ BC æqualis sit, & ista basi sub altitudine FH seu BK comprehendatur BKIC, & diagonius IB occurrat HF continuatæ in G, & compleatur parallelogrammum BCED basi BC & altitudine BD, tum complemẽta BDEC, BKHF æquabuntur: ideoque area BKHF divisa per datam BC dabit latitudinem ortivam BD quæsitam.

Vt usum nostrorum tum theorematũ, tum problematum illustrionem efficiamus aliquot problemata præterea ad hũc numerum aggregavimus.

PROBLEMA 26.

Quadratum dati rectilinei multiplum construere.



Perfacile id quidem futurum est, si problemata nostra de dati rectilinei in triangulum parallelogrammumve permitatione cum ultima prop. lib. 2 *Euc.* conjungas. Proponatur enim sexangulum ABCDEF cujus latus summum CD basi AF parallelum sit, hoc ut in quadratum transmutetur, reducito sexangulum in trapezium AGHF, cumque ex thesi opposita latera sint parallela, inter altitudinem CI & semissem oppositorum laterum HG & AF media proportionalis sit KF unde descriptum quadratum FKLM dato sexangulo, æquabitur rursum quadratum FL sive á linea FQ ipsi æquali descriptum quadratum.

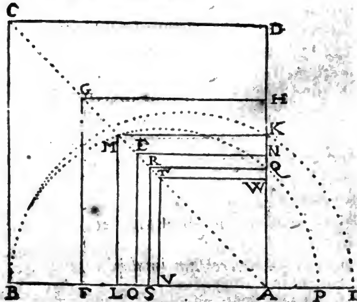
FQ PO ejusdem erit duplum, tum rectæ subtēsæ ab M ad Q ponatur æqualis FR, quadratum hinc descriptum FRST erit triplum ejusdem, hinc quadratum ab FP esset ejusdem quadrupulum, ab OR quintuplum, ab FS sextuplum.

In secundo diagrammate exhibetur dato quinquangulo B æquale triangulū XWV, cujus altitudo W 2, inter hujus semisſē & basin XV media proportionalis est XY latus quadrati XYZI dato quinquangulo æqualis, cujus duplum constructū est aliud X 43. cujus demonstratio facillima est antecedentium nō oblito.

PROPOSITIO. 27.

Dati quadrati quadratum submultiplum construere.

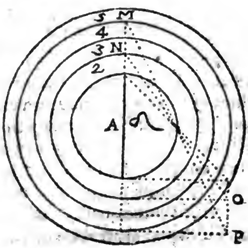
Sive invenire quadratum continens dati quadrati $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$.



Datū esto quadratū ADCD, bisecetur diagonius AC in E, latus ei æquale esto AH, quadratū ab ea descriptū AHGF erit dati quadrati dimidium. si $\frac{1}{3}$ postulabis inter totam BA & ejus trientem AI sit media proportionalis AK, quadratum ab ea descriptum erit dati $\frac{1}{3}$. atque ita deinceps in cæteris modo consimili ANEO $\frac{1}{4}$, AQRS $\frac{1}{5}$ atq; ita in cæteris modo consimili. si tamen aliquam partem postules totius, quæ simul sit alterius jam constructi dimidium, ut si totius $\frac{1}{6}$ exigam, quia jam $\frac{1}{3}$ totius exhibita est in quadrato AKMI., bisecato hujus diagonium AM ejusque semissi æquale statuatur latus AW, unde quadratum descriptum Aw TV erit hujus trientis dimidium, itaque totius $\frac{1}{6}$, quemadmodum petebatur.

PROBLEMA 28.

Circulum dati circuli multiplum describere.



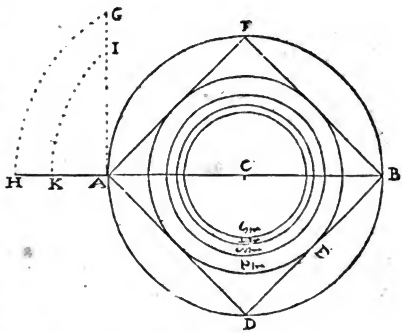
Latus quadrati circulo inscripti est radius circuli prioris dupli. Ita in subjecto diagrammate radius secundi circuli est æqualis lateri quadrati primo inscripti, huic in termino diametri secundi excitetur perpendicularis æqualis radio primi, recta á centro ejus verticem cōnectens est radius tertij circuli qui sit triplus primi. Et si radius in terminos hujus tertij perpendicu-

laris

laris constituatur is radius erit quarti, qui primi sit quadruplus. Idem per diametros quoque peragetur, si diametrum primi perpendicularem statuas in extrema diametro secundi, recta reliquum terminum cum hujus vertice connectens. ON erit erit diameter circulij tertij, primi triplum comprehendentis: atque ita porró. Nam inde res tota dependet, quod figuræ similes homologorum laterum rationem habeant duplicatam, sive quam quadrata ab homologis lateribus. in circulis autem diameter & peripheriæ sunt latera homologa: ea propter circuli inter se erunt ut quadrata diametrorum, aut peripheriarum inter se: quod alioquin é 2 propos. lib. 12 *Euc.* derivari & assumi potuit.

PROBLEMA 29.

Circulum dati circuli submultiplum describere.



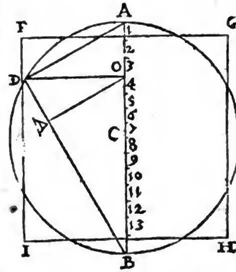
Exponatur circulus super diametro AB, cētrūq; C huic circulo pro data ratione postulatur alius submultiplus $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ aut quamlibet aliam particulam continens. si $\frac{1}{2}$ postuletur inscribatur ei quadratum AIBD, hujus latus BI erit diameter circuli dimidij, si $\frac{1}{3}$ petatur diameter AB in partes tres concidatur, linea inter hunc trientem & totam AB proportione media, ut AG, est diameter circuli $\frac{2}{3}$ prioris complexi. si AB continuetur quinta sui parte in K recta AI inter KA & AB proportione media est diameter circuli subquintupli. atque ita deinceps secundum datam quamlibet rationem circulos eadem tibi via construere licebit.

PROBLEMA 30.

Dato circulo æquale quadratum construere.

 Problema

Problema ſublime & arduum , quodque plurimos exercuit, neque á quoquam hactenus apodictice & ſecundum artis præcepta per circinū & regulam explicari potuerit: ſunt tamē á quibuſdam modi inventi qui á vero haud ita longe abeant diverſi, quorum aliquot hic referre lectori ſtudioſo haud ingratum fore arbitror. Inter quos modus Archimedeus primū obtineat locum, is enim rationem perimetri ad diametrum ad iſtos minimos numeros revocavit ut diametri $3\frac{1}{7}$ majores ſint ipſius circuli ambitu, & diametri $3\frac{10}{71}$ minores, quod cum demonſtratione & veritate plané conſentit.



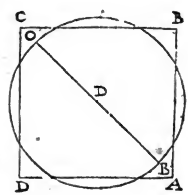
Vt itaque majorem Archimedeorum terminorum rationem ſecuti quadratum circulo æquale conſtruas diametrum ejus in 14 æquas partes ſecato, ut AB ſectam vides, & in tertij termino O perpēdicularem OD excitato, cujus verticem D cum reliquo diametri termino B connectat DB, quadratū ab iſta DB deſcriptum datum circulum quam proximé æquabit.

Vel quod idem eſt, Media proportionalis inter totam diametrum & $\frac{11}{14}$ eiuſdē eſt latus quadrati dato circulo ut proximé ęqualis.

PROBLEMA 31.

Dato quadrato circulum æqualem deſcribere.

Et in iſto quoque problemate Archimedeam rationem ſequemur. Dato quadrato ABCD circulus æqualis deſcribendus eſto. Hic primum per antecedēs problema quadratum dato circulo ęquale conſtrues, atque in illo diagrammate latus quęſitū eſto BD, tūq; huic dati quadrati lateri æqualis illic ponatur AB recta ex A puncto perpendicularis occurrat diametro in O ajo. EO diametrum eſſe circuli dato quadrato æqualis, quod ex analogia problematis prēmiſsi demōſtrari facile poteſt.



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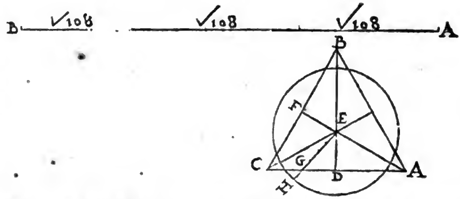
PROBLEMA 32.

Peripheriam datæ rectæ æqualem constituere



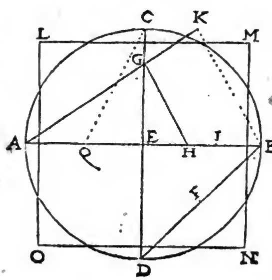
Data esto linea AB in peripheriam ipsi æqualem inflectenda. secretur data recta in tres partes æquales, é quibus triangulũ æqui laterum construatur ABC, cujus centrũm E est in concursu perpendicularium AF BD, tumque CD bisecetur in G recta EG sui quadrante continuata in H erit radius circuli dicto triangulo æquilatero isoperimetri, namque et ista per constructionem expositæ lineæ AB equalis est.

Hinc ut quadratum eidem circulo æquale describas, invenito mediam proportionalem inter radium circuli EH & dimidium datæ lineæ *AB*, ea erit latus quadrati dato circulo æqualis.

ἐπιχείρημα hoc est inventum doctissimi Cardinalis Cusani, et belle cum numerorum δοκιμασία consentit, quos post paulo arbitros hujus rei capiemus, nam licet quamvis non ad amussim cum ijs congruat tamen intra terminos ab Archimede præstitutos intercidit.

PROBLEMA 33.

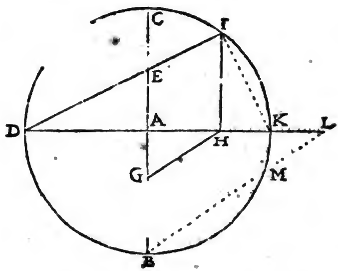
Circulum quadrare ex invento Vietæo.



Neque vero sic silentio nobis transeundũ fuit subtilissimum Magni vietæ ἐπιχείρημα, quod responsorum octavo retulit: longe enim propius ad veritatem alludit, quam utraque præcedentium. Circulus sub E centro descriptus quadrifariam secetur à duabus diametris.

AB

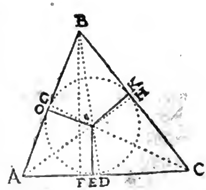
AB DC sese perpendiculariter intersecantibus, & ponatur EG æqualis dimidiæ inscriptæ DB, & agatur infinita AG: secetur autem radius EB per 11 propos. 2. lib. *Euc.* secundũ mediã & extremam rationem in H, & sit EH minus segmẽtum: hinc fiat ductis parallelis, ut AH ad AG, sic AB ad AK, hæc quarta proportionalis erit latus quadrati circulo ut proximé æqualis: quale hic In LMNO videre est.



Sed idem quoque peripheriã in directum exporrigere haud minori factionis concinnitate docuit. Si enim circulus á duabus normalibus diametris quadrifariam dividatur, & radius AB secundum mediam & extremã rationem secetur in G, ut BG majus segmentum sit radij proportionaliter secti: & recta á termino diametri D per E medium radij AC continuata occurrat peripheriæ in I, unde perpendicularis demittatur IH & connectatur GH, BL contra GH parallela intercipiet AL rectam æqualem quadranti peripheriæ datæ BDCK. Itaque media proportionalis inter diametrum DK & ipsam AL erit latus quadrati dato circulo æqualis.

PROBLEMA 34.

Invenire radium circuli in datum triangulum inscripti.



Datum esto triangulum datorum laterum AC 14, AB 13, BC 15, in quod inscriptus sit circulus E GK quæritur ejus radij longitudo. problematis hujus solutio é 55 & 61 nostra propositione libri secundi repetenda est, & habet factionem valde expeditam. Nam cum rectæ ab angulis per circuli centrum eductæ BD CG & AH eosdem bisecent, etiam latera ipsis subtensa secabunt ratione crurũ. itaque segmenta AD DC proportionalia sunt cruribus AB BC. atque ex hac analogia deprehendes AD $6\frac{1}{2}$, DC $7\frac{1}{2}$. Cumque triangulum GOA triangulo AOE, & EOC ipsi KOC, itemque KOB ipsi GOB æquilaterum sit, consequens est GA & KC simul æquari basi AC: quare GA AC CK simul erunt partium 28, quæ de omnium laterum summa 42 deductæ relinquunt 14 pro quantitate linearum GB & BK, quarum singulæ propterea erunt partium 7. atque ideo KC seu EC 8, unde subducta DC $7\frac{1}{2}$ relinquam facit DE $\frac{1}{2}$, jam per 84 propositionem libri secundi inventa esto perpendicularis BF 12, AF 5,

AFS, FC 9. hinc EC 8 subducta de FC dabit EF 1, quę addita ad ED $\frac{1}{2}$ dabit FD 1$\frac{1}{2}$: vnde proportio quemadmodum DF 1$\frac{1}{2}$ ad FB 12, sic ED $\frac{1}{2}$ ad radium EO 4: qui duplicatus dabit integram diametrum partium 8. Radius circuli est altitudo trium triangulorum verticibus in centro coeuntium & ipsa trianguli latera pro basibus habentium, quamobrem semisse omnium laterum dati trianguli 21 per radium 4 multiplicato, dabitur area trianguli ABC 84.

Longè facilius unica proportione concludes hoc modo.

Vt trianguli perimeter ad basin, sic perpendicularis in eam demissa ad radium circuli inscripti.

Namque rectangulum à base & perpendiculari comprehensum vel à radio inscripti circuli & toto ambitu sunt dupla dati trianguli. quare inter se aequalia: ideoque etiam cruribus reciproce proportionalia. erit itaq; in exposito paradigmate, ut 42 perimeter ABC ad basin AC 14, sic perpendicularis BF 12 ad radium OE 4.

Vel, si aream trianguli per ploblema sequens invenias, eamque ad dimidiam perimetrũ applices quotus erit radius optatus. area per sequens problema invenietur 84, hac per dimidiam dati trianguli perimetrum divisa dabit in quoto radium optatum 4. demonstratio quoque inde patet, quia rectangulum sub dimidia perimetro & radio areae triangulari aequale sit.

Est & tertia mihi via, in qua neque perpendiculari neque area trianguli in consilium adhibita, tantũ unica proportione radij quantitatem concludo: sed & hoc, & alia hujus generis complura nostra data occasione in lucem & utilitatem philomathũv aliquando proferemus.

Ad id quod sequitur Lemma 1.

Si quatuor rectae sint proportionales rectangulum extremarum aequatur rectangulo mediarum.

In enim in elementis demonstratum est, & in numeris quoque habet facilẽ demonstrationem.

Lemma 2.

Si quatuor numeri sint proportionales factus mediorum in primum aequatur facto extremorum in eundem primum multiplicato.

Sunto quatuor proportionales 3 4 6 8, factus ab extremis 3 & 8 est 24, factus á medijs 4 & 6 per antecedes lemma erit itidem 24: quare uterque per eundem scilicet primum 3 multiplicatus dabit 72. Idem vero est sive factum ab extremis per primum, sive primi quadratum per extremum multiplices. numeri enim ijdem, quocunque ordine inter se multiplicentur facient eundem.

Lemma 3.

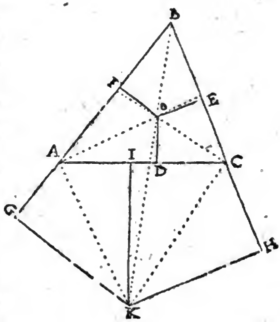
Latus facti á duobus quadratis aequatur facto laterum.

Vt 49 & 9 quadrati faciunt 441 quadratum, cujus latus 21 æquatur facto á lateribus 7 & 3.

PROBLEMA 35.

Datis lateribus trianguli ejus aream invenire.

Quod theorematice ita concipere & enuntiare licuit. *Si de dimidio collectorum laterum dati trianguli latera sigillatim subducantur, latus continuè facti è dimidio & reliquis est area trianguli.*



Proponatur triangulum ABC cujus latera AB 15 BC 13 AC 14 partium sint, summa collectorum laterum erit 42, dimidiū 21, hinc latera singula sigillatim subducta relinquunt 6, 8, 7, numerus ab his reliquis & dimidio continua multiplicatione factus 7056, unde latus erutum dabit 84 aream dati trianguli. Cujus factionis veritas ita demonstratur. Bisectis enim angulis A & C, in linearū bisecantium concursu O erit centrum inscripti circuli, unde perpendiculares OD OE OF radii erūt æquales per 34 problema, itemq; laterum segmenta cōtermina EC & CD, & AF AD: tum continuentur latera BA BC alternis basis segmentis, AG æqualiter ipsi CD, atque CH ipsi AD, ab eorum terminis G & H lateribus perpendiculares excitatæ concurrant in K, unde ad terminos basis agantur rectæ KA KC, tum AI statuatur æqualis segmento DC, & ducatur KI, denique connectatur BK, quæ utrumque angulum ad B & K bisecabit (quia triangula BHK BGK æquilatera sint, cum duo latera GB BK, HB BK æqualia & angulum ad G & H rectum habeant) atque ideo per centrū O transibit. Atque ita continuatum latus BG omnium laterum semissi æquabitur, namque BF ipsi BE, FA ipsi AD, AG ipsi DC sive EC æqualis est: non secus quoq; continuatum BH omnium laterum semissem esse constabit. Sed cū KG & KH perpēdiculares æquales sint, & CH major quā AG, efficitur quadratū CK quoq; majus esse quadrato AK pro differentia quadratorū CH & AG, sive ipsis æqualiū CI AI: atqui cū differentia quadratorum é basis segmentis ęquatur differentiæ quadratorum á cruribus, tū recta à vertice in id punctū perpendicularis cadit, eritque ipsi KG æqualis, quia IA & AG æquātur. Ideoque angulus IAK angulo GAK æqualis, totusq; GAI erit bisectus: sed & BAC bisectus est: quare OAK rectus est, & reliqui FAO GAK recto æquales: atqui FAO FOA uni recto quoque æquantur:

igitur

igitur FOA GAK inter se æquabũtur, & triangula FAO AGK æquiangula atq; similia erũt: ideoq; ut OF ad FA, sic AG ad GK: quare per 1 lemma rectangulum sub FA & AG æquatur rectãgulo extremorũ FO & GK. ergo ut quadratũ FO ad AG in AF, sic FO ad GK, & propter earum parallelismum sic quoque erit BF ad BG. quamobrem ut 16 quadratum ab FO ad 48 factum ab FA & AG, ita FB 7 ad FG 21 laterum omnium semissem: & inverse ut 21 ad 7, sic 48 ad 16 quadratum FO. verum per lemma secundum quadratũ primi 21 videlicet 441 in ultimũ 16, qui est quadratis ab OF, æquatur facto á medijs FB 7 & 48 (ab FA & AG comprehenso) in primum 21 multiplicato, hoc est numerus á tribus reliquis 6, 7, 8 & omnium laterum semisse continué factus, æquatur quadrato semisis ejusdem 441 in quadratum radij FO 16 multiplicato: atqui per tertium lemma latus facti á duobus quadratis, æquatur facto ab ipsorum lateribus, quare si de collectorum laterum dimidio latera singula sigillatim subducantur, latus numeri continue facti é dimidio & reliquis erit area dati trianguli.

Quotquot hanc damonstrationem è Iordano aut Tartalea descripserunt omnes eandem insistunt viam, & pernescio quam planoplanorum stereometriam lectorem circumducunt: longe aliter quam veteres illi mathematici, qui studiose declinabant illam μετάβασιν εἰς ἄλλο γένος, eamque vel applicatione ad communem altitudinem tollunt, vel proportione dissolvunt, vel rationum compositione interpretantur, quemadmodum apud Archimedem Apollonium aliosque sæpe videre est. Sed istam ἀλογίαν Petrus Ramus (quem hominum λογικώτατον vocat. vir subtilissimus Franciscus Vieta quondam libellorum supplicum regius magister) primus notavit. cujus gratia olim hanc obscuritatem & caliginem discussimus & plana tractavimus plane, hoc est in suam classem & ordinem reduximus; quam ad rem hoc lemmatio prævio nobis opus erit.

Lemma.

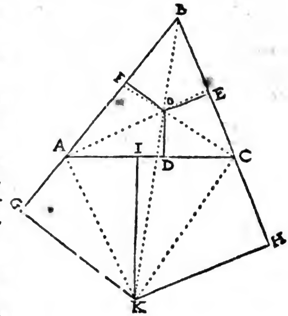
Si quatuor rectæ sint proportionales tria rectangula sub prima & secunda, secunda & tertia, tertia & quarta erunt continue proportionalia.

Sunto proportionales 3, 6, 4, 8 erit itaque ut 3 ad 6, sic rectangulum sub 3 & 6, ad rectangulum sub 4 & 6; & ut 4 ad 8, sic rectangulum sub 6 in 4 ad 4 in 8: quare ex æquo ut 3 in 6 ad 6 in 4, sic 6 in 4 ad 4 in 8: videlicet ut 12 ad 24 sic 24 ad 48.

Theorema porrò ipsum ad areæ triangularis investigationem comparatum quod non Geometrica enuntiatione, sed δεδομένῳ formula hactenus circumfertur, nunc illud reformatum benè Geometricè ita concipio & enuntio.

Si de dimidio collectorum laterum dati trianguli latera sigillatim subducantur erit ut rectangulum sub dimidio & differentia quacunque ad aream trianguli, sic eadem area ad rectangulum sub reliquis differentijs comprehensum.

Exponatur



Exponatur enim triangulum ABC, & ab O inscripti circuli centro radij perpendiculares in latera sunto OF OD OE, segmenta itaque FB BE, FA AD, DC CE æquabuntur: & ab eodem centro rectæ OA OB OC ad angulorum apices eductæ eosdem bisecabũt: tumque anguli sub basin quoq; bisecentur à rectis AK KC in puncto K cõcurrentibus, & jungatur OK: inde perpendiculares GK IK ex K demissæ æquabuntur, ob crus commune AK & angulos ad A æquales quibus subtenduntur, & reliquum latus AG reliquo AI quoque æquale erit. simili ratione IK KH, & IC CH æquari demonstrabis: quare GB & BH trianguli perimetro quidem, sed & ipsa inter se quoque æquantur, quia triangula GBK HBK æquilatera sunt: est enim GK ipsi KH æqualis, quoniam eidem KI æquantur: & KOB commune crus in directum exporrigitur. Cum enim externus trianguli angulus GAC utrique interno & opposito ad B & C æqualis sit, dimidius KAC, hoc est ipsi æqualis KOC (nã quia in quadrangulo AOCK oppositi anguli KAO KCO singuli duorum rectorum dimidij, hoc est recti sunt, quatuor puncta AKCO erunt in circulo) illorum semisibus OBC OCB quoque æquabitur; atque ideo cum tertio angulo BOC duos rectos æquabit: quare KO OB in directũ ex oporriguntur. Et triangula KBG KBH latus KB commune, KG KH æqualia, & angulos ad G & H rectos habentia erunt æquilatera: propterea que GB BH latera æqualia dimidiam trianguli perimetrum æquabunt. Porrò autem cum angulus OAK rectus sit, & reliqui duo GAK FAO, itemq; FAO & FOA acuti in triangulo rectangulo uni recto quoque æquentur, subducto communi FAO, reliquus reliquo æqualis erit, & triangula rectangula FAO AGK ideo æquiangula & similia: quare latera KG GA AF FO proportionalia, & rectangulum sub extremis KG FO rectangulo mediorum GA in AF æquale erit. Sed cum FO GK perpendiculares parallelæ sint, erit quoque ut BF ad BG, sic OF ad GK: itaque per lemma præmissum, ut rectangulum sub BF in BG ad BG in FO, sic BG in FO, ad FO in GK, hoc est ut jam demonstravamus ad GA in AF; est autem rectangulum sub BG semisse omnium laterum in FO radium inscripti circuli æquale areæ trianguli: quare ea proportione media est inter dicta rectangula, quod demonstrasse oportuit.

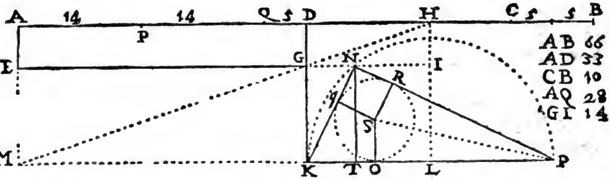
Cum ex hac demonstratione pateat rectangulum sub dimidio BG in differentiam BF, aream trianguli ABC, & rectangulum sub reliquis differentijs GA in AF esse continue proportionales, logistæ statim conceperunt factum extremorum æquari quadrato numeri medij, atque inde exiitit ille numerus quatuor numerorum continua multiplicatione factus, quod ut ab Arithmetico bene conceditur ita à Geometra jure repudiatur. habemus alia consectaria ex eodem hoc fonte nostræ demonstrationis derivata, qua occasionem oportunam nacti benevolo lectori non invidebimus, interim istud jam ex adversarijs nostris deprompsisse sufficiat. Potuit verò ipsum theorema ita quoque enunciari.

Si de

Si de dimidio collectorum laterum dati trianguli latera sigillatim subducantur, rectangulum comprehensum sub media proportionali inter dimidium & differentiam quamlibet, itemque sub media proportionali inter reliquas differentias æquatur dato triangulo sed. tũ præmissum à nobis lemma paulo aliter quoque concipi debuit.

PROBLEMA 36.

Dati trianguli rectangula perimetro & inscripti circulo radio ipsum triangulum invenire.



E data AB triangulum rectangulum esto construendum in quod circulus inscribi possit sub diametro CB. Haud ardua erit hujus solutio per ea quæ ad 55 propositionem libri 2 annotavimus. Rectangulum enim ADGE sub radio seu dimidia CB, & AD dimidia permetro comprehensum areæ dati trianguli ęquatur, diameter autem tota CB de perimetro AB subducta reliquam facit AC, cujus dimidium AQ æquatur quæsiti trianguli basi recto subtensæ, ad cujus semissem DH applicatũ parallelogrammum, ADGE exhibet latitudinem ortiuam GK perpendicular[i] ab angulo recto in dictam basin ęqualem: Et quia perpendicularis ea proportionalis est inter segmenta basis, statuatur GK seu NT proportionalis inter KT TP segmenta basis KP, seu AQ. rectæ KN NP á vertice N cum terminis basis K & P connexæ comprehendent in semicirculo KPN trianlum cujus perimeter datæ rectæ AB æqualis dati inscribendi circuli capax sit. Demonstratio ex constructione perspicitur. Itaque numeris quoque idem comprobare constitui.

Per 43 propos. lib. 1. *Euclid.* complementa AEGD GILK æquantur, ideoque ut GI 14 ad EG 33, sic GD 5 ad GK $11\frac{11}{14}$, ac tãta quoque est perpendicularis NT, factus ab $11\frac{11}{14}$ & dimidia basi 14 dabit aream trianguli 165. Hinc per 5 prop lib. 2 *Euclid.* inveniantur segmenta basis KT $14 - \sqrt{57\frac{19}{196}}$, TP $14 + \sqrt{57\frac{19}{196}}$, inde dabuntur crura recti KN $19 - \sqrt{31}$, NP $19 + \sqrt{31}$. facti ab istis KN NP cõprehensi 330 dimidium 165 æquatur areæ constructi trian guli: & basis KP 28, differentia á summa crurum KN & NP 38 erit 10, cujus dimidium 5 pro NR vel SR radio inscripti circuli, quemadmodum postulabatur.

Quod si collibitum sit eandem aream secundum superioris problematis modum quoque invenire, licebit hoc modo. Latera in unã summam collecta 66

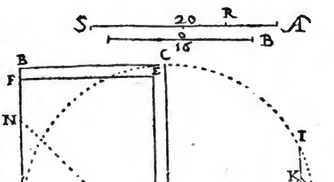
 ejus

ejus dimidium 33, unde ipsa sigillatim subducta relinquunt tres differentias 5, $14+\sqrt{31}$. $14-\sqrt{31}$. numeri & á dimidio & his tribus continué facti latus dabit aream trianguli 165, ut supra.

Quia lectori non tam obvia forsan demonstratio futura, atque autori nostro videtur, demonstrationis filum ad istam structuram ita pertexam. Data recta AB, diameter inscribendi circuli CB, differentia earum CA, dimidium AQ. Et fiat ut ½ AQ ad AD, sic QD ad GK: tum ipsi AQ æqualis statuatur KP, inter cujus segmenta KT TP proportione sit media KG, seu NT: rectæ KN NP connexæ angulum rectum comprehendent, quia in semicirculo, & latera tria KNP datæ AB æqualia erũt. Inscribatur enim circulus & è centro S perpendiculares sunto SQ SR. Hic cum ex fabrica sit ut ½ AQ ad AD hoc est ut KP ad AB, sic QD ad NT, atqui ut KPN ambitus ad basin KP, sic perpendicularis NT ad radium RS seu (per 55 datam, sic QD ad NR prop. lib. hujus 2) ipsi æqualem NR: quare ex æquatione perturbata, ut ambitus KPN ad AB & sumptis primi ac secundi dimidijs ut KP & NR ad AD, hoc est KP & QD, sic QD ad NR. quare rectangulum sub KP cum NR in NR, æquatur rectangulo sub KP cum QD in QD atque ideo NR & QD æquantur: nam si QD major esset alterũ rectãgulum illius QD in KP plus QD majus esset altero, si minor minus. quare cum dimidius ambitus KP plus NP æquetur dimidiæ lineæ AD, totus toti æquabitur: quod demonstrasse oportuit.

PROBLEMA 37.

Datam rectam ita secare ut quadrata segmentorum dato quadrato æqualia sint.



Data linea recta AS ita secanda sit in R ut SR & RA æque possint datę B. Ex data B construatur ABCD. & AO AN segmenta dimidia sunto datæ AS quorum terminos connectat NO, quo latere describatur quadratum AF EG, reliquus gnomo BCDGEF erit differentia quadratorum, hinc DH sit æqualis lateri AG, & HP perpendicularis ęqualis ipsi GD, quare rectangulum AHPQ dicto gnomoni ęquabitur, media proportionalis inter ejus latera AH HP seu HM, sit HI bisecta in K, & construatur triangulum æquicrurum KHL, subtensa KL addita & subducta dimidio datæ AS dabit segmenta optata SR RA, quarum potentia datæ B quadrato æqualis sit.

Exposita AS sit partium 20, & B 16, cum AO & AN æquentur recta angulo recto subtensa NO cui AG æquatur, erit $\sqrt{200}$, hæc de AD 16 subducta reliquam facit DG seu FB vel HM $16-\sqrt{200}$: addita autem dabit totam AH

$16+\sqrt{200}$

$16 + \sqrt{200}$; rectangulum itaque ab AH & PH vel HM comprehensum erit 56 æquale quadrato HI, ipsaque HI $\sqrt{56}$, dimidium HK $\sqrt{14}$; & quia HKL triangulum rectangulum æquicrurum sit dabitur KL $\sqrt{28}$, quę ad dimidiam AS addita & eadem subducta dabit segmenta SR $10 + \sqrt{28}$, AR $10 - \sqrt{28}$. horum segmentorum quadrata sunt $128 + \sqrt{11200}$ & $128 - \sqrt{11200}$, quorum summa 256 æquatur quadrato lineæ B 16; quare factum est quod oportuit.

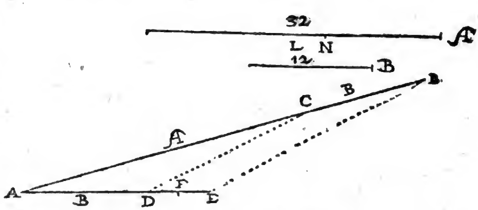
Vt lectorem benevolum hoc quoque προσθηκιδίῳ demerear, nō quidem illam autoris in hac fabrica vestigia legendo, sed ad fontem ipsum digitum intendendo. Est inquam hoc problema re ipsa idem cum antecedente, sed alia formula propositum. saltem ad illud commodissimè revocari potest.

Datur enim hic B basis anguli recti, cujus crurum summa sit ◡AS. quamobrem subducta base B de ◡AS relinquetur diameter circuli in dicto triangulo inscribendi, quemadmodum illic docuit: quare & cætera illi similia erunt: sequere fabricam omnia consentire deprehendes. sed ut leviter defungar: exhibet nobis autor segmenta SR $10 + \sqrt{21}$ RA $10 - \sqrt{28}$, rectangulum sub ipsis comprehensum 72 est duplum area trianguli, cujus SR RA & B latera sint, dimidium 36 ipsa area, ea per dimidiam laterum summam 18 divisa dabit 2 in quoto pro radio circuli inscripti. ac tanta quoque est dimidia differentia inter basin B 16 & crurū summam ◡AS 20.

Potui aliam etiam concinniorem & parabiliorem factionem afferre, si per operas typographicas & diagrammatum sculptorem liberum fuisset: quamobrem in tempus magis oportunum & hanc, & alias differre cogimur.

PROBLEMA 38.

Datam rectam dividere in duo segmenta, ut minus segmentum cum alia data æque possint segmento majori.



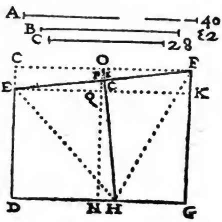
Proponitur recta AM ita secanda in N ut quadratum minoris segmenti AN cū quadrato alterius datæ B, æquentur quadrato reliqui segmenti MN. datis duabus A & *B* inveniatur tertia proportionalis DE, cujus semissis FE ad dimidiam A, hoc est AL addita & ab eadem subducta dabit MN majus segmentum optatum & NA minus. Hujus veritatem in numeris hoc modo licet experiri, ut data A 32 ad B 12, ita *B* 12 ad DE $4\frac{1}{2}$, cujus dimidium $2\frac{1}{4}$ DF ad dimidiam *A* 16 additum dabit MN $18\frac{1}{4}$, & ab eadem subductum dabit N*A* $13\frac{3}{4}$. Hic enim quadratum N*A* $189\frac{1}{16}$ cum

cum quadrato lineę B 144 æquant quadratum majoris segmenti MN $333\frac{1}{11}$: quod fuit propositum.

Quia autoris diagramma demonstrationi haud satis accommodatum est, necesse erit pro eo aliud animo effingamus. Postulatur enim hic triãgulum rectangulum cujus crus unum circa angulum rectum sit data B, alterum autem segmentũ minus ipsius A.M, & basis majus. concipito igitur circulum quemcunque. quem tangat data B, & inde in ejusdem concavum pertingat data A, ejus segmentum circulo inscriptum dabit segmẽtum optatum minus, reliquum majus, demonstratio è tali diagrammate cuilibet erit obvia.

PROBLEMA 39

Datã rectam ita secare ut quadrata segmentorum datis quadratis aucta inter se æquentur.

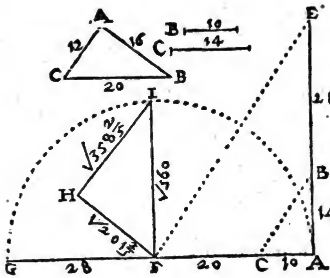


Data esto A in duas partes ita secanda ut quadratum segmenti unius cum quadrato datæ B æquetur quadrato segmenti reliqui cum quadrato C. expositę A æqualis statuatur DG, á cujus terminis perpendiculares sunto DE GF, quarum vertices conjungat EF, á cujus medio I perpendicularis IH secabit datam DG in H: ajo segmentum GH & datam GF æque posse segmento DH & datæ DE: cũ enim EC CH, FC CH æquales sint & CH perpendicularis, EH & FH angulis ad C æqualibus & æquicruris subtensi æquales erunt. sed ED & DH æque possunt ipsi EH: & FG & GH ipsi FH, quare quadrata ab ED & DH quadratis FG & GH erunt æqualia. & propterea DG secta est in H quemadmodum petebatur. Hoc quantumvis Geometrica factione expeditum sit, in numeris tamen paulo difficilius est. continuetur igitur DE in C ut DC GF æquales sint, & connectatur CF: cui HC perpendicularis continuata occurrat in O atque inde perpendicularis sit ON in subjectam DG: hinc jam ad numerorum investigationem expedita est via. triangula enim EFK EPQ similia sunt quia PQ & FK parallelæ: sed, & EPQ & POI itidem similia, quia angulos ad verticem P æquales, ad I & Q rectos habent: & ONH ipsi OPI ob communem angulum ad O simile est: quare EFK ONH triangula quoque erunt similia. Porró autem quadrata EK 1600 & FK 16 addita dabunt EF subtensam $\sqrt{1616}$, hujus dimidiũ FC; $\sqrt{404}$. unde proportio ut EK 40 ad EF $\sqrt{1616}$, sic FI $\sqrt{404}$ ad FO seu ipsi ęqualem NG $20\frac{1}{5}$: Et rursum ut EK 40 ad KF 4 sic ON 32 ad NH $3\frac{1}{5}$: subducta itaque NH $3\frac{1}{5}$ de NG $20\frac{1}{5}$ erit reliqua HG 17: quare & HD 23.

PRO-

Problema 40.

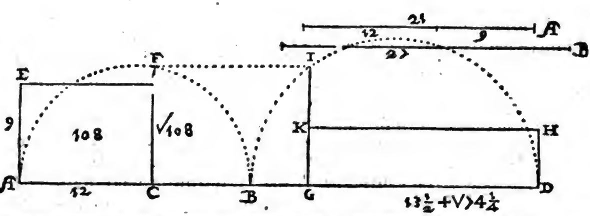
Dato triangulo triangulum simile in data ratione construere.



Exponatur triangulum ABC cui aliud optatur simile in ratione data B ad C. fiat ut B ad C, sic basis trianguli CB ad BE cui æqualis ponatur FG. tum inter FG & basin trianguli CB sive ei æqualem FC inveniatur media proportionalis FI. cujus demonstratio é 12 & 19 propos. lib. 6. *Euclid.* facile derivari potest. Namque ut prima trium continué proportionalium CF ad ultimam FG, sic figura super primam FC seu CB descripta ad figuram similem super homologo latere FI descriptam.

Si numeris idem comprobare postules, invenito aream dati trianguli HFI per problema huius libri 35. Summa laterum $\sqrt{3225\frac{1}{7}}$: dimidum $\sqrt{806\frac{2}{7}}$ inde singula latera deducta relinquunt $\sqrt{22\frac{2}{7}}$ $\sqrt{208\frac{1}{7}}$ & $\sqrt{89\frac{1}{4}}$, numerus ab his reliquis & dimidio continué factus est [illegible], hujus latus $134\frac{2}{7}$ exhibet areā trianguli IFH, & ABC 96; atqui ut B 10 ad C 14, sic 96 se habet ad $134\frac{2}{7}$. Quare factū est quod oportuit. Hoc autem casu quia auguli A & H recti sunt, facilore via tete expedire potuisti.

Problema 41.



Datas duas rectas ita secare ut rectangulum sub segmentis unius aquale sit rectangulo sub segmentis reliqua.

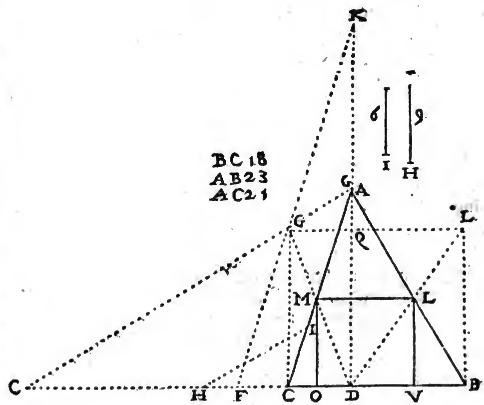
Datæ sunto AB BD; secato minorem AB pro libitu in C, linea FC inter hæc

 segmenta

ſegmenta proportione media, quoque proportione media ſtatuatur inter ſegmẽta ſecundæ BD, qualis hic eſt GI: recta enim per verticem F contra ſubjectam AB parallela occurrat ſecundo ſemicirculo in I, unde demiſſa perpendicularis IG exhibebit optata ſegmenta BG GD. rectangulum enim ſub ipſis comprehensũ DGKH rectangulo AELC æquabitur.

PROBLEMA 42.

Dato triangulo parallelogrammum inſcribere ſub laterum ratione data.



Proponatur triangulum ABC in quod inſcribendũ ſit parallelogrammam OMLV, cujus laterum ratio ſit eadem quæ H ad I. fa: igitur ut H ad I, ſeu HC ad CI, ſic baſis CB ſive ei æqualis CH ad IG; & cõſtruatur parallelogrã-mum CGLB, atq; á G & L ad D locũ perpẽdicularis ex A vertice demiſſæ connectantur rectæ GD DL, hinc ab M & L ſectionum punctis demiſſæ perpendiculares MO LN comprehenden OM LN rectangulum optatum. Hic perſpicuum eſt rectangulum CGLB rectangulo NL MO ſimile eſſe, quia á rectis GD DL in ſimilia ſegmenta dividuntur: quare ut GC ad BC (hoc eſt ut I ad H) ſic MO ad VO: poſſem veró idem etiam aliter demonſtrare, ſed per numeros hoc problema explicare longe videtur jucundiſſimum. agatur itaque per angulum parallelogrammi G linea FK parallela contra latus CG donec concurrat cum perpendiculari DA continuata in K: atque ideo triangula ADC KDF GCF erunt ſimilia; quamobrem ut AD √498 ad DC 6½, ſic GC 12 ad CF √15, huc addita CD dabit totam DF √15 + 6½, ſive quod idem ſit 6½ + √15. ſecundo ut DC 6½ ad AD √398, ſic DF 6½ + √15 ad DK √398 + 12. Tertio ut KD √398 + 12 ad GC 12, ſic AD 6½ ad latitudinẽ MO 18 -- √127. Deniq; ut DK √398 + 12 ad BC vel DG 18, ſic AD √398 ad longitudinẽ LM vel NO 28 — √287.

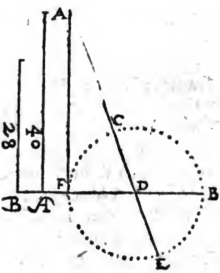
$\frac{3}{4}\frac{3}{4}\frac{7}{4}\frac{8}{4}\frac{2}{4}\frac{2}{3}$. Infra hujus generis exempla plura in medium afferemus.

Cum illud in sua demonstratione ἀναπόδικτον assumat autor perpendiculares MO & LN æquales esse, vel quod idem est parallelam ML occurrere lateri AB in communi sectione ejus & diagonij DC Haud ingratum lectori me facturum sum arbitratus si & fabricam & demonstrationem legitime pertexerem, hoc modo.

Data ratio laterum inscribendi parallelogrammi sit CB ad BL, datumque triangulum ACB. fiat igitur ut AD ad DQ, sic AP ad PD, & per punctum P sit ML parallela contra basin CB, & dimissis perpendicularibus MO, LN absolvatur parallelogrammum, MLNO. Ajo ejus latera habere rationem datam. Nam cum sit per fabricam ut AD ad DQ, sic AP ad PD, erit quoque sumpta illic communi altitudine BC, & hic ML, ut ad AD in BC ad DQ in BC, sic AP in ML ad PD in ML. & alternis, ut AD in BC ad AP in ML, sic DQ in BC ad PD in ML. Atqui AD in BC *est ad AP in ML ut triangulum* ABC *ad triangulum ALM, sed cum ista sint similia erunt ut quadrata à lateribus homologis ML & CB, quamobrem ex æquo etiam ipsa parallelogramma inter se erunt ut quadrata ab homologis lateribus, atque ideo in [illegible]. quamobrem erit ut* CB *ad* BL, *sic ML seu OV ad VL. Quemadmodum postulabatur.*

PROBLEMA 43.

Datis duabus rectis alterã earum continuare ut oblongum è continuata & continuatione æquetur quadrato reliquæ.



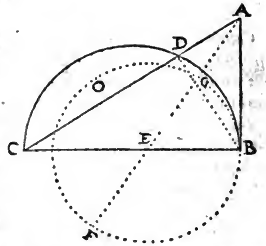
Data B ita continuanda esto. ut oblongum ex ipsa continuata & continuatione æquetur quadrato datæ A. Datæ A & B comprehẽdant angulum AFB, & FB ipsi B æqualis fiat diameter circuli CFEB datum AF in F termino contingentis recta AE ab A vertice per centrum educta dabit AC quæsitam, cujus veritas é 36 propos. 3 lib. *Euc.* perspicua est.

Numerorum veró abacus ita habet. quadratũ dimidiæ FB 196 cum quadrato AF 1600 dabunt 1796, hujus latus $\sqrt{1796}$ erit subtensa AD: unde subductus radius CD 14 relinquet continuationem optatam AC $\sqrt{1796}-14$. idem ad DE additus dabit totam continuatam AE $\sqrt{1796}+14$. Rectangulum ab his comprehensum æquale est quadrato AC 1600, quemadmodum oportuit.

PROBLEMA 44.

Dato trianguli rectanguli crure recti & basis segmento alterno ipsum triangulum invenire.

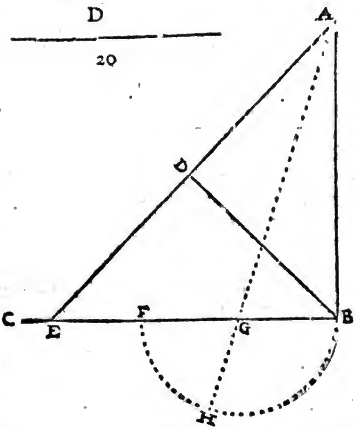
 Namque



Namque hic in triangulo rectãgulo CAB datur AB crus recti ad B, & cum basis AC dividatur á perpendiculari BD datur CD basis segmentum reliquo cruci conterminum.

Exponatur linea AB perpendicularis super infinitam BC, & invenienda sit diameter BC ut semicirculus super eam descriptus CDB á recta AC verticem perpendicularis A cum C diametri termino connectente absumat inscriptam CD æqualem datæ: sumatur itaque datæ semissis cui æqualis sit EB, eoque intervallo describatur peripheria FO GB, & ab A per ejus centrũ transfigatur AF, huic deinde æqualis statuatur AC. Ajo semicirculum super diametro CB descriptum absumere CD æqualem diametro GF. Nã cũ per antecedens problema rectãgulum CA in AD & FA in AG eidem quadrato ex AB æqualia sint, inter se quoque æqualia erunt, atque ideo latitudinem æqualem habebunt DA & GA: quare si ab æqualibus AC AF æquales lineæ detrahantur, reliquæ CD FG æquales erunt. quare factum est quod oportuit. Numeri quoque huic demonstrationi consentiunt. Sit enim AB 28 DC 40, erit linea AF vel ACB √1184+20, AD √1184—20, & CB √√1894400+800, BD √.√1894400--800. itaq; subducto quadrato BD de quadrato BC relinquitur quadratum DC 1600, & ipsa longitudo DC 40.

Neque alia ratio est problematis hujus.



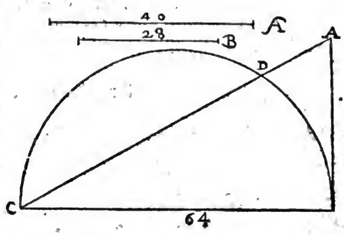
Datur *AB* perpendicularis in rectam BC, á cujus vertice A ad subjectam BC agenda sit recta AE, ut perpendicularis ex angulo B demissa absumat segmẽtum DE datæ cuicumque D æquale. Solutio jam in promptu est ex ipso problemate, ponatur enim BF æqualis datæ D, & super ea tãquam diametro describatur semicirculus BHF, & agatur recta per ejus centrum AH, cui çqualis ponatur AE. ajo perpendicularem ex angulo B in AE demissam absumere DE segmentum datæ lineæ æquale, quia EDB puncta in semicircu-

lo sunt

lo sunt, ut jam ante demonstratum fuit. Arithmeticus calculus talis est. sit AB 30 & D 20, quadrata igitur AB 900 & *BG* 100 addita dabunt quadratum AG 1000, & ipsam *A*G $\sqrt{1000}$, huc addita HG exhibet totam *A*H sive AE $\sqrt{1000} + 10$, unde subducta DE, quam datæ D æquari demonstravimus, relinquitur AD $\sqrt{1000} - 10$: hujus quadrati de quadrato AB subducti latus dabit DB $\sqrt{\sqrt{400000} - 200}$. Idem quadratum AB de quadrato AE deductum relinquet quadratum EB cujus latus pro EB exhibet $\sqrt{\sqrt{400000} + 200}$.

PROBLEMA 43.

Dato trianguli rectanguli crure recti & ratione quam habet segmentum ipsi contermi-num ad crus reliquum invenire basin.

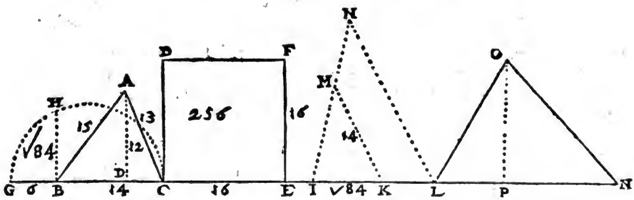


Hoc est in diagrammate proposito, Data diametro circuli CB á termino C agēda est recta CA, ut ratio inscriptę *C*D ad perpendicularem AB sit eadem quæ A ad B. Problema hoc ita solvetur. querēdus tibi est semicirculus á cujus diametri termino uno recta A inscripta & continuata abmat segmentum á recta reliquo termino perpendiculari æquale ipsi B. tumque per proportionem reliqua concludes, ut diameter invēta ad datā B*C*, sic inscripta ad inscriptam CD, ea continuata ob laterum proportionem absumet AB in ratione data. ut sit quemadmodum *A* ad B sic CD ad AB. Si per Arithmeticum abacum id experiri libeat posita CB 64, dataque ratione A 40 ad B 28, deprehendes CD $\sqrt{\sqrt{51708204\frac{596}{2401}} - 4179\frac{29}{49}}$, & A*B* $\sqrt{\sqrt{12415139\frac{21}{25}} - 2048}$.

PROBLEMA 44.

Triangulum dato quadrato quidem æquale & alteri triangulo dato simile construere.

 Propo-

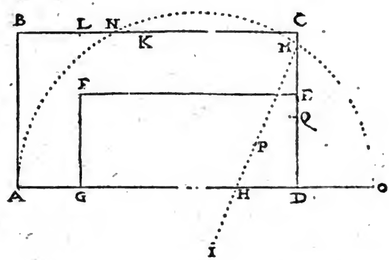


Proponatur triangulum ABC cui aliud quidem simile NOL sit describendū, verum hac lege ut ejus area dato quadrato CDFE ęquetur: constructio obvia est é 25 propos. 6. lib. *Euclid.* Idem quoque hoc modo absolvitur. Inquirito latus quadrati dicto triangulo ABC æqualis, ut BH, inde tribus HB DC CB, seu IK IL IM inveniatur quarta proportionalis LN, hęc erit latus quæsiti trianguli ipsi BC homologum dato quadrato æqualis. Laterū autem quantitatem secundum numeros expositos deprehēdes NL $\sqrt{597\frac{1}{7}}$, IO $\sqrt{515\frac{1}{7}}$, NO $\sqrt{685\frac{5}{7}}$: quare perpendicularis ex O demissa erit $\sqrt{438\frac{6}{7}}$, quæ per dimidium basis NI $\sqrt{149\frac{1}{7}}$ multiplicata dabit areā triāguli $\sqrt{65536}$, hoc est 256, æqualē dato quadrato á latere 16.

PROBLEMA 45.

A dato parallelogrammo rectangulo duabus parallelis æqualiter ab utroque latere distantibus ejus dimidium absumere.

Exponatur rectangulum parallegrammnm A B C D unde absumendum sit aliud DEFG ejus dimidio æquale, ut tamen segmentum A G tantum sit quantum segmentum EC, hoc est ut parallela EF tanto intervallo absit á BC, quanto GF ab AB. Bisecetur itaque latus CD in Q, & bisegmēto DQ ponatur æqualis DO, deinde inter AB latus reliquum & RO continuationem sit



media proportionalis DM, hujus quadratum dimidium erit dati parallelogrammi ABCD. dehinc latitudini DC æquale sit segmentum CK, quare BK laterum erit differentia , ea bisecetur in L, & bisegmento differentię BL æqualis sit HD. tumque recta MH continuetur in I intervallo HD, ut tota continuata sit

MI.

MI. tanta erit longitudo quæsiti parallelogrammi; eadem porro HD subducta de HM relinquet PM latitudinem quæsitã. Ponatur itaque ipsi MI æqualis DG, & rectæ PM æqualis DE & compleatur parallelogrammum GDEF, ajo CE & AG inter se æquari, cujus veritas per logistarum abacos ita comprobatur. Esto AD 14, DC 8, area ab his comprehensa 112, dimidium 56, hujus numeri latus exhibet ipsam DM $\surd$56. Et rursum differentia inter 8 & 14 est BK 6, dimidium BL seu ipsi æqualis DH 3: sed HD & DM æque possunt ipsi HM, quare HM erit $\surd$65, cui addita HI 3 dabitur MI $\surd 65 - 3$. tanta inquam erunt latera, DG & DE, si enim DE $\surd 65 - 3$ subduxeris de DC 8 reliqua erit EC $11 - \surd 65$. Et si DG $\surd 65 + 3$ subducas de AD reliqua AG erit itidem $11 - \surd 65$ priori EC æqualis, Denique etiam spatiũ sub his lateribus $11 + \surd 65$ & $11 - \surd 65$ comprehensum videlicet 56 æquale est dimidio dati ABCD 112. Quare factum est quod oportuit.

Quamvis & temporis angustia circumscribamur, & figuris ad alienum arbitrium deformatis utamur, non possum tamen quin huic problemati in gratiam lectoris facem alliceam.

Concipiamus itaque AO continuatam esse ex Latere AD & DC, atque inter ejus segmenta lineam DM quæ dimidium dati rectanguli possit, proportione esse mediam, & segmento DO æquari CE vel AG: ajo ambitum AGFECB dati parallelogrammi dimidio æquari. Namque cum per fabricam DO æqualis sit AC vel CE erit, ut BC & EC ad MD, sic MD ad CE, quare norma GFBB A dimidium dati parallelogrammi comprehendit.

Hæc fabrica & demonstratio in omnibus parallelogrammis & quacunque ratione ita secandis locum habet. Diagramma demonstrationi nostræ minus appositum, at diligens & industrius lector facile ex dictis reliqua assequetur, & sibi diagramma deformabit.

Liber

LVDOLPHI à CEVLEN HILESHEIMENSIS

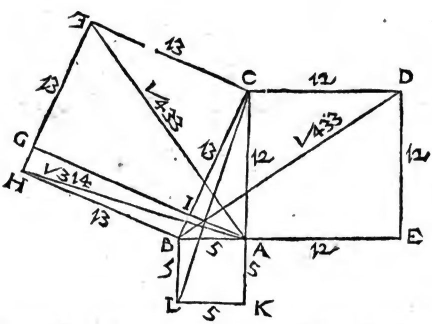
Liber quartus,

DE

Δεδομένων Geometricorum per numeros ſolutione.

CVm itaque hactenus Geometricam linearum factionem ubique uſurpaverimus, ſupereſt ut deinceps ipſas propoſitiones libri ſecũdi, aliaſq; præterea numeris quoq; ſubijciamus: atq; ita non tantum lineis in docto pulvere ductis verum etiam calcula in logiſtarum abaco earum veritatem comprobemus. Namque ut ab Eutocio è vetere poeta veriſsime citatur ταῦτα γὰρ τὰ μαθήματα δοκοῦντ᾽ εἶμεν ἀδελφά Artes iſtæ tanquam germanæ ſorores ſunt. Ideoque hæc quæ ſequuntur zetemata dedomenῶn formula concepi, ut philomatis occaſionem ſubminiſtrarem figuratorũ numerorũ affectionẽ & ſymptomata cognoſcendi, inque iſtis ſeſe exercendi, ut numeri iſti quorum tractatio obſcurior & intricatior hactenus habita fuit ita minus aſpera, aut ab uſu remota eſſe re ipſa cõprobetur. Quamobrem á Pythagorea hecatombe, hoc eſt á propoſitione noſtra ſecunda & viceſima libri ſecundi, ſive 47 lib. 1. Euclidis auſpicium facere cõſtitui, cum iſta ad abacum noſtrum quam maxime ſit neceſſaria, ut quod illic geometricé quidem & veré demonſtratum eſt, hic cum numeris accuraté conſentire doceamus.

ZETEMA.



Iam ſupra nobis demonſtratum eſt quadratum á crure anguli recti BAKL æquari rectangulo HGIB: itemque quadratum reliqui cruris reliquo rectangulo GFCI, hoceſt quadratum baſis HBCF quadratis crurum BLKA AEDC æquari. Cum autem per 24 prop. libri noſtri ſecundi triangula ſint dimidia parallelogrammorum eandem baſin & altitudinem

tudinem habentium, parallelogrammum HGIB erit duplum trianguli ABH, & GFCI trianguli FCA. itemque quadratum ACFE duplum trianguli BCD. quare triangulū HBA erit $12\frac{1}{2}$, BCD 72. Id verò numeris explorandum sit secundum areæ triangularis investigationem supra demonstratam: quamobrem illud nobis hic propositum esto: investigetur primum area trianguli HBA cujus latera dantur 1375 $\sqrt{314}$. sūma omniū $18+\sqrt{314}$, hujus semissis $9+\sqrt{78\frac{1}{2}}$ inde singula latera subducta relinquēt tres differētias. $9-\sqrt{78\frac{1}{2}}$, $\sqrt{78\frac{1}{2}}-4$, $\sqrt{78\frac{1}{2}}+4$: factus á prima differentia in semissē dabit $2\frac{1}{2}$, tū secūda differentia in tertiam multiplicata dabit $62\frac{1}{2}$: denique horum duorum factorum factus erit $156\frac{1}{4}$, hujus latus $12\frac{1}{2}$ optatam dati trianguli ABH aream exhibet, cujus duplum 25 æquatur areæ quadrati ABLK.

Rursum dentur latera trianguli FCA, quod æquatur dimidio quadranguli FGIC, FC 13, CA 12, AF $\sqrt{433}$ area eadem ratione inventa erit 72. abaci hujus formulam in curiosorum gratiam integram adscri psi.

$25+\sqrt{433}$ laterum summa.
$12\frac{1}{2}+\sqrt{108\frac{1}{4}}$ summæ semissis.
$\sqrt{433}$ 13. 12. latera.

$12-\sqrt{108\frac{1}{4}}$. $\sqrt{108\frac{1}{4}}-\frac{1}{2}$. $\sqrt{108\frac{1}{4}}+\frac{1}{2}$ differentiæ.

$12\frac{1}{2}+\sqrt{108\frac{1}{4}}$ $\sqrt{108\frac{1}{4}}+\frac{1}{2}$
$12\frac{1}{2}-\sqrt{108\frac{1}{4}}$ $\sqrt{108\frac{1}{4}}-\frac{1}{2}$

$156\frac{1}{4}$ $108\frac{1}{4}$
$108\frac{1}{4}$ $-\frac{1}{4}$

48 factus primus 108 108 factus secundus.
48

864
432

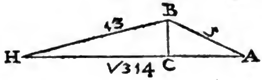
5184. factus tertius. 5184

72 area trianguli.

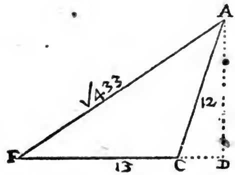
Est itaque 72 area dicti trianguli, cujus duplum 144 ęquatur oblongo FGCI, huc addita 25 oblongum BHGI, dabit aream totius quadrati BHFC 169. eandēque summam conflabunt quoque quadrata á cruribus BA 5, AC 12. itaque numerorum epilogismus geometricæ demonstrationi accurate consentit.

Iterum

Iterum assumamus triãgula ABC, FAC quorum areæ etiam aliter ope perpendicularis inveniantur. demittatur itaque ex obtuso angulo in subjectã basin AH perpendicularis BC ea per præcepta nostra invenietur $\sqrt{11\frac{1}{3}\frac{1}{3}\frac{1}{4}}$ quæ per dimidiam basin AH $\sqrt{78\frac{1}{2}}$ multiplicata dabit areã trianguli $12\frac{1}{2}$, ut supra.



Iam ad aream trianguli AFC inveniẽda esto in basin FC perpendicularis AD $11\frac{1}{13}$ ea per dimidiam basin FC $6\frac{1}{2}$ multiplicata dabit quæsitam aream trianguli FAC 72, itidem ut supra.



Vt plura hujus rei exempla in medium afferamus, assumamus triãgula ACD AFD in diagrammate propositionis 26, quæ super eadem basi intra easdem parallelas posita, ideoque æqualia sunt, quod numerorum epilogismo docendum sit. trianguli primi ADC gæodæsia non admodum operosa est ob angulum rectum ADC. cujus crura æqualia sunt partium 12, quare basis AC $\sqrt{288}$. multiplicato itaque dimidio crure 6 per basin 12 dabitur area 72: cui ob demonstrata triangulum AFD necessario æquatur. latera autem hæc sunt AD 12, AF $\sqrt{1168}$, DF $\sqrt{544}$ quorum omnium summa $\sqrt{1168}+\sqrt{544}+12$ ejus dimidium $\sqrt{292}+\sqrt{136}+\sqrt{6}$, unde singula latera sigillatim subducta dabunt tres differentias $\sqrt{136}+6-\sqrt{292}$, $\sqrt{292}-\sqrt{136}+6$, $\sqrt{292}+\sqrt{136}-6$. numeri ab his differentijs continua multiplicatione facti latus 72 dabit aream trianguli. Ecce totius operis typum.

$$\begin{array}{l}\sqrt{292}+\sqrt{136}+6\\ \sqrt{292}-\sqrt{136}+6\\ \hline 292-136+36+\sqrt{42048}\\ \text{id est. } \sqrt{4208}+192\end{array}\qquad\begin{array}{l}\sqrt{136}+6-\sqrt{292}\\ \sqrt{136}-6+\sqrt{292}\\ \hline 136-36-292+\sqrt{42048}\\ \text{id est. } \sqrt{42048}-192\end{array}$$

$$\begin{array}{l}\sqrt{42048}+192\\ \sqrt{42048}+192\\ \hline 42048\\ -36864\\ \hline 5184\end{array}\qquad\begin{array}{l}5184\\ \hline 7|2 \text{ area trianguli.}\end{array}$$

Hic

Hic secunda differentia per semissem multiplicata fecit numerum $\sqrt{42048}+192$, inde prima differentia per tertiam $\sqrt{42048}-129$, numeri ab utroque facti 5184 latus 72 est area dati trianguli AFD quemadmodum oportuit.

In eodem diagrammate triangulum DGI est æquealtum trianguloLNM, omniumq; laterum trianguli D G I summa est $\sqrt{1168}+\sqrt{468}+14$: dimidium $\sqrt{292}+\sqrt{117}+7$. Laterum à semisse differentiæ $\sqrt{117}+7-\sqrt{292}$, $\sqrt{117}-7+\sqrt{292}$, $\sqrt{292}-\sqrt{117}+7$.

$$\begin{array}{l} \sqrt{117}+7+\sqrt{292} \\ \sqrt{117}+7-\sqrt{292} \\ \hline 117+49-292+\sqrt{22932}. \\ \text{id est } \sqrt{22932}-126 \end{array} \qquad \begin{array}{l} \sqrt{292}-\sqrt{117}+7 \\ \sqrt{292}+\sqrt{117}-7 \\ \hline 292-117-49+\sqrt{22932} \\ \text{id est } \sqrt{2293}+126 \end{array}$$

$$\begin{array}{c} \sqrt{22932}-126 \\ \sqrt{22932}+126 \\ \hline 22932 \\ 15876 \\ \hline 7056 \end{array} \qquad \begin{array}{c} 7056 \\ \hline 8|4 \\ \hline \end{array} \text{ area triãg. LNM}$$

In diagrammate propositionis 24 exponuntur tria triangula æqualia HKD, HMD, HCD, latera trianguli HKD sunt 18, $\sqrt{567}$, $\sqrt{243}$ secundi HMD sunt 18 $\sqrt{819}$ $\sqrt{279}$: tertij HCD sunt 18 $\sqrt{3159}$ $\sqrt{1953}$. Horum triangulorum æqualitas ita per numeros deprehenditur.

Area HKD invenitur multiplicata perpendiculari KD $\sqrt{243}$ in dimidiam basin HD9, factus enim $\sqrt{19683}$ area optata.

Secundo area trianguli HMD inveniatur additis lateribus.

summa collectorum laterum $18+\sqrt{819}+\sqrt{279}$

dimidium $9+\sqrt{204\frac{3}{4}}+\sqrt{69\frac{3}{4}}$

differentiæ $9-\sqrt{204\frac{3}{4}}+\sqrt{69\frac{3}{4}}$. $\sqrt{204\frac{3}{4}}-9+\sqrt{69\frac{3}{4}}$. $\sqrt{204\frac{3}{4}}+9-\sqrt{69\frac{3}{4}}$.

$$\begin{array}{l} 9+\sqrt{204\frac{3}{4}}+\sqrt{69\frac{3}{4}} \\ 9-\sqrt{204\frac{3}{4}}+\sqrt{69\frac{3}{4}} \\ \hline 81-204\frac{3}{4}+69\frac{3}{4}+\sqrt{22599} \\ \text{id est } \sqrt{22599}-54 \end{array} \qquad \begin{array}{l} \sqrt{204\frac{3}{4}}+9-\sqrt{69\frac{3}{4}} \\ \sqrt{204\frac{3}{4}}-9+\sqrt{69\frac{3}{4}} \\ \hline 204\frac{3}{4}-81-69\frac{3}{4}+\sqrt{22599} \\ \text{id est } \sqrt{22599}+54 \end{array}$$

$$\begin{array}{c} \sqrt{22599}+54 \\ \sqrt{22599}-54 \\ \hline 22599 \\ -2916 \\ \hline 19683 \end{array} \qquad \sqrt{19683} \text{ area trianguli HMD.}$$

Tertio

Tertiò eodem etiam modo inquiratur area trianguli HCD.

laterum omnium summa $18+\sqrt{3159}+\sqrt{1539}$

dimidium $9+\sqrt{789\frac{3}{4}}+\sqrt{384\frac{3}{4}}$

differentiæ $9-\sqrt{789\frac{3}{4}}+\sqrt{384\frac{3}{4}}$. $\sqrt{789\frac{3}{4}}-9+\sqrt{384\frac{3}{4}}$. $\sqrt{789\frac{3}{4}}+9-\sqrt{384\frac{3}{4}}$.

$9+\sqrt{789\frac{3}{4}}+\sqrt{384\frac{3}{4}}$ $\qquad$ $\sqrt{789\frac{3}{4}}-9+\sqrt{384\frac{3}{4}}$

$9-\sqrt{789\frac{3}{4}}+\sqrt{384\frac{3}{4}}$ $\qquad$ $\sqrt{789\frac{3}{4}}+9-\sqrt{384\frac{3}{4}}$

$81-789\frac{3}{4}+384\frac{3}{4}+\sqrt{124659}$ $\qquad$ $789\frac{3}{4}-81-384\frac{3}{4}+\sqrt{124659}$

id est $\sqrt{124659}-324$ $\qquad$ id est $\sqrt{124659}+324$

$\sqrt{124659}-324$

$\sqrt{124659}+324$

124659

104976

19683 $\qquad$ $\sqrt{19683}$ area trianguli HCD.

Denique iterum libeat etiam secundum hanc viam investigare aream trianguli HKD.

summa laterum $18+\sqrt{567}+\sqrt{243}$

dimidium $9+\sqrt{141\frac{3}{4}}+\sqrt{60\frac{3}{4}}$.

differentiæ $9-\sqrt{141\frac{3}{4}}+\sqrt{60\frac{3}{4}}$. $\sqrt{141\frac{3}{4}}-9+\sqrt{60\frac{3}{4}}$. $\sqrt{141\frac{3}{4}}+9-\sqrt{60\frac{3}{4}}$.

$9+\sqrt{141\frac{3}{4}}+\sqrt{60\frac{3}{4}}$ $\qquad$ $\sqrt{141\frac{3}{4}}-9+\sqrt{60\frac{3}{4}}$

$9-\sqrt{141\frac{3}{4}}+\sqrt{60\frac{3}{4}}$ $\qquad$ $\sqrt{141\frac{3}{4}}+9-\sqrt{60\frac{3}{4}}$

$81-141\frac{3}{4}+60\frac{3}{4}+\sqrt{19683}$ $\qquad$ $141\frac{3}{4}-81-60\frac{3}{4}+\sqrt{19683}$

id est $\sqrt{19683}$. $\qquad$ id est $\sqrt{19683}$

$\sqrt{19683}$

$\sqrt{19683}$

19683 $\qquad$ hujus latus $\sqrt{19683}$ area trianguli HKD ut supra.

Quare trium triangulorum HCD HKD HMD areæ æquales sunt. omniū enim calculus eundem exhibet numerum areæ indicem $\sqrt{19683}$, hoc est in numeris absolutis $140\frac{29611}{100000}$ ut proxime.

Præterea ad istam exemplorum congeriem quoque 80 nostram propositionē libri secundi aggregare libet. in qua triangulo ABC æquatur parallelogrammū CDFE. latera autem trianguli sunto AB 14, BC 20, AC 21. hinc distantia HF ad perpendicularem BH 26. Datis itaque lateribus trianguli ABC datur area $\sqrt{18098\frac{7}{16}}$, & perpendicularis AG seu HB $\sqrt{180\frac{63}{64}}$. & HF datur 26, quare dabitur subtensa BF $\sqrt{856\frac{63}{64}}$. Hinc subducta HI 10 de HF 26 dabitur reliqua IF 16: atque ideo subtensa DF $\sqrt{436\frac{63}{64}}$. Dantur itaque latera trianguli BFD æqualis triangulo ABD cujus area $\sqrt{4524\frac{39}{64}}$ dimidia est quadranguli BHID. Id vero in datis lateribus trianguli BDF calculo explorandum est.

laterum summa $10 + \sqrt{856\frac{63}{64}} + \sqrt{436\frac{63}{64}}$

dimidium $5 + \sqrt{214\frac{63}{256}} + \sqrt{109\frac{63}{256}}$.

diff.) $\sqrt{214\frac{63}{256}} + \sqrt{109\frac{63}{256}}$. $\sqrt{214\frac{63}{256}} - 5 + \sqrt{109\frac{63}{256}}$. $\sqrt{214\frac{63}{256}} + 5 - \sqrt{169\frac{63}{256}}$

$5 + \sqrt{214\frac{63}{256}} + \sqrt{109\frac{63}{256}}$	$\sqrt{214\frac{63}{256}} - 5 + \sqrt{109\frac{63}{256}}$
$5 - \sqrt{214\frac{63}{256}} + \sqrt{109\frac{63}{256}}$	$\sqrt{214\frac{63}{256}} + 5 - \sqrt{109\frac{63}{256}}$
$25 - 214\frac{63}{256} + 109\frac{63}{256} + \sqrt{10924\frac{39}{64}}$	$214\frac{63}{256} - 25 - 109\frac{63}{256} + \sqrt{10924\frac{39}{64}}$
id est $\sqrt{10924\frac{39}{64}} - 80$	id est $\sqrt{10924\frac{39}{64}} + 80$

$$\begin{array}{l} \sqrt{1092\frac{39}{64}} - 80 \\ \sqrt{1092\frac{39}{64}} + 80 \\ \hline 10924\frac{39}{64} \\ -6400 \\ \hline 4524\frac{39}{64} \end{array}$$

$4524\frac{39}{64}$ hujus facti latus $\sqrt{4524\frac{39}{64}}$ est area trianguli *BAD*, cujus duplum $\sqrt{18098\frac{7}{16}}$ area parallelogrammi BFEA.

Ejusdem generis exemplum nobis sit é diagrammate propositionis 82 libri secundi, in isto enim parallelogrammum DFGH æquatur triangulo OPD, cujus latera OD 14, PD 10, OP 15, ideoque perpendicularis KC $\sqrt{93\frac{447}{784}}$: & area $\sqrt{4584\frac{15}{16}}$. atque ideo parallelogrammum FDGH super data linea A hoc est latere DH 17 descriptum eidem æquari debet.

Id per numeros indagabimus hoc modo. quadrangulum ABDC super dimidia basi OD dicto triangulo quoque çquatur, estque CD partium 7. jam quçrendum est nobis latus AC: datur AK 9 & perpendicularis KC $\sqrt{93\frac{447}{784}}$. ideoque subtensa AC vel BD $\sqrt{174\frac{447}{784}}$. hinc perpendicularis ab I sit IM, est itaque HM æqualis ipsi AK 9, quare tota DM 26. quare subtensa DI datur $\sqrt{69\frac{447}{784}}$. Dantur igitur latera trianguli DHI, cui simile est ECD, datur autem CD 7, dantur itaque per proportionem quoque reliqua latera CE hoc est DF $\sqrt{29\frac{2767}{4724}}$: & DE $\sqrt{[illegible]\frac{3343}{4624}}$. denique invenietur quoque diagonius DG $\sqrt{192\frac{2767}{4724}}$ Dantur igitur latera trianguli OPD, cujus area sit $\sqrt{1146\frac{15}{64}}$. quia æquatur dimidio trianguli DGH,

Laterum

Laterum summa $17 + \sqrt{\tfrac{890575}{4624}} + \sqrt{\tfrac{136863}{4624}}$

summæ dimidium $8\tfrac{1}{2} + \sqrt{\tfrac{890575}{18496}} + \sqrt{\tfrac{136863}{18496}}$

differētiæ $8\tfrac{1}{2} - \sqrt{\tfrac{890575}{18496}} + \sqrt{\tfrac{136836}{18496}}$. $\sqrt{\tfrac{890575}{18496}} - 8\tfrac{1}{2} + \sqrt{\tfrac{136763}{18496}}$. $\sqrt{\tfrac{890575}{18496}} + 8\tfrac{1}{2} - \sqrt{\tfrac{136863}{18496}}$

$$\begin{array}{l} 8\tfrac{1}{2} + \sqrt{\tfrac{890575}{18496}} + \sqrt{\tfrac{136863}{18496}} \\ 8\tfrac{1}{2} - \sqrt{\tfrac{890675}{18496}} + \sqrt{\tfrac{136863}{18496}} \\ \hline 72\tfrac{1}{4} - \tfrac{890575}{18496} + \tfrac{136863}{18496} + \sqrt{\tfrac{136863}{18496}} \end{array}$$

id est, $31\tfrac{1}{2} + \sqrt{\tfrac{136863}{64}}$

$$\begin{array}{l} \sqrt{\tfrac{890575}{18496}} - 8\tfrac{1}{2} + \sqrt{\tfrac{136863}{18496}} \\ \sqrt{\tfrac{890575}{18496}} + 8\tfrac{1}{2} - \sqrt{\tfrac{136863}{18496}} \\ \hline \tfrac{890575}{18496} - 72\tfrac{1}{4} - \tfrac{136863}{18496} + \sqrt{\tfrac{136863}{64}} \end{array}$$

id est, $\sqrt{\tfrac{136863}{64}} - 31\tfrac{1}{2}$

$$\begin{array}{l} \sqrt{\tfrac{136863}{64}} + 31\tfrac{1}{2} \\ \sqrt{\tfrac{136863}{64}} - 31\tfrac{1}{8} \\ \hline \tfrac{136863}{64} - 992\tfrac{1}{4} \end{array}$$

id est $1146\tfrac{15}{64}$, cujus latus $\sqrt{1146\tfrac{15}{64}}$ area dati trianguli, ut supra.

Denique novissimũ exemplum nobis esto é propositione 71 ejusdem libri secundi. Vbi duo triangula BCE CGH exponuntur reciproca cruribus æqualium angulorũ C & H. Hic latera prioris dantur BC 10, BE $\sqrt{500}$, EC $\sqrt{200}$. Secundi autem CG $12\tfrac{1}{2}$, CH $\sqrt{484\tfrac{1}{4}}$, GH $\sqrt{128}$. Vtriusque aream æquari numeri in abaco docebunt, hoc modo.

Summa laterum trianguli BCE

$$10 + \sqrt{500} + \sqrt{200}$$

dimidium $5 + \sqrt{125} + \sqrt{50}$

differentiæ $5 - \sqrt{125} + 50$. $\sqrt{125} - 5 + \sqrt{50}$. $\sqrt{125} + 5 - 50$

$$\begin{array}{l} 5 + \sqrt{125} + \sqrt{50} \\ 5 - \sqrt{125} + \sqrt{50} \\ \hline 25 - 125 + 50 + \sqrt{5000} \end{array}$$

id est, $\sqrt{5000} - 50$

$$\begin{array}{l} \sqrt{125} - 5 + \sqrt{50} \\ \sqrt{125} + 5 - \sqrt{50} \\ \hline 125 - 25 - 50 + \sqrt{5000} \end{array}$$

id est, $\sqrt{5000} + 50$

$$\begin{array}{l} \sqrt{5000} - 50 \\ \sqrt{5000} + 50 \\ \hline 5000 - 2500 \end{array}$$

id est 2500, cujus latus 50 est area trianguli BEC.

Summa laterum trianguli G H C

$12\frac{1}{2} + \sqrt{484\frac{1}{4}} + \sqrt{128}$

dimidium $6\frac{1}{4} + \sqrt{121\frac{1}{16}} + \sqrt{32}$

differentiæ $6\frac{1}{4} - \sqrt{121\frac{1}{16}} + \sqrt{32}$. $\sqrt{121\frac{1}{16}} - 6\frac{1}{4} + \sqrt{32}$. $\sqrt{121\frac{1}{16}} + 6\frac{1}{4} - \sqrt{32}$.

$6\frac{1}{4} + \sqrt{121\frac{1}{16}} + \sqrt{32}$

$6\frac{1}{4} - \sqrt{111\frac{1}{16}} + \sqrt{32}$

$39\frac{1}{16} - 121\frac{1}{16} + 32 + \sqrt{5300}$

id est, $\sqrt{5000} - 50$

$\sqrt{121\frac{1}{16}} - 6\frac{1}{4} + \sqrt{32}$

$\sqrt{121\frac{1}{16}} + 6\frac{1}{4} - \sqrt{32}$

$121\frac{1}{16} - 39\frac{1}{16} - 32 + \sqrt{5000}$

$\sqrt{5000} + 50$

$\sqrt{5000} + 50$

$\sqrt{5000} - 50$

$5000 - 2500$

id est 2500, cujus latus 50 est area trianguli CGH. sunt itaque CGH, BEC triangula æqualia, quod ostendisse oportuit.

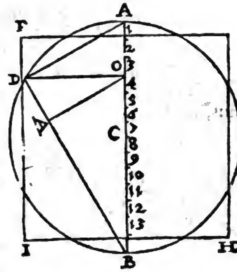
2 ZETEMA.

Quantum Archimedæa circuli quadratura à vero absit explorandum esse.

In libro quem de circulo et adscriptis publicavi solide et accurate docui diametri rationem ad ejusdem circuli circumferentiam esse majorem quã unum ad $3\frac{14159265358979323846}{100000000000000000000}$, minorem verò quam $3\frac{14159265358979323847}{100000000000000000000}$ et quamvis hi termini tam ingentes satis accurati sint ad geodæsiam omnium circulorum quacunque tandem amplitudine ijdem in toto hoc terrarum orbe vel dari vel fingi possint, insatiabilis tamen quædam veri investigandi cupido me incitavit, ut eosdem terminos ampliarem et ulterius procederem: assumpto itaque hujus operis socio *Petro Cornelij* meo discipulo, huc rem produxi, ut deprehenderim posita diametro *1* peripheriam esse $3\frac{14159265358979323846264338327950}{100000000000000000000000000000000}$ minorem vero, & $3\frac{14159265358979323846264338327951}{100000000000000000000000000000000}$ majorem vero.

Quam ob rem ratione diametri ad perimetrum ita accurate expressa circulorum areæ vero quam proximæ exhiberi, atque ideo quoque latera quadratorum ipsis æqualium facile cognossi possunt.

Huc

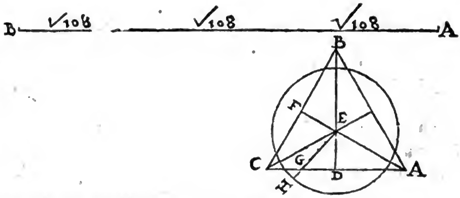


Huc itaque advocetur diagramma problematis tricesimi libri antecedentis et exploremus utrum quadratum FGHI illius fabricæ exposito circulo æquetur: et statuamus diametrũ AB 14 decempedarum, ea per $3\frac{1}{7}$ multiplicata dabit integram peripheriam decempedarũ 44, jam semiperipheria 22 per radium 7 multiplicata exhibebit aream circuli 154 decempidarum quadratarum, hinc inquiratur BD latus quadrati circulo æqualis, hoc modo, multiplicato diametrum AB 14 per OB 11 facti 154 latus $\sqrt{154}$ dabit longitudine lineam BD, cujus quadratum dato circulo æquatur. Sed idem Archimedes evidentissimis rationibus ostendit rationem peripheriæ ad diametrum esse minorem tripla sesquiseptima: et secundum terminos à nobis supra expressos posita diametro partium 14 perimeter erit $43\frac{9}{1}\frac{8}{0}\frac{2}{0}\frac{2}{0}\frac{9}{0}\frac{7}{0}\frac{1}{0}$ minor vero, $43\frac{9}{1}\frac{8}{0}\frac{2}{0}\frac{2}{0}\frac{9}{0}\frac{7}{0}\frac{2}{0}$ major vero, hujus semsis $21\frac{9}{1}\frac{9}{0}\frac{1}{0}\frac{1}{0}\frac{4}{0}\frac{8}{0}\frac{5}{0}$ per radium 7 multiplicatus dabit aream circuli 153 decempedarum 135 pedum $11\frac{1}{7}$ digitorum quadratorum: vel 153 decempedarum quadratarum, 11 pedum scamnatorum, $3\frac{1}{2}$ digitorum scamnatorum. Hac enim mensura gæodetę utuntur: est autẽ pes scamnatus spatium oblongum à latitudine pedis unius et decempedæ longitudine comprehensũ, simili modo digitus scamnatus latitudine digiti longitudine decempedæ. scamnatum autem digitum indigitavimus exemplo Columellæ qui agrum minore latitudine et majore longitudine instar scamni exporrectum agrũm scamnatum vocat. eâdem notione pedes scamnatos, digitos scamnatos in opere Hypomnematum Principis nostri usurpavimus. notandum itaque est pedem scamnatum esse 12 pedes quadratos; digitum scamnatum 144 digitos quadratos, hoc est unicum pedem quadratum. Nam quam hic vocamus decempedam eam non pedibus decem (ut vocis etymon innuit) sed duodecim in longum patere, pro vetere nostrorum gæodetarum more intelligimus. Subducta itaque hac area de illa Archimedea differentia erit 8 pedum et $132\frac{4}{7}$ digitorum quadratorum. tanto igitur spatio quadratum hoc propositum circulum excedit.

3 ZETEMA.

Quam longè Cusani quadratura circuli à vero absit.

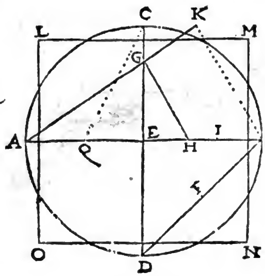
T   é ductu



E ductu fabricę problematis 32 libri tertij, quo in loco Cusani constructionem docuimus, haud difficile est invenire quãtitatẽ radij EH $\sqrt{24\frac{3}{6}\frac{9}{4}}$: quare tota diameter fuerit $\sqrt{98\frac{7}{1}\frac{}{6}}$, cujus peripheria sit $\sqrt{972}$ videlicet tripla $\sqrt{108}$. Numeris hisce ad numeros explicabiles reductis dabitur ratio diametri ad peripheriam, quæ 1 ad $3\frac{1}{1}\frac{4}{0}\frac{2}{0}\frac{3}{0}$. quamobrem ista paulo propius ad verum collineat quam superior Archimedæa, á vero tamen adhuc longe abest, quamvis intra terminos ab Archimede præstitutos intercidat. Cæterum ut ex hac diametri ad perimetrum ratione aream invenias multiplicato radium $\sqrt{24\frac{3}{6}\frac{9}{4}}$ per semiperipheriam $\sqrt{243}$ facti latus $77\frac{3}{1}\frac{0}{0}\frac{9}{0}\frac{6}{0}$ exhibebit aream circuli: atqui per ea quæ supra demonstravimus ad istum radium area circuli vera esset major $77\frac{3}{1}\frac{1}{0}\frac{2}{0}\frac{6}{0}$, & minor quam $77\frac{3}{1}\frac{1}{0}\frac{2}{0}\frac{7}{0}$. Cusanus itaque iustum areæ modum excedit.

4 ZETEMA.

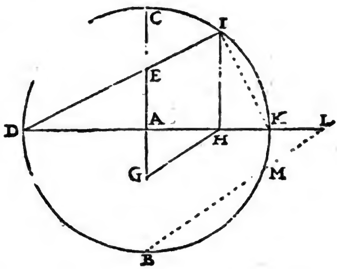
Quam prope Vietæa circuli quadratura ad verum accedat



Revocemus diagramma propositionis 33 libri 3, sitq; AB diameter partium 2, erit itaque DB latus quadrati inscripti $\sqrt{2}$, dimidium BF $\sqrt{\frac{1}{2}}$ cui æqualis posita est EG. radius porrò AE bisectus est in Q quare EQ est $\frac{1}{2}$, & CE 1, itaq; CQ hoc est QI $\sqrt{1\frac{1}{4}}$. et EI $\sqrt{1\frac{1}{4}} - \frac{1}{2}$: itaque IB, hoc est ipsa EH $1\frac{1}{2} - \sqrt{1\frac{1}{4}}$. atque tota AH $2\frac{1}{2} - \sqrt{1\frac{1}{4}}$. Præterea datis AE & EG $\sqrt{\frac{1}{2}}$ dabitur AG $\sqrt{1\frac{1}{2}}$. Hinc ob similitudinem triangulorum AGH, ABK, erit quemadmodum AH $2\frac{1}{2} - \sqrt{1\frac{1}{4}}$ ad AG $\sqrt{1\frac{1}{2}}$, sic AB 2 ad AK latus quadrati circulo æqualis $\sqrt{1\frac{1}{2}} + \sqrt{\frac{1}{5}}$, cujus quadratum $1\frac{4}{5} + \sqrt{1\frac{4}{5}}$ exhibet spa-

tium

tium quadratum L M N O dato circulo æquale. ſeu quod idem eſt in numeris abſolutis $3\frac{1}{10}\frac{4}{100}\frac{1}{1000}\frac{6}{10000}\frac{4}{100000}$. atqui area circuli ſecundum accuratiorem æſtimationem eſt major quam $3\frac{1}{10}\frac{4}{100}\frac{1}{1000}\frac{5}{10000}\frac{9}{100000}$ minor autem quam $3\frac{1}{10}\frac{4}{100}\frac{1}{1000}\frac{6}{10000}\frac{0}{100000}$. Quamobrem Vietæa quadratura proximé omnium ad verum accedit, ut ſi area circuli ſtatuatur decempedarum mille tantillum à vero abſit. Supereſt ut deinceps quoque expendamus rationem qua lineam rectam peripheriæ dati circuli æqualem conſtruit.



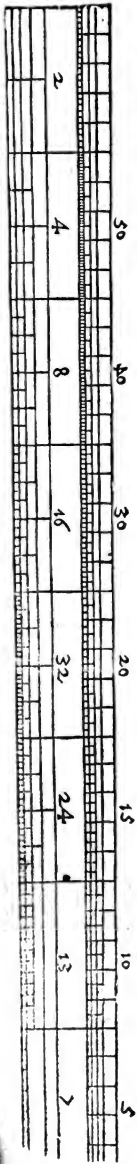
Quam ob rem iterum ſtatuamus diametrum eſſe partium 2 ut ſupra. Hic radius CA biſectus eſt in E, perque id punctum recta inſcripta DI: conſtat itaque DE eſſe $\sqrt{1\frac{1}{4}}$, cujus exceſſus ſuper AE ſit GB $\sqrt{1\frac{1}{4}} - \frac{1}{2}$. Inde proportio, ut DE $\sqrt{1\frac{1}{4}}$ ad DA 1, ſic DK 2 ad DI $\sqrt{3\frac{1}{5}}$. Item, ut DC $\sqrt{1\frac{1}{4}}$ ad DA 1, ſic DI $\sqrt{3\frac{1}{5}}$ ad DH $1\frac{3}{5}$ inde ſubducta DA, reliquam facit AH $\frac{3}{5}$. Hinc ſuducatur GB $\sqrt{1\frac{1}{4}} - \frac{1}{2}$ de AB 1 relinquetur AG $1\frac{1}{2} - \sqrt{1\frac{1}{4}}$. denique ad extremum, ut GA $1\frac{1}{2} - \sqrt{1\frac{1}{4}}$ ad AH $\frac{3}{5}$, ſic AB 1 ad AL $\frac{9}{10} + \sqrt{\frac{9}{20}}$ eſt autem AL totius datæ peripheriæ pars quarta: itaque ea per diametrũ multiplicata dabit areã circuli $1\frac{4}{5} + \sqrt{1\frac{4}{5}}$. ut ſupra, tantum enim ibidẽ comprobavimus eſſe quadratum dato circulo æquale. Quare ratio Vietæa omnium quotquot hactenus extant accuratiſſima eſt.

5 ZETEMA.

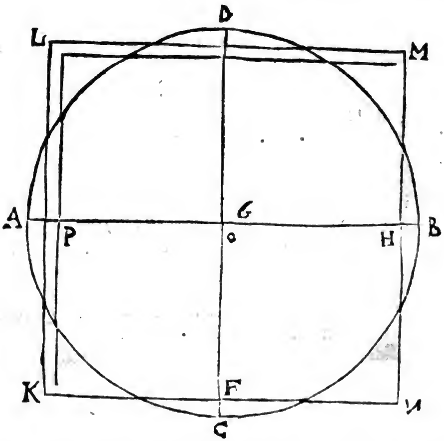
Quadratum circulo mechanicè quam proximè æquale conſtruere.

Vt quadratum circulo mechanicè quam proxime conſtrui poſſit, idque tantum organice, ut mechanicis inſerviam (eſt enim hoc hominum genus geometricæ fabricæ et horum lineamentorum plane expers) paretur linea ærea unius pedis longitudine, cujus typum hic vides, eaque tota ſecetur in partes decem, earumque ſingulæ in alias quaſcunque particulas æquales pro arbitrio quam accuratiſſime ſubdividatur: regula ita comparata non tantum huic negotio ſed etiam in alijs quam plurinus Geodetis uſui erit.

 Sed



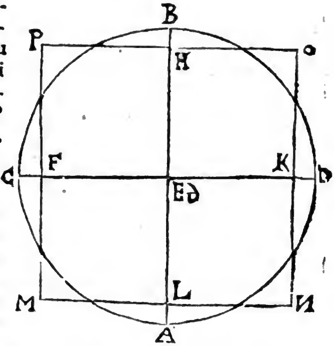
Sed ut ad rem veniam. Iam ſupra oſtendi, ſuppoſita diametro partium duarum aream eſſe $3_{1}\frac{1}{0}\frac{4}{0}\frac{1}{0}\frac{5}{0}\frac{9}{0}\frac{2}{0}\frac{6}{0}\frac{5}{0}\frac{3}{0}\frac{5}{0}$ conſequens itaque eſt quadratum circulo æquale aream habere eandem, quare ejus latus erit $\sqrt{3_{1}\frac{1}{0}\frac{4}{0}\frac{1}{0}\frac{5}{0}\frac{9}{0}\frac{2}{0}\frac{6}{0}\frac{5}{0}\frac{3}{0}\frac{6}{0}}$, hoc eſt $1_{1}\frac{7}{0}\frac{7}{0}\frac{2}{0}\frac{4}{0}\frac{5}{0}\frac{3}{0}\frac{8}{0}\frac{5}{0}$, ſi cui utile videatur is hanc lateris inveſtigationem etiam ulterius continuare poterit. Hujus lateris ſubſidio facile invenientur latera quadratorum omnium datis circulis æqualiũ: ſunt enim circuli ut quadrata ab ipſorum diametris: itaque etiam quadrata ipſis circulis æqualia ſunt interſe, ut quadrata diametrorum.



Exponatur itaque circulus cui quadratum æquale conſtruendum ſit, datus circulus duabus normalibus diametris AB, C D ſecetur quidrifariam, tumque circino explorato in quaſnã expoſitæ regulę partes & partium particulas ea incidat, hic deprehẽdes digitos $3\frac{2}{1}$, jã per proportionẽ cætera concludas, ut diameter 2 ad $1_{1}\frac{7}{0}\frac{7}{0}\frac{2}{0}\frac{4}{0}$ latus quadrati ipſi æqualis, ſic diameter data $3\frac{2}{1}$ ad $2_{1}\frac{8}{0}\frac{3}{0}\frac{6}{0}$, qui ſunt digit 2 & $\frac{8}{1}\frac{3}{0}$ paulo minus. tũq; circinũ divaricato ut intervallũ crurũ ſint integri pollices 2 & $\frac{8}{1}\frac{3}{0}$ pollices tertij hoc latus erit quęſitũ quadrati circulo ut proximé æqualis: quare iſtis lateribus conſtructum quadratum KLMN dato circulo æquabitur. Par ratio eſt cęterorum omnium.

Eſto

Esto exemplum ſecundum, ſitque diameter circuli dati deprehēſa digitorum $2\frac{1}{3}$. unde per proportionem invenies latus quadrati æqualis $2\frac{6}{100}$, ſeu $2\frac{3}{50}$. tanto intervallo diducta circini crura dabunt latus quadrati circulo æqualis MN : quadratūque MNOP ab ea deſcriptum circulo dato ęquale.

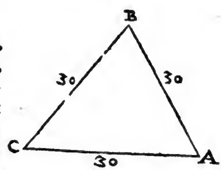


In majoribus vero illis ſolis numeris quæſitum ſolvetur. vt ſi quæratur latus quadrati æqualis circulo cujus diameter ſit 532 decempedarum. jam cum, ut ſupra monuimus, circuli ſint ut quadrata á diametris, vel etiam ut quadrata à peripherijs, proportionem inſtitues hoc modo, vt 4 quadratum diametri ad aream ſui circuli 3.1415926535, Ita 283024, quadratum datæ diametri 532, ad aream ſui circuli 222286.5297, tantus itaque erit circulus ab iſta diametro deſcriptus: quare hujus numeri latus 471.4727 erit latus quadrati dato circulo ęqualis.

Idem aliter, vt diameter 2 ad 1.77245385 latus quadrati ſuo circulo æqualis, ita data diameter 532 ad latus quadrati æqualis circulo ſuper data diametro deſcripti 471.4727, ut ſupra.

5 ZETEMA.

Datis lateribus trianguli æquilateri aream invenire.



Dantur latera AB BC, CA decempedarum 30, quæritur area, id per proportionem ſolvere licet, hoc modo. Area trianguli æquilateri cujus ſingula latera ſint 1 eſt $\sqrt{\frac{3}{16}}$. figuræ autem ſimiles ſunt ut quadrata homologorum laterum: quare ut 1 quadratum á latere ad $\sqrt{\frac{3}{16}}$, ita 900 quadratum á latere dato ad aream $\sqrt{151875}$, quæ in numeris abſolutis valet decempedas 389.71143 quadratas. ſeu decempedas 389, pedes 102, digitos quadratos $64\frac{2}{3}$, hoc eſt ut gæodætarum more loquar ad 389 decempedas quadratas, 8 pedes ſcamnatos, & $6\frac{2}{3}$ digitos ſcamnatos.

Vt latus è dato numero 151825 accurate eruerem decem circulos adjeci, latusq; inventum per 100000 diviſum dedit aream optatam 389.71143. quod ſi ad latus inveſtigandum circulos 12 adjeciſſem, latus erutum per 1000000 fuiſſet dividendum, atque ita in cæteris.

operis formula hæc eſt.

~~2~~13
~~13347~~27
~~5~~ ~~811647~~135
~~6745461567664~~51
~~1515750000000000~~
3|8|9|7|1| 1|4|3
~~676~~894422228
77779944
7 7 9
7

71143
144
284572
284572
71143
102|44592
144
1|78368
17|8368
44|592
digiti 64|21248

Quare area dati trianguli erit
389 decempedarum, 102 pedum, 64$\frac{2}{7}$ digitorum quadratorum

Vel ita more gæodætarum.

71143
12
142286
71144
pedes 8|53716
12
1|074
5|37
digiti 6|444

redeunt ut ſupra 8 pedes ſcamnati 6$\frac{2}{7}$ digiti ſcamnati

12 Idem

Idem etiam aliter

12		$\surd$151875
12		$\surd$ 20736
124		911256
12		455625
144		1063123
144		303750
576	5	$\surd$3149280000 digiti ſcamnati
576	$\cancel{2}\cancel{8}400$	
144	$\cancel{6}\cancel{1}\cancel{3}\cancel{6}\cancel{7}\cancel{7}\cancel{9}76$	
	$\cancel{3}\cancel{1}\cancel{4}\cancel{9}\cancel{3}\cancel{8}\cancel{0}\cancel{0}\cancel{0}\cancel{0}$	
$\surd$20736	5/ 6/ 1| 1| 8	ſunt ergo digiti ſcamnati
	$\cancel{1}\cancel{0}\cancel{1}\cancel{2}2222$	
	1 1 2	
	1	

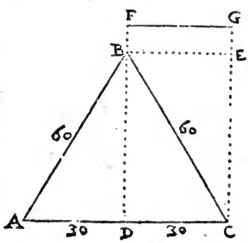
$$\begin{array}{r} \cancel{1} \\ \cancel{5}\cancel{0}\cancel{7}6 \\ \cancel{5}\cancel{6}\cancel{1}\cancel{1}\cancel{8} \\ \cancel{1}\cancel{2} \end{array}\left\{\begin{array}{r} \cancel{1}\cancel{0}\cancel{1}8 \\ \cancel{4}\cancel{6}\cancel{7}\cancel{6} \\ 12 \end{array}\right\{$$ 389 decempede, 8 pedes, 6 digiti.

Notabis decempedas $\surd$ 151875 reductas ad digitos ſcamnatos dare 56118, eoſque diviſos per 12 exhibere in quoto 4676 pedes ſcamnatos. cumq; duodecim pedes ſcamnati æquent unam decempedam quadratam, iſtos 4676 per 12 diviſos dare 389 decempedas, pedes ſcamnatos 8, digitos ſcamnatos 6 ut prius.

5 ZETEMA.

Latera trianguli æquilateri ſingula ſunto decempedarum 60, quæritur area.

Reſpondeo 1558$\frac{84572}{100000}$ decempedas habere in quadrum, ſive quod idem ſit 1558 decempedas 121 pedes, 112$\frac{3}{4}$ digitos quadratos. Ad aream trianguli æquilateri inveſtigandum in edito quodam libro pſeudotheorema tale aliquando legiſſe me memini. Latus per ſui ſemiſſem multiplicatum exhibet aream trianguli æquilateri. quod ipſum quanto facilius quæſitum præſtat, tanto longius à vero abit.

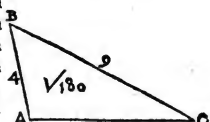


Neque ullo modo ad ſcopum collineat. ideoque hujuſmodi præceptis multi turpiſſime decepti ſunt, cum per geodætas ignariſſimos agrorum magnitudinem exploraverunt, ſiquidem emptores plus longe perſolverint quam par foret. atque ita juſtum pretum exceſſerunt. in hoc enim exemplo verum agri modum ſuperat decempedis 241 $\frac{}{1}\frac{1}{0}\frac{5}{0}\frac{4}{0}\frac{2}{0}\frac{8}{0}$. Sed ut hanc pſendographiam poſito diagrammate reſellam. Eſto triangulum æqui laterum ABC, quod dato agro ſivile concipiatur huic parallelogram mum DBEC æquealtum ſuper dimidia baſi DC deſcriptum per 38 & 41 propoſ. lib. 1 *Euclidis* æquabitur itaque altitudine BD $\sqrt{2700}$ multiplicata per DC 30, dabitur area trianguli ABC $\sqrt{2430000}$, quæ ſunt decempedæ quadratæ 1558 $\frac{}{1}\frac{8}{0}\frac{4}{0}\frac{5}{0}\frac{7}{0}\frac{2}{0}$, ut ſupra, continuetur autem eadem perpendicularis in F ut DF lateri BC æqualis ſit: quare rectangulm ab FD 60 et DC 30 comprehenſum erit 1800 decempedarum, quam aream juſto agri modo majorē vel oculis eſt perſpicuum, excedit enim parallelogrammo FBEG hoc eſt 241 $\frac{}{1}\frac{1}{0}\frac{5}{0}\frac{4}{0}\frac{2}{0}\frac{8}{0}$ decempedis quadratis. Iſtud pſeudotheorema in libro quodam in ſuperiore Germania germanico idiomate editum deprehendi. Sed aliud longe infelicius et à vero magis alienum offendi in libello Antuerpiæ typis excuſo anno ſaluiis 1547.

Quæritur area trianguli æquilateri, cujus ſingula latera ſint 28 decempedarum. Hic vera & accurata area erit $\sqrt{115248}$, hoc eſt paulo minus $339\frac{1}{2}$ decempedis. Verum ille iſtam ipſam, queſtiunculam ſolvit hoc modo. Latus dati trianguli æquilateri quadratum facit 784, huc latus ipſum addito ſumma erit 812, cujus dimidium 406 eſt area dati trianguli: quàre iſta quoque regula verum hic excedit $66\frac{1}{2}$ decempedis Et ſi latera ſingula ſtatuantur 60 ut in ſuperiore exhiberet aream 1830. quare cum utrumque hoc theorema modum excedat, tamen hoc noviſſimum etiam maximé.

6 ZETEMA.

Datur area trianguli æquilateri 200 pedum quadratorum quæruntur ipſa latera.



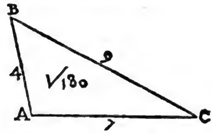
Reſpondeo 21 $\frac{}{1}\frac{4}{0}\frac{9}{0}\frac{1}{0}\frac{8}{0}$, ſiue $\sqrt{}\sqrt{213333\frac{1}{3}}$ ſingula in longitudinem patere. ſimilitudine hoc zetema ſolves. quia figuræ ſimiles ſunt inter ſe ut quadrata ab homologis lateribus. atque inde proportio, ut area trianguli æquilateri $\sqrt{}\frac{}{1}\frac{3}{0}$ ad quadratum ſui lateris 1, ſic area triãguli 200 ad quadratũ ſui lateris $\sqrt{213333\frac{1}{3}}$ cujus latus erit $\sqrt{}\sqrt{213333\frac{1}{3}}$, hoc eſt 21 $\frac{}{1}\frac{4}{0}\frac{9}{0}\frac{1}{0}\frac{4}{0}$ & paulo amplius. ad hujus generis zetemata ab eodem autore theorema tale proponitur. Datam aream per 8 multiplicato, fiunt 1600, ad factum unitatem addito, & á ſummæ latere quadrato unitatem rurſum ſubducito, reliquumque numerum per 2 dividito, quotus erit optatum quæſiti trianguli latus. Hujuſmodi præceptiunculæ

tiunculæ periculosæ, neque ullo solido fundamento sustructę sunt. Ab ipso affertur exemplum trianguli cujus area sit 210 pedum, cujus latus concludit esse pedum 20, quæ mera hallucinatio est. namque area trianguli æquilateri cujus latus sit 20 pedum tantum erit $\sqrt{}$3000 hoc est 173$\frac{205}{1000}$, non autem 210 quemadmodum ille vult.

7 ZETEMA

Datis lateribus trianguli AB 4, BC 9, AC 7 decempedarum quæritur area.

Respondeo $\sqrt{180}$, hoc est decempedarum 13$\frac{2}{7}$ paulo amplius.

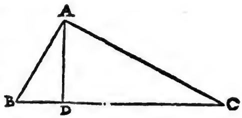


In libro Germanico idiomate conscripto regula talis extat, maximorum duorum laterum semissis per tertij lateris dimidiū multiplicatus dat aream. ita hic addas 9 & 7, summæ 16 dimidium 8 per dimidium tertij 2 dat aream 16. ab his pseudographemasi, tanquam scopulis periculosissimis sedulo est tibi cavendum.

8 ZETEMA.

In triangulo rectangulo datorum laterum AB 8 BC 17 AC 15 ex angulo recto A demissa est perpendicularis AD, quæritur ipsa & insuper segmenta basis BD, DC.

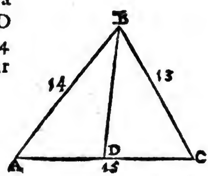
Respondeo valere AD 7$\frac{1}{17}$, BD 3$\frac{9}{17}$, DA 13$\frac{4}{17}$. Nam per 8 propos. lib. 6. *Euc.* & 63 nostram libri secundi, ut BC 17 ad AC 15, AB 8 ad AD 7$\frac{1}{17}$. Item ut BC 17 ad AB 8, sic AB 8 ad BD 3$\frac{13}{17}$. Et quemadmodum BC 17 ad CA 15, sic CA 15 ad CD 13$\frac{4}{17}$. Denique si aream ejusdem trianguli postules, multiplicato dimidium cruris AB in reliquum crus AC, factus 60 erit area dati trianguli. vel etiam si dimidiam basin BC 8$\frac{1}{2}$ multiplices cum perpendiculari AD 7$\frac{1}{17}$ factus 60 erit area optata.



9 ZETEMA.

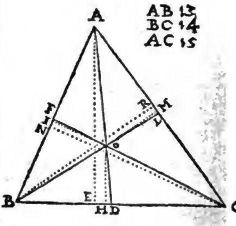
Trianguli datorum laterum AB 14 BC 13 AC 15 angulus B bisecat linea BD, quæruntur basis segmenta AD DA.

Respondeo AD 7$\frac{7}{9}$, et DC$\frac{7}{9}$ Namque per 61 nostrā propositionem libri secundi secatur basis AC in D ratione crurum AB 14 ad BC 13. Itaque ut 72 ad 14 sic 15 ad AD 7$\frac{7}{9}$ quamobrem pro DC relinquentur 7$\frac{2}{9}$. eritque 14 ad 13 sic 7$\frac{7}{9}$ ad 7$\frac{2}{9}$.



10 ZETEMA.

Trianguli ABC anguli bisecantur lineis AD, BL & CF quarum omnium sectio communis punctum O, quæritur quantitas segmentorum AO, OD, CO, OF, BO, OL.



Respondeo, AO $\sqrt{65}$, OD $\sqrt{16\frac{1}{4}}$: CO $\sqrt{80}$, OF $\sqrt{16\frac{6}{7}\frac{4}{7}}$: BO $\sqrt{52}$, OL $\sqrt{16\frac{4}{7}}$. Hic tibi ante per zetema proximum erunt quærenda laterum segmenta, eorum magnitudinem ad dextram numeris expressam vides. Porrò autē per 55 nostram prop. lib. 2. perpēdiculares OH IO OM ex concursu O in subjecta latera æquari: & præterea segmenta contermina CH CM, itemque AM AI, IB BH, quę per citatū theorema facillime inveniri quæuntq;, est earū quantitas tanta quātum annotatam vides. Cum itaque laterum segmenta BD DC, CL LA, AF FB nota sint, itemque ED LR NF, facile invenies ipsas angulorum bisectrices BL AD & CF; nam quadrata AE perpendicularis & intersegmenti ED æquantur quadrato lineæ AD, hoc est 144 & $2\frac{1}{4}$ addita conflant summam $146\frac{1}{4}$ cujus latus dabit pro AD $\sqrt{146\frac{1}{4}}$. Hujus autem segmenta AO OD per similitudinem triangulorum AED & HOD investigantur hoc modo, vt AE 12 ad AD $\sqrt{146\frac{1}{4}}$, sic OH 4 ad OD $\sqrt{16\frac{1}{4}}$ ea deducta de AD reliquam facit AO $\sqrt{65}$. Idem aliter OG parallela OI contra basin BC, desecat segmentum EG æquale ipsi OH, ut AE sit partium 8 GE 4. quarum ratio est ut 2 ad 1: quamobrem AD secundum hanc rationem secta dabit segmēta eadem quæ prius.

AE 12
BR $11\frac{1}{3}$
CN $12\frac{1}{5}\frac{8}{9}$

BD $6\frac{1}{2}$
DC $7\frac{1}{2}$
AF $6\frac{2}{2}\frac{1}{9}$
FB $6\frac{8}{2.9}$

BL $\sqrt{1 \cdot 5\frac{6}{8}\frac{7}{8}}$
CF $\sqrt{167\frac{6}{8}\frac{7}{4}\frac{3}{8}}$
AD $\sqrt{146\frac{1}{4}}$

BD $6\frac{1}{2}$
CF $7\frac{1}{2}$
HD $\frac{1}{2}$
DE $1\frac{1}{2}$

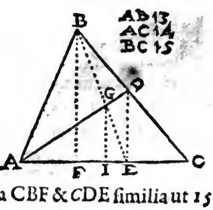
AL $7\frac{2}{5}$
LC $7\frac{7}{9}$
ML $\frac{7}{9}$
RL $1\frac{8}{4\,9}$

NF $\frac{3}{3}\frac{7}{5}\frac{6}{7}$
IF $\frac{8}{2\,9}$

perp. OM, OM, OF 4
HC. CM 8
AM AI 7
IB IH 6

11 ZETEMA.

Trianguli ABC lateribus datis angulóque A á linea AD bisecto, quæritur quanta sit perpendicularis DE ex D in latus AC demissa.



Respondeo DE $6\frac{2}{9}$, & EC $4\frac{2}{3}$. Quæratur enim per zetema 9 linea DC $7\frac{7}{9}$, deinde perpēdicularis ex B demissa sit BF, erunt itaque triangula CBF & CDE similia ut 15 ad BF 12, sic CD $7\frac{7}{9}$ ad DE $6\frac{2}{9}$.

ZETE-

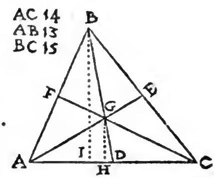
12 ZETEMA.

Quæritur in eodem diagrammate, si agatur BE *quanta sint segmenta* BG, *GE, AG, GD.*

Respondeo BG $\sqrt{}\,55\frac{690}{4489}$, GE $\sqrt{}\,28\frac{17332}{40401}$, AG $\sqrt{}\,81\frac{3303}{4489}$, GD $\sqrt{}\,4\frac{268012}{363609}$ Quæratur por præcedentem AD $\sqrt{}\,125\frac{67}{81}$: deinde BE ex qudratis BF & FE. tumque secato BE ratione crurum *B*A & AE dehinc concludetur per proportionem hoc modo ut EB ad E*F*, sic GE ad EI. & ut *A*E ad AD sic AI *A*G & sic IE ad GD. Denique etiam si postules segmenta perpendicularis AF, quæ ab *A*D intersecatur in, H fiat ut AB ad AF sic BH ad HF. Atque hinc observato quanta commoda è propositione 3 lib 6 *Euc.* in Geometriam redundent.

13 ZETEMA.

In exposito triangulo ABC datorum laterum rectæ ab angulis eductæ latera bisecantes sese in G intersecant quæruntur ipsarum segmenta FG GC, BG GD, AG GE, denique ipsa perpendiculares ex puncto G in latera demissæ, ut GH.



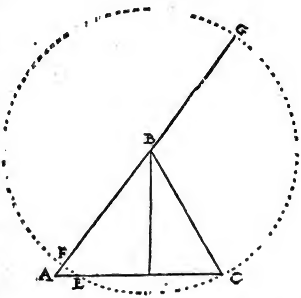
Laterum quantitatem hic à latere adscriptam vides, quantam in superiore exemplo usurpavimus. Respondeo itaque BD valere 148, GD $\sqrt{}\,16\frac{4}{9}$, GB $\sqrt{}\,65\frac{7}{9}$: AE $\sqrt{}\,126\frac{1}{4}$, GE $\sqrt{}\,14\frac{1}{36}$, GA $\sqrt{}\,56\frac{1}{9}$: CF $\sqrt{}\,168\frac{1}{4}$, FG $\sqrt{}\,18\frac{25}{36}$, GC $\sqrt{}\,74\frac{7}{9}$. denque rectæ GH ex G perpendiculares in AC 4 indidem perpendicularis in BC $3\frac{11}{15}$, in *A*B est $4\frac{4}{13}$. Solutionis via hac est. Primum omnium inquirito perpendicularem BI 12, segmentum basis AI 5 subductum de AD 7 lateris dimidio, relinquet ID 2, hic itaque ID & IB æqué possunt ipsi BD, quæ propterea erit $\sqrt{}\,148$. eandem operis rationem secutus inveni. es lineas AE FC ea quantite quam supra notavimus. Ad investigationem autẽ segmenti GH habes triangula duo similia BID GHD ubi majoris trianguli latera omnia dantur, in minore autem perpendicularem GK esse partium 4 infra demonstrabitur. itaque laterum analogia talis erit. Vt ut BI 12 ad BD $\sqrt{}\,148$, sic HG 4 ad GD $\sqrt{}\,16\frac{4}{9}$: quare reliqua GB erit $\sqrt{}\,65\frac{7}{9}$ eodemque modo etiam reliquarum segmenta assequeris: sed perpendicularem GH esse partium 4 ita constat. cum enim triangula ADB CDB ęquæalta sint & in æquali basi erunt æquali: at eandem ob causam ADG DGC inter si quoque æquantur: quamobrem si ab illis hæc subducantur reliqua ABG CBG itidem æqualia erunt. haud alia ratione ABE ACE, & BEG GCE æquantur, ideoque etiam reliqua ABG ACG itidem æquantur: quare tria triangula ABG, BGC, CGA æqualia totum triangulium in tres trientes dispescunt ut & cætera triangula singula verticibus in centro gravitatis coeuntia tertiam totius partem habent.

Cumque totius area sit 84 par tertia ejus 28 divisa per $7\frac{1}{2}$ dimidiũ ipsius AC dabit perpendicularem GH 4. et per $7\frac{1}{2}$ dimidium BC dabit in quoto $3\frac{11}{15}$. denique per semissem BA $6\frac{1}{2}$ pro perpendiculari ex G in latus AB demissa exhibet $4\frac{4}{13}$.

Ad investigationem bisecantium BD, AE, CF nulla perpendiculari opus esse alibi demonstravimus. Neque verò ad segmentorum BG, GD anquisitionem eam adhiberi opus fuit. Cum enim ostensum sit AGC esse totius partem tertiam, sequitur AGD esse totius partem sextam; quamobrem triangula BADG AD æquealta ad A erunt ut bases BD ad DG. atqui ABD est tripla GAD itaque GD erit pars tertia totius BD. et BG $\frac{2}{3}$ totius BD. ut quare at 3 ad 1 sic BD $\sqrt{148}$ ad G $\sqrt{19\frac{4}{9}}$. Et ut 3 ad 2 sic BD $\sqrt{148}$ ud BC $\sqrt{65\frac{7}{9}}$. atque ita in reliquis semper enim BG ipsius GD, et AG ipsius GE, & CG ipsius GF erit dupla.

14 ZETEMA.

Data base crurum summa et distantia perpendicularis ab altrutro basis angulo invenire triangulum.



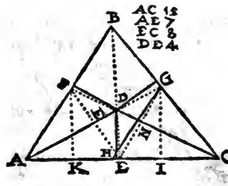
Basis propositi trianguli AC esto partium 15 summa laterum AB & BC 27, perpendicularis ex angulo verticis B demissa incîdat basin in D, sitque segmentum DA $8\frac{2}{5}$, quæruntur latera AB, BC.

Respondeo, AB esse partiũ 14 BC. 13 concipiatur enim triangulum ABC é cujus vertice minoris cruris intervallo descriptus sit circulus & reliquum latus AB continuatum in G. erit itaque FG diameter, & per 36 propos. lib. 3 *Euclid*. rectangulum sub GA & AF comprehensum rectangulo CAE æquale. Et cum per 3 propos. 3 lib. *Euc*. EC bisecta sit á perpendiculari BD, deturque DC dabitur AD $8\frac{2}{5}$ & AC 15, erit reliqua AE $1\frac{4}{5}$. rectangulum autem sub AC 15 & AE $1\frac{4}{5}$ est 27, id per AG lineam divisum dabit unitatem in quoto pro AF, ea de AG 27 subducta relinquit FG 26, dimidinm BG 13, pro crure minore BC itaque pro majore AB 14.

15 ZETEMA.

In triangulum ABC inscriptus est circulus cursus centrum D, unde perpendicularis DE demissa facit basis segmenta AE 7, EC 8 quæruntur latera AB BC.

Respon-



Respondeo AB 13, BC 14. Ad hujus generis zetematum solutionem figuram præparabis ut hic uides vbi deinde notabis triangula EDC DGC esse æquilatera. itemque ADE & ADF. deinde rectam GL perpendicularem esse diagonio DC, & EE ipsi AD. quare triangulo DGC DGL similia erunt. & altrinsecus DNF DNF per 8 propos. libri 6 *Euc.* angulos autem ad L & N rectos ex 11 & 13 propos. lib. 1. *Euclid.* constare poteſt. datur autem DC $\sqrt{80}$ é potentia daorum DE & EC. unde per proportionem concludatur LG, vt DC $\sqrt{80}$ ad CG 8, sic DG 4 ad GL $\sqrt{12\frac{4}{7}}$, cujus duplum est ipsa EG $\sqrt{51\frac{3}{7}}$. Haud aliter invenienda erit EF $\sqrt{48\frac{1}{6}\frac{6}{7}}$. unde jam facile erit perpendicularium FK GI quantitatem eruere, & segmentorum AK IC. Hinc recta GH acta est parallela lateri AB ideoque GHI FAK triangula similia sunt, cum angulis ad A & H æquales, ad K & I rectos habeant per antecedentia autem dantur latera trianguli AFK, & datur latus IG nnde reliqua GH & HI per proportionem concludentur. Nam ut FK $6_{5}\frac{2}{7}$ ad FA 7, sic GI $6\frac{2}{7}$ ad GH $7\frac{3}{7}$. similiter concludes segmentum HI $3\frac{2}{3}\frac{7}{7}$ quod cum IC compositnm exhibet totam HC $8\frac{4}{7}$. Vnde ad extremum optata latera AB & BC concludentur propter similitudinem triangulorum HGC ABC. quare ut HC $8\frac{4}{7}$ ad HG, $7\frac{3}{7}$ sic AC 15 ad AB 13. Et quemadmodum HC $8\frac{4}{7}$ ad CG 7, sic AC 15 ad BC 14.

DC $\sqrt{80}$
AD $\sqrt{95}$
LG $\sqrt{12\frac{4}{7}}$
EG $\sqrt{51\frac{3}{7}}$
LD $\sqrt{3\frac{1}{7}}$
LC $\sqrt{51\frac{1}{7}}$

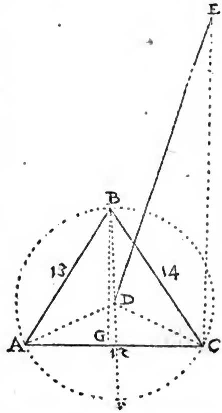
Perpendiculares

GI $6\frac{2}{7}$
FK $6_{5}\frac{2}{7}$
FE $\sqrt{48\frac{1}{6}\frac{6}{7}}$
AN $\sqrt{36\frac{6}{6}\frac{1}{7}}$
ND $\sqrt{3\frac{6}{6}\frac{1}{7}}$
HI $3\frac{2}{3}\frac{7}{7}$
IC. $4\frac{4}{7}$
HC $8\frac{4}{5}$
FN $\sqrt{12_{5}\frac{4}{7}}$
AF 7
GC 8.

16 ZETEMA.

Datis triangulæ pyramidis lateribus cujus ſtātia tria latera æquantur, perpendicularem à vertice in basin demiſſe iuvenire.

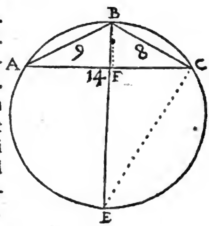
 Exem-



Exemplum ab autore tale concipitur et propònitur. In campi planitie triangula trabs perpendicularis excitata est à cujus vertice ad trianguli subjecti angulos rectæ omnes æquales videlicet $29\frac{6}{7}\frac{5}{8}$ partiũ, quæritur trabis illius altitudo. Respondeo $28\frac{7}{9}$. Namque cum ab angulis trianguli Iacentis ad verticem colossi sive trabis se par sit intervallum, ipse colosus centro circuli triangulo circumscripti erit perpendicularis: quamobrem diametro inventa zetema hoc facile selvitur. Diameter porrò in omnibus triangulis invenitur hoc modo. sit BF circuli diameter et perpendicularis ex eodem angulo BG. erunt itaque triangula BAC, BFC similia, cum anguli ad A et F in eadem sectione per 21 prop. 3 lib. *Euclid.* æquentur, et ad C et G recti sint. Latera autem trianguli ABG facile innotescunt é datis lateribus ttianguli A B C. unde, per proportionē diameter B F concludetur hoc modo. Quemadmodum BG $11\frac{1}{7}$ ad BA 13, sic B C 14 ad BF $16\frac{1}{4}$: dimidium $8\frac{1}{8}$ pro radio BD, DC, vel D A: datur autem latus EC $29\frac{6}{7}\frac{5}{8}$, quod angulo recto subtenditur, quamobrem quadratum DC $42\frac{2}{6}\frac{1}{4}$ subductum de quadrato CE $462\frac{5}{7}\frac{4}{7}\frac{0}{8}\frac{9}{4}$ relinquit quadratum colossi perpendicularis DE $42\frac{9}{7}\frac{3}{7}\frac{1}{8}\frac{8}{4}$ cujus latus $20\frac{7}{7}\frac{2}{8}$ seu $28\frac{7}{9}$ ipsa quæsita altitudo est.

17 ZETEMA.

Datis trianguli lateribus circuli circumscripti diametrum invenire.



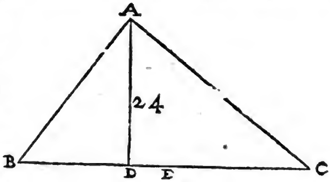
Sunto latera AC 14, AB 9, BC 8, quæritur quanta sit diameter BE. Respondeo $\sqrt{224\frac{2}{2}\frac{2}{6}\frac{4}{7}\frac{}{7}}$. Hujus Zetematis solutionem superori immixtam, hic distincte iterum exhibemus. solventut autem ista talia per 21 propos. lib. 3 *Euclid.* er per 13 lib. 2 *Euclid.* Inveniatur primo perpendicularis BF $\sqrt{23\frac{1}{7}\frac{0}{8}\frac{3}{4}}$. Cum itaque duo triangula ABF, BEC similia sint erit ut BF $\sqrt{23\frac{1}{7}\frac{4}{8}\frac{3}{4}}$ ad BA 9, sic BC 8 ad diametrum circuli BE $\sqrt{224\frac{2}{2}\frac{2}{6}\frac{4}{7}\frac{}{7}}$. Et rursum ut BF ad AF sic BC ad CE $\sqrt{160\frac{2}{2}\frac{2}{6}\frac{4}{7}\frac{}{7}}$, eandem tibi invenire licebit si quadratum BC subducas de quadrato BE reliqui enim latus ipsam CE tibi quoque exhibebit.

18 ZETEMA.

In triangulis rectangulis data perpendiculari ab angulo recto in basin cum ipsa basi latera invenire.

Basin

Basis *B*C sit partium 50, perpendicularis ab angulo recto AD 24 quæruntur latera A*B*, AC. Respondeo, AC 40. AB 30. Statuatur enim AD 24 proportionalis inter segmenta basis hoc mode subducto quadratum perpendicularis AD 575 de quadrato dimidie basis AC 625 reliqui 49 latus 7 additium ad dimidiam basin

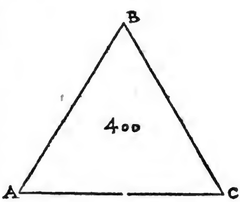


AC 25 dabit segmenrum majus DC 32, et ab eodem dimidio subductum dabit BD segmentum minus 18. porro autem cum *B*D et DA æquæ posſint ipsi BA, & DD, DA ipsi AC, estæ quoque dabuntur in eadem mensura. AB quidem 30, & AC 40: cæterum perpendicuiarem AD proportione mediam esse inter segmenta basis BD, DC éx 8 propos. 6 *Euc.* constat. eoque quadratum ejus æquatur rectangulo segmenrorum. sed rectangulum segmentorum BD, DC cum quadrato DE æquatur quadrato bisegmenti ED per 5 propos. lib. 2 *Eucl.* quare subducto quadrato AD de quadrato EB relinquitur quadratum intersegmenti ED, quod ad semissem EC additum et a semisse altero EB subductum dabit segmenta inæqualia BD DC. reliqua jam sunt planiora.

19 ZETEMA.

Data area trianguli æqui lateri quæruntur latera.

Exponatur triangulum ABC cujus area sit 400 dece pempedarum quæritur lateaum magnitudo. Respondeo $\sqrt{\sqrt{853333\frac{1}{3}}}$ invenito ante cujuslibet æquilateri trianguli trianguli aream, ut si latus assumamus 1 ejus area eri $\sqrt{\frac{3}{16}}$ und proportio ut area $\sqrt{\frac{3}{16}}$ ad quadratum lateris 1. sic area 400 ad quadratum à suo latere $\sqrt{853333\frac{1}{3}}$. Cujus latus $\sqrt{\sqrt{853333\frac{1}{3}}}$ sunt enim triangula æquilatera inter se similia. figuræ autem simule per 19 propos. lib. 6 *Euclid*, sunt ut quadrata homologorum laterum. Hujus periculum recténe in secus operatus sis ita facies invenito perpendicularem à B in basin AC demissam ea erit $\sqrt{\sqrt{480000}}$. hæc per $\sqrt{\sqrt{5333\frac{1}{3}}}$ dimidium basis AC multiplicata dabit $\sqrt{\sqrt{25600000000}}$ hoc est 400 pro area trianguli quemadmodum oportint.



20 Zetema.

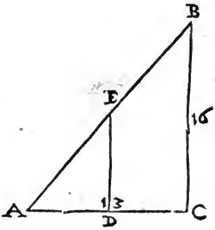
Data trianguli laterum ratione et area ipsa latera invenire.

Eſto triangulum cujus area ſit $\sqrt{75\frac{15}{16}}$. ratio autem laterum ſit ut 2, 3, 4, quæruntur ipſa latera. Invenito aream his ipſis lateribus 4, 3, 2 cõprehenſam ea erit $\sqrt{8\frac{7}{16}}$. Vnde proportio, quẽadmodum $\sqrt{8\frac{7}{16}}$ ad 16 quadratum à lateræ 4, ita area data $\sqrt{75\frac{15}{16}}$ ad quadratum 48 cujus radix $\sqrt{48}$ eſt latus maximum ipſi 4 homologum, ſic medium eadem analogia concludes $\sqrt{27}$, & minimum $\sqrt{12}$. demonſtratio ſimilis ſuperiori.

21 ZETEMA.

Trianguli ABC angulus C, rectus ejus crura AC 13, BC 16 unde, parallela DE deſectũ eſt triangulum ADE cujus area ſit 24: quæruntur latera AD, DE, AE.

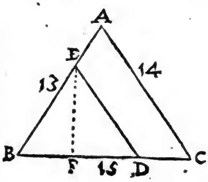
Reſpondeo AD $\sqrt{39}$, DE $\sqrt{59\frac{1}{13}}$, AE $\sqrt{98\frac{1}{13}}$. Inveniatur enim area trianguli ABC multiplicata baſi AC 13 in dimidiã altitudinẽ B C 8, factus 104 erit area quæſita. vnde proportio ut 104 ad 169 quadratum ab *AC*, ſic 24 area trianguli AED ad quadratum AD 39, cujus radix $\sqrt{39}$ pro latere dicto. atque ita in cæteris. demonſtratio antecedenti germana eſt. Operis periculum facies ſi dimidiã AD $\sqrt{9\frac{3}{4}}$ multiplices per altitudinem DE $\sqrt{59\frac{1}{13}}$ habebis aream AED $\sqrt{576}$, ſeu 24 quemadmodum decuit.



22 ZETEMA.

De triangulo ABC datorum laterũ AB 13, AC 14, BC 15 decempedarum abſumptum eſt ſpatium BED datæ magnitudinis 40 decempedarum, idque linea contra AC latus parallela, quæruntur latera.

Reſpondeo BD $\sqrt{107\frac{1}{7}}$, BE $\sqrt{80\frac{10}{21}}$, DE $\sqrt{93\frac{1}{3}}$. Sunt enim ABC EDB triangula ob paralleliſmum laterum ED AC ſimilia. area autem trianguli ABC eſt 84. Vnde proportio, quemadmodum 84 ad quadratum latere *AC* 196, ſic 40 ad quadratum lateris ED $93\frac{1}{3}$: ideoque ipſa linea ED $\sqrt{93\frac{1}{3}}$. eadem via aſſequeris quantitatem rectarum BE & BD. cujus periculum facies ſi

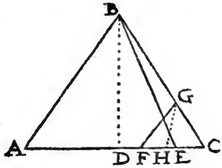


perpen-

perpendicularem EF $\sqrt{59\frac{1}{7}}$ multiplices per semissem basis BD $\sqrt{1\frac{2}{7}}$, factus enim 40 est area trianguli EBD, quemadmodum optabatur.

23 ZETEMA.

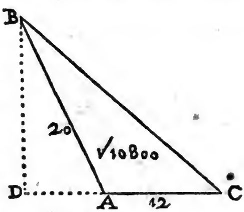
Datis lateribus trianguli ABC ut CB 273, decempedarum AB 294, AC 315 unde dissectum sit triangulum GCF jugerum 7 decempedarum 210, ita ut GF ipsi AB parallela sit quærantur hujus trianguli latera.



Investigato primum aream totius trianguli ABC decempedarum 37044 quadratarum: cum autem nobis hic jugerum constet 600 decempedis quadratis, trianguli absumendi area erit decempedarum 4410. jam spatium istud absumendum 4410 per dimidiam perpendicularem divisum dabit in quoto EC $37\frac{1}{2}$ basin trianguli EBC absumendo æqualis & dato ABC æquealti. tumque inter AC & EC media proportionalis sit FC $\sqrt{11812\frac{1}{2}}$, ab F ducta parallela FG desecabit triangulum FGC dato spatio æquale 7 jugerum & 210 decempedarum. reliqua latera facile ex similitudine concludentur, FG $\sqrt{10290}$, CG $\sqrt{8872\frac{1}{2}}$. ut operis veritatem comprobes anquirito perpendicularem GH $\sqrt{6587\frac{3}{7}}$. hujus dimidium $\sqrt{1646\frac{6}{7}}$ per basin FC $\sqrt{11812\frac{1}{2}}$ multiplicatum dabit aream trianguli FGC 4410, ut petebatur.

24 ZETEMA.

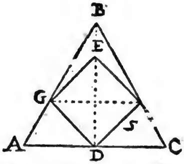
Data trianguli area & lateribus duobus cum specie anguli ab ipsis comprehensi tertium latus invenire.



Exponatur area trianguli ABC $\sqrt{10800}$ decempedarum quadratarum, & AB decempedarum 20, AC 12, quæritur quantum sit futurum latus tertium BC? Respondeo 28. demittatur enim perpendicularis à vertice B, quæ isto casu cadat extra: cum itaque area trianguli sub dimidia basi & altitudine comprehendatur, quod é 38 propos. lib. I. *Euc.* facile demonstrari potest: ideo etiam contra areâ $\sqrt{10800}$ per dimidiam basin 6 divisâ dabitur perpendicularis BD $\sqrt{300}$ cujus potentia 300 de potentia AB 400 subducta relinquit potentiam lateris AD 100, atque ideo ipsa AD est partium 10. hæc ad AC addita facit totam DC 22. sed cum BD & DC potentiæ sint 784, ipsa BC erit 28.

25 ZETEMA.

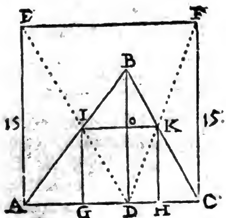
Si in triangulum æquilaterum ABC inscriptum sit quadratum DFEG tribus angulis D, F, G latera trianguli contingens, & GF diagonio contra basin AC parallela, sitq; latus quadrati partium 5, quæruntur latera trianguli.



Respondeo singula latera esse $\sqrt{50}+\sqrt{16\frac{2}{3}}$. cũ enim ex thesi diagonius GF sit parallela basi AC, erit triangulum BGF triangulo ABC simile, ideoque æquilaterum, atqui posito latere GD partium 5 diagonius GF seu ED datur $\sqrt{50}$, cui çquãtut GB BF quare GH semissis GF erit $\sqrt{12\frac{1}{2}}$. hujus quadratum de quadrato lineæ GB $\sqrt{50}$, relinquit quadratum $37\frac{1}{2}$, ideoque ipsa BH est $\sqrt{37\frac{1}{2}}$. huc addita semidiagonius HD exhibet totam perpendicularem BD $\sqrt{37\frac{1}{2}}+\sqrt{12\frac{1}{2}}$: Vnde proportio, ut BH $\sqrt{37}$ ad BG $\sqrt{50}$, Sic BD $\sqrt{37\frac{1}{2}}+\sqrt{12\frac{1}{2}}$ ad BC $\sqrt{50}+\sqrt{16\frac{2}{3}}$ latus trianguli ABC optatum.

26 ZETEMA.

In datum positione triangulum quadratum inscribere cujus basis in base trianguli reliquique duo anguli in reliquis lateribus terminentur.



Datur triangulum ABC, cujus latera sint expositæ mensuræ AB 14, BC 13, AC 15, quæritur latus quadrati HGIK, super base AC construatur quadratum AEFC, et à vertice B sit perpendicularis in basin BD. inde à D ad angulos quadrati E & F sunto rectæ DE DF intersecantes latera AB BC in punctis I & K, tumque connexa IK erit latus optati quadrati inscribendi. demissis itaque perpendicularibus IG KH erit GHKI maximum quadratum quod quidem ita dato triangulo inscribi possit. Lateris quantitatem secundum expressam mensuram invenies hoc modo. cum triangula AED & IGD: item CBD, CHK & BOK: nec non DFC, DKH: & BDA IAG atque BOI triangula similia sint, etiam lateribus proportionalia erunt. porro autem perpendicularis BD est $11\frac{1}{5}$, DC $6\frac{3}{5}$, AD $8\frac{2}{5}$. AE aut CF 15: quamobrem ut FC 15 ad CD $6\frac{3}{5}$, sic KH ad HD: sed FC ad CD habet rationem quam 15 ad $6\frac{3}{5}$ seu 25 ad 11, hoc est $2\frac{3}{11}$. Et rursum quemadmodum BD $11\frac{1}{5}$ ac DC $6\frac{3}{5}$, sic KH ad HC. quare KH ad HC habet rationem quam $11\frac{1}{5}$ ad $6\frac{3}{5}$, seu quam 56 ad 33, quæ est ratio $1\frac{23}{33}$. quare cum duæ magnitudines eidem tertiæ sint proportionales inter se quoque proportionales erunt: unde efficitur rationem segmentorũ lineæ DC, videlicet DH ad HC esse ut $1\frac{23}{33}$ ad $2\frac{3}{11}$. Itaque

linea

linea DC secundum hanc rationem divisa dabit segmentum DH $2\frac{538}{655}$, & HC $3\frac{102}{131}$. unde proportio ut DC $6\frac{3}{5}$ ad CF 15, sic DH $2\frac{538}{655}$. ad HI $6\frac{108}{131}$ pro latere quadrati inscripti. Operis veritatem etiam comprobare tibi licebit hoc modo, ut AE 15 ad AD $8\frac{2}{5}$ sic IG ad GD. quare ratio IG ad GA est ut 75 ad 42: hoc est $1\frac{11}{14}$. & ut BD $11\frac{1}{5}$ ad DA $8\frac{2}{5}$, sic IG ad GA, quare IG ad GA habet rationem 56 ad 42 seu $1\frac{1}{3}$. quamobrem AD secta secundum hanc rationem $1\frac{11}{14}$ ad $1\frac{1}{3}$ exhibet segmentum AG $4\frac{106}{131}$, & GD $3\frac{387}{655}$. Tumque instituatur proportio vt AD $8\frac{2}{5}$ ad AD 15, sic GD $3\frac{387}{655}$, ad GI $6\frac{108}{131}$ latus quadrati quantum supra exhibuimus. Denique etiam si segmenta GD & DH componas tantundē quoq; efficitur, videlicet $6\frac{108}{131}$. ita tribus vijs eadem veritas explorata sibi cōgruit. Si verò quadratū tale cōstruas super latere AB 14, ejus latus erit $6\frac{6}{13}$.

Vltimo si super latere trianguli minimo BC 13 tale quadratum construxeris, illud quadrati inscripti latus erit $6\frac{162}{337}$.

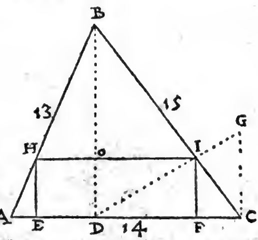
Ex hac inductione illud liquet, si maximum quadratum triangulo ita inscribendum sit ut latus ejus unum sit segmentum basis trianguli & reliqui anguli reliqua latera contingant, tum minimum trianguli latus pro base assumendū, quale hic fuit BC, id enim maximum quadrati latus nobis exhibuit.

Cum in rectangulis & obtusangulis triangulis unicum duntaxat tale quadratum super recti vel obtusi base describi possit, consectariolum istud in illis locum non habet. Verumtamen Zetematis hujus solutionem minus operosam (quæ non paulo intricatior esset si perpendicularis BD numero surdo definiatur) è demonstratione nostra problematis. 24 libri superioris tanquam generali fonte tibi unica proportione derivare licebit, hoc modo.

Vt basis cum perpendiculari ad basin, sic perpendicularis ad latus quadrati inscribendi. Esto BD *ut ante* $11\frac{1}{5}$ AC 15. *ergo ut* $26\frac{1}{5}$ *ad* AC 15, *sic* BC $11\frac{1}{5}$ *ad* GD *seu KH* $6\frac{108}{131}$, *atque ita cæteris.*

27 Zetema.

Datis lateribus trianguli cui inscriptum est oblongum EFIH, laterisque stantis ad iacens ratione data quæ 5 ad 14, quæruntur latera.



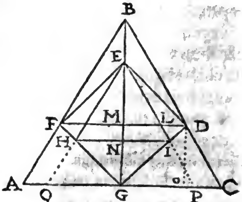
Respondeo HE $3\frac{9}{17}$ & EF $9\frac{15}{17}$. Namque ut 14 ad 5 sic sit AC ad perpendicularem CG, unde á vertice *B* perpendicularis sit in eandem BD, & connectatur DG recta IF ex I perpendicularis in basin AC erit latitudo optata, & IH parallela contra basin AC erit longitudo ejusdem parallelogrammi quæsita. Lineas autem ipsas per numeros explicabis hoc modo. ratio GC ad CD, ea via quam superiore zetemate exhibuimus inventa esto $\frac{5}{9}$ & BD ad DC, $1\frac{1}{3}$: secundū has rationes divisa est DC in F: quamobrem exhibentur segmenta DE $6\frac{6}{17}$, FC $2\frac{11}{17}$. Vnde proportio ut *D*C 9 ad CG 5 sic DF $6\frac{6}{17}$, ad FI $3\frac{9}{17}$: quare longitudo parallelogrammi HI vel EF erit $9\frac{15}{17}$. Veritatem operis iterata segmentorum AE ED inventione explorare tibi licebit, ut in superiore zetemate vidisti. Et si libeat utere & in hoc in & antecedente zetemate ea, quam libro tertio exhibuimus factione.

Factio

Factio autoris arithmetica ex illo problemate quo nos ablegat aque est intricata ideo quẽadmodũ superiore zetemate optatum compendiose exhibuimus, ita quoque In istoc libet experiri. quare cum ◡AC ad CG 5 ratio data. fiat ut BD 12 & CG 5 hoc est 17 ad CG 5, sic BD 12 ad IF $3\frac{9}{17}$. Et quemadmodum 17 ad ◡AC 14, sic BD 12 ad HI $9\frac{15}{17}$.

28 ZETEMA.

Si quadrato ab angulo tanquam communi vertice inscriptum sit triangulum æquilaterum basi HI contra diagonium FD parallela, itemque aliud circumscriptum lateribus inscripti parallelis per quadrati angulos FG D trãseuntibus; datis lateribus circumscripti in exposita mensura partium 16, quæruntur quadrati & trianguli inscriptorum latera.



Respondeo latus quadrati FG esse $\sqrt{288}-\sqrt{96}$: & latus trianguli huic quadrato inscripti EI $\sqrt{3072}-48$. zetema hoc haud ita difficultur é 23 zetemate propter figurarum similitudinem solvetur. Namque ut illic latus trianguli æquilateri $\sqrt{50}+\sqrt{16\frac{2}{3}}$ ad latus quadrati simili modo inscripti 5, sic AB 16 ad EF latus inscripti quadrati $\sqrt{288}-\sqrt{96}$. Porrò ad inventionem EH lateris trianguli in quadratum inscripti, continuato crura EH, & EI usque basin AC in Q & P, à quibus comprehenditur triangulum æquilaterum QEP: cujus latera EQ EP ex Data perpenpiculari EG facile cognoscuntur: namque ut BG $\sqrt{192}$ ad BC 16, sic EG diagonius quadrati $24-\sqrt{192}$ ad EP $\sqrt{768}-16$ jam divido EP secundum rationẽ BD ad DC datur autẽ BD $24-\sqrt{192}$ quia æqualis est diagonio quadrati FD, datur itaque etiam reliqua DC $\sqrt{192}-8$. unde invenies segmentum EI latus inscripti trianguli $\sqrt{3072}-48$.

Idem aliter. Inquerito quantum latus trianguli æquilateri in 23 zetematis paradigmate foret. idque deprehẽdes $\sqrt{150}-\sqrt{50}$ Vnde proportio, quemadmodum 5 latus quadrati circumscripti ad latus trianguli æquilateri eidem simili modo inscripti $\sqrt{150}-\sqrt{50}$, ita $24-\sqrt{192}$ hoc latus quadrati ad latus trianguli æquilateri EH $\sqrt{3072}-48$, ut supra.

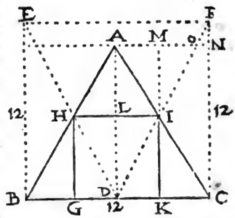
Si cui libeat hoc zetema Algebricé solvere, statuat pro latere trianguli æquilateri FED 1℞, ideo MD semidiagonius quadrati, vel ipsi æqualis DO erit $\frac{1}{2}$℞ unde proportio, vt GB $\sqrt{192}$ ad BC 16, sic DO $\frac{1}{2}$℞ ad DC $\sqrt{\frac{1}{3}}$℥. ergo tota BC erit $\sqrt{\frac{1}{3}}$℥ + 1℞: sed eadem datur partium 16, quamobrem $\sqrt{\frac{1}{3}}$℥ + 1℞ æquantur 16, si utrũque æquationis membrum dividas per $\sqrt{\frac{1}{3}}+1$, fit 1℞ æquale $24-\sqrt{192}$. pro latere trianguli FD, quæ eadem diagonius est quadrati DEFG, ergo semidiagonius MD $12-\sqrt{48}$ cujus quadratum $192-\sqrt{27648}$ duplicatum dat $384-\sqrt{110592}$, unde erutum latus exibet magnitudinem optati lateris ED $\sqrt{288}-\sqrt{96}$. Denique etiam latus trianguli quadrato circumscripti ex eadem algebrica positione quoque erues.

BD DC,

BD DC, EI IP proportionales esse ita patet, cum BC EP lineæ parallelæ sint, erit ut BD ad DG, sic EI ad IG, & ut GD ad DC, sic GI ad IP: ex æquo igitur ut BD ad DC, sic EI ad IP.

29 ZETEMA.

Dato triangulo æquilatero cui maximum quadratum inscriptum sit posito trianguli latere partium 12 quæruntur ejus latera.



Respondeo HG esse $\surd 1728 - 36$. Clavius è doctissimo Federico Comandino, cujus & diligentiæ & industriæ universa mathesis multũ debet, nobis ad finem libri sexti *Euclid.* affert inventum generale, quomodo cuilibet dato triangulo super datũ latus quadratum inscribatur hoc modo. Demittatur perpendicularis AD ex angulo A optato lateri BC opposito, eadem secetur in L, ut segmenta AD & LD eam habeant inter se rationem quam perpendicularis AD ad basin BC, tum LD latus erit quadrati triangulo inscripti. itaque HI parallela per L punctum contra basin BC acta, & ab H & I demissis perpẽdicularibus HG IK comprehendes quadratum GHIK quale optabatur. Vt hic secundum meam constructionem, cujusmodi supra 24 zetemate exhibui, perpendicularem AD ita in L divisam esse demonstrem, notato triangula DFC, OFN, DAO, DIK & MOI inter se omnia similia eorumque latera homologia ideo proportionalia esse. quare ut AD ad CF seu ipsam basin BC, sic IM ad IK, hoc est ipsis æquales AL ad LD quod demonstrandum fuerat. Latus autem inscripti quadrati per numeros haud difficulter hinc expedies. Addita enim perpendiculari AD ad basin BC fiunt $12 + \surd 108$. Inde jam proportio, quemadmodum $12 + \surd 108$ ad basin BC 12, sic AD $\surd 108$ ad optatum LD $\surd 1728 - 36$.

Lineas AD FC, MI IK proportionales esse, non statim inde sequitur quia ipsa in triangulis similibus æqualibus angulis subtenduntur, hoc enim idem eveniret etiamsi MK paulo propius vel longius abesset à perpendiculari AD: sed per æquationem fuit concludendum.

Verum cum istud Comandini *ἐπιχείρημα consectarium dumtaxat sit, ex illa generali triãguli & parallelogrammi adscriptione quã supra libro tertio ad problema 42 retulimus èque generali luce multa specialia comme clare & facile deriventur, non ingratum lectori arbitror futurum, si hanc parallelogrammi & trianguli adscriptionem quàm generalissime quemadmodum eam olim concepi (valde enim me generalia delectant) hoc loco proponam. demonstrationem hanc referes ad digramma Zetematis 27.*

In datum positione triangulum parallelogrammum in dato angulo dataque laterum ratione inscribere.

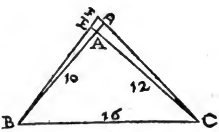
Oportet autem angulos parallelogrammi, proximis trianguli angulis ad basin majores esse. Exponatur triangulum ABC in quod sit inscribendum parallelogrammum cujus angulus angulo C vicinus, detur ADB major quam angulus trianguli ACB ad basin super qua parallelogrammum sit statuendum, & BDC major reliquo BAC, laterum ratio sit data quæ basis AC ad lineam CG, sitque CG parallela contra DB, recta à puncto D ad G verticem lineæ G

connexa

connexa incidat BC *in I, unde IH sit contra basin AC parallela, tumque ab ejus terminis HE, IF contra ipsam parallelæ comprehendent parallelogrammum EHIF, sub datis angulis quidem ob parallelismum linearum HE,* BD, *IF. et præterea in data ratione AC ad CG: namque ob similitudinem triangulorum* BID, *GIC, ut BD ad GC, sic BI ad IC, et ob parallelismum OI & DC, ut BI ad IG sic BO ad OD: quare ex æquo, ut BD ad* BO, *sic GC ad DO; & sic AC ad HI: ex æqualitate igitur, ut AC ad CG sic HI ad OD, seu IF: quare parllelogrammū EFHI triangulo ABC inscriptum datos habet angulos laterumque rationem datam. quod erat faciendum.*

30 ZETEMA.

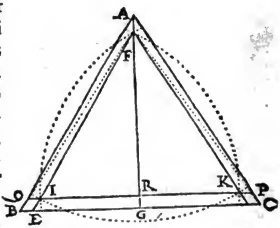
Ab angulis ad basin dati obtinsanguli trianguli BAC in continuata obtusi anguli crura demissa sint perpendiculares CD, BE, eædemque ad mutuum occursum continuatæ in F, quæritur quantæ sint continuationes EF, FD.



Sunto latera BC 16, BA 10, AC 12. Respondeo EF esse $\sqrt{\frac{5}{1}\frac{2}{7}\frac{6}{3}\frac{5}{6}}$, et DF $\sqrt{\frac{2}{5}\frac{8}{9}\frac{0}{7}\frac{9}{3}}$. Namque per 83 propos. lib. 2 inveniantur perpēdiculares CD & BE, et crurum cōtinuationes EA $\frac{1}{2}$, AD $\frac{3}{7}$. Deinde cum triangula quatuor BFD, BEA, CEF, DAC sint inter se similia, sunt enim rectangula, et angulum unum acutum habent communem vel sibi ad verticem positum. Erit ideo ut DC $\sqrt{143\frac{1}{2}\frac{6}{7}}$ ad AC 12, sic DB $10\frac{3}{7}$ ad BF $\sqrt{112\frac{2}{3}\frac{5}{9}\frac{6}{9}}$: unde subducta EB $\sqrt{99\frac{3}{4}}$, relinquetur EF $\sqrt{\frac{6}{1}\frac{2}{3}\frac{6}{9}\frac{5}{6}}$. Denide quemadmodū BE $\sqrt{99\frac{3}{4}}$ ad AB 10, sic AC $12\frac{1}{2}$ ad FC $\sqrt{356\frac{2}{9}\frac{5}{7}\frac{6}{7}}$ unde subducta DC reliquam facit DF $\sqrt{\frac{2}{9}\frac{8}{9}\frac{0}{7}\frac{9}{7}}$. Cujus periculū facere, quin et alijs modis idem quoque solvere tibi licebit.

31 ZETEMA.

Dato latere triarguli æquilateri ABC partium 29, intra quod aliud triangulum æquilaterum consistat ut à vertice exterioris trianguli A ad angulos interioris remotissimos I & K, rectæ AI AK sint 26, tantumque itidem intervallum sit ab angulo C ad F & I, à B *ad F et K; quærantur latera trianguli interioris.*



Respondeo $\sqrt{1397\frac{1}{4}} - 14\frac{1}{2}$. Investigetur primò perpendicularis exterioris trianguli AG $\sqrt{630\frac{3}{4}}$. deinde etiam perpendicularis FG $\sqrt{465\frac{1}{4}}$, quia enim BFC trianguli latera æqualia dantur ex thesi 26, et basis B C 29 subducto quadrato B G de quadrato B F relinquitur quadratum perpendicularis FG, unde et ipsa magnitudine datur. Hinc continuatis lateribus FI, FK usque in basin BC constituetur triāgulum æquilaterū EFD cujus laterū magnitudo invenietur per proportionē hoc modo. Vt AG $\sqrt{630\frac{3}{4}}$ ad AC 29, sic FG $\sqrt{465\frac{1}{4}}$ ad FD, vel ED $\sqrt{621}$

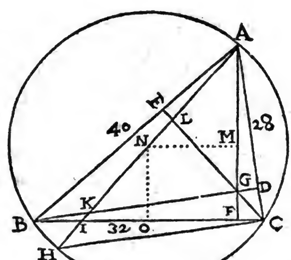
Hac

Hac si multetur latus BC 28, utraque BE et DC simul dantur $29 - \sqrt{621}$, cujus dimidium DC $14\frac{1}{2} - \sqrt{155\frac{1}{4}}$ additũ ad ED dabit totam CE vel DB $14\frac{1}{2} + \sqrt{155\frac{1}{4}}$ quibus æquales item sunt AQ, AP, PQ. et ED quoque ipsi DF. Hinc jam ut AB 29 ad AG $\sqrt{630\frac{3}{4}}$, sic AQ seu PQ $14\frac{1}{2} + \sqrt{155\frac{1}{4}}$ ad RA $\sqrt{157\frac{1}{3}\frac{1}{6}} + \sqrt{116\frac{7}{16}}$ hinc deducta AF $\sqrt{630\frac{3}{4}} - \sqrt{467\frac{3}{4}}$ reliquam facit perpendicularem FR. Vnde novissima proportio, quemadmodũ AG $\sqrt{630\frac{3}{4}}$ ad AC 29, sic perpẽdicularis FR $\sqrt{1047\frac{1}{3}\frac{5}{6}} - \sqrt{157\frac{1}{3}\frac{1}{3}}$ ad quæsitam magnitudinem lateris FK $\sqrt{1397\frac{3}{4}} - 14\frac{1}{2}$. Et si quoque aream trianguli FIK investigare hic libeat, multiplicato perpendicularem FR $\sqrt{1047\frac{1}{3}\frac{5}{6}} - \sqrt{157\frac{1}{3}\frac{1}{6}}$ cũ dimidia base IR $\sqrt{349\frac{7}{16}} - 7\frac{1}{4}$ factus $\sqrt{484150\frac{1}{6}\frac{5}{4}} - \sqrt{220382\frac{5}{6}\frac{5}{4}}$ crit trianguli EIK area optata.

In istius Zetematis datis id redundat quod triangulum interius IFK *autor supponat æquilaterum, cum id non dandum sed ex reliquis datis fuerit colligendum. Ipsum vero Zetema multo potuit solvi expeditius faciliusque hoc modo. Quia latera singula trianguli ABC dantur partium 29, ipsa perpendicularis AG erit* $\sqrt{630\frac{3}{4}}$. *Et cum trianguli æquicruri* BFC *crura B*F F*C dantur partium 26, & basis BC 29, dabitur quoque perpẽdicularis* FG $\sqrt{465\frac{1}{4}}$. *Hinc propter similitudinem triangulorum AGC* FGC, *ut AG* $\sqrt{620\frac{3}{4}}$ *ad GC* $14\frac{1}{2}$ *sic FG* $\sqrt{465\frac{1}{4}}$ *ad GD* $\sqrt{155\frac{1}{4}}$, *quæ subducta de GC reliquam facit DC* $14\frac{1}{2} - \sqrt{155\frac{1}{4}}$. *Eademque GD duplicata exhibet totam ED* $\sqrt{621}$, *cui FD quoque æquatur: sed & KD ipsi DC æqualis est, cum latera interioris trianguli* FIK *exterioris trianguli lateribus æquidistanter parallela sint, rhombus ergo est figura KDCP: quamobrem de* FD $\sqrt{621}$ *subducta DC* $14\frac{1}{2} - \sqrt{155\frac{1}{4}}$ *exhibet latus quæsitum F*K $\sqrt{1397\frac{3}{4}} - 14\frac{1}{2}$.

31 ZETEMA.

Diameter circuli circa datum triangulum ABC *circumscripti educta ab angulo A intersecutur à perpendicularibus à reliquis angulis in ejus crura demissis. quæruntur latera trianguli KLG ab intersegmentis diametri & perpendicularium comprehensi.*



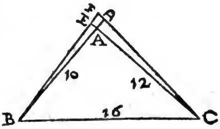
Respõdeo latus KL valere $\sqrt{317\frac{2}{6}\frac{3}{7}\frac{6}{5}\frac{0}{8}\frac{7}{7}}$, GK $\sqrt{495\frac{2}{2}\frac{3}{6}\frac{1}{5}\frac{1}{7}}$, LG $\sqrt{242\frac{1}{1}\frac{3}{3}\frac{3}{7}\frac{4}{7}}$, positis lateribus trianguli AB partium 40, BC 32, AC 28. jam Primum ab A perpendicularis AF demissa in basin BC per G cõmunem sectionem perpendicularium CE BD quoque transibit: harum trium perpendicularium magnitudo per antecedentia præcepta inveniatur, AF $\sqrt{773\frac{7}{16}}$, DB $\sqrt{1010\frac{1}{4}\frac{0}{5}}$, EC $\sqrt{495}$, & diametri AH $\sqrt{1621\frac{4}{4}\frac{2}{5}\frac{1}{5}}$, radij NH $\sqrt{405\frac{2}{4}\frac{2}{5}\frac{9}{5}}$, segmenti FC $3\frac{1}{4}$, FB $28\frac{3}{4}$.

tumque

connexa incidat BC *in I, unde IH sit contra basin AC parallela, tumque ab ejus terminis HE, IF contra ipsam parallelæ comprehendent parallelogrammum EHIF, sub datis angulis quidem ob parallelismum linearum HE,* BD, *IF. et præterea in data ratione AC ad CG: namque ob similitudinem triangulorum* BID, GIC, *ut BD ad GC, sic BI ad IC, et ob parallelismum OI & DC, ut BI ad IG sic BO ad OD: quare ex æquo, ut BD ad* BO, *sic GC ad DO; & sic AC ad HI: ex æqualitate igitur, ut AC ad CG sic HI ad OD, seu IF: quare parllelogrammũ EFHI triangulo ABC inscriptum datos habet angulos laterumque rationem datam. quod erat faciendum.*

30 ZETEMA.

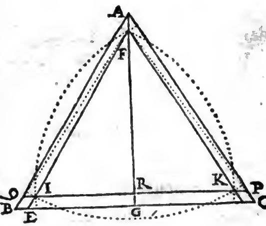
Ab angulis ad basin dati obtinsanguli trianguli BAC in continuata obtusi anguli crura demissæ sint perpendiculares CD, BE, eædemque ad mutuum occursum continuatæ in F, quæritur quantæ sint continuationes EF, FD.



Sunto latera BC 16, BA 10, AC 12. Respondeo EF esse $\sqrt{}\,\frac{5}{1}\frac{2}{5}\frac{6}{5}\frac{5}{6}$, et DF $\sqrt{}\,\frac{2}{5}\frac{8}{9}\frac{0}{7}\frac{9}{5}$. Namque per 83 propos. lib. 2 inveniantur perpẽdiculares CD & BE, et crurum cõtinuationes EA $\frac{1}{2}$, AD $\frac{3}{5}$. Deinde cum triangula quatuor BFD, BEA, CEF, DAC sint inter se similia, sunt enim rectangula, et angulum unum acutum habent communem vel sibi ad verticem positum. Erit ideo ut DC $\sqrt{}\,143\frac{1}{2}\frac{6}{7}$ ad AC 12, sic DB $10\frac{3}{5}$ ad BF $\sqrt{}\,112\frac{2}{3}\frac{5}{9}\frac{6}{9}$: unde subducta EB $\sqrt{}\,99\frac{3}{4}$, relinquetur EF $\sqrt{}\,\frac{6}{1}\frac{2}{3}\frac{4}{9}\frac{5}{8}$. Denide quemadmodũ BE $\sqrt{}\,99\frac{3}{4}$ ad AB 10, sic AC $12\frac{8}{2}$ ad FC $\sqrt{}\,356\frac{2}{5}\frac{5}{7}\frac{6}{7}$ unde subducta DC reliquam facit DF $\sqrt{}\,\frac{2}{5}\frac{8}{9}\frac{0}{7}\frac{9}{5}$. Cujus periculũ facere, quin et alijs modis idem quoque solvere tibi licebit.

31 ZETEMA.

Dato latere triarguli æquilateri ABC partium 29, intra quod aliud triangulum æquilaterum consistat ut à vertice exterioris trianguli A ad angulos interioris remotissimos I & K, rectæ AI AK sint 26, tantumque itidem intervallum sit ab angulo C ad F & I, à B *ad F et K; quæruntur latera trianguli interioris.*



Respondeo $\sqrt{}\,1397\frac{1}{4}$ — $14\frac{1}{2}$. Investigetur primò perpendicularis exterioris trianguli AG $\sqrt{}\,630\frac{3}{4}$. deinde etiam perpendicularis FG $\sqrt{}\,465\frac{3}{4}$, quia enim BFC trianguli latera æqualia dantur ex thesi 26, et basis B C 29 subducto quadrato B G de quadrato B F relinquitur quadratum perpendicularis FG, unde et ipsa magnitudine datur. Hinc continuatis lateribus FI, FK usque in basin BC constituetur triãgulum æquilaterũ EFD cujus laterũ magnitudo invenietur per proportionẽ hoc modo. Vt AG $\sqrt{}\,630\frac{3}{4}$ ad AC 29, sic FG $\sqrt{}\,465\frac{3}{4}$ ad FD, vel ED $\sqrt{}\,621$

Hac

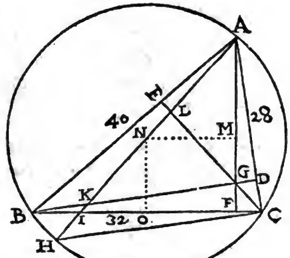
Hac si multetur latus BC 28, utraque BE et DC simul dantur 29 — $\surd 621$, cujus dimidium DC $14\frac{1}{2}$ — $\surd 155\frac{1}{4}$ additũ ad ED dabit totam CE vel DB $14\frac{1}{2} + \surd 155\frac{1}{2}$ quibus æquales item sunt AQ, AP, PQ. et ED quoque ipsi DF. Hinc jam ut AB 29 ad AG $\surd 630\frac{3}{4}$, sic AQ seu PQ $14\frac{1}{2} + \surd 155\frac{1}{4}$ ad RA $\surd 157\frac{11}{36} + \surd 116\frac{7}{36}$ hinc deducta AF $\surd 630\frac{3}{4}$ — $\surd 467\frac{1}{4}$ reliquam facit perpendicularem FR. Vnde novissima proportio, quemadmodũ AG $\surd 630\frac{3}{4}$ ad AC 29, sic perpẽdicularis FR $\surd 1047\frac{15}{36}$ — $\surd 157\frac{11}{36}$ ad quæsitam magnitudinem lateris FK $\surd 1397\frac{3}{4}$ — $14\frac{1}{2}$. Et si quoque aream trianguli FIK investigare hic libeat, multiplicato perpendicularem FR $\surd 1047\frac{15}{36}$ — $\surd 157\frac{11}{36}$ cũ dimidia base IR $\surd 349\frac{7}{16}$ — $7\frac{1}{4}$ factus $\surd 484150\frac{15}{64}$ — $\surd 220382\frac{15}{64}$ crit trianguli EIK area optata.

In istius Zetematis datis id redundat quod triangulum interius IFK autor supponat æquilaterum, cum id non dandum sed ex reliquis datis fuerit colligendum. Ipsum vero Zetema multo potuit solvi expeditius faciliusque hoc modo. Quia latera singula trianguli ABC dantur partium 29, ipsa perpendicularis AG erit $\surd 630\frac{3}{4}$. *Et cum trianguli æquicruri* BFC *crura* BF FC *dantur partium 26, & basis BC 29, dabitur quoque perpẽdicularis* FG $\surd 465\frac{3}{4}$. *Hinc propter similitudinem triangulorum AGC* FGC, *ut AG* $\surd 620\frac{3}{4}$ *ad GC* $14\frac{1}{2}$ *sic* FG $\surd 465\frac{3}{4}$ *ad GD* $\surd 155\frac{1}{4}$, *quæ subducta de GC reliquam facit DC* $14\frac{1}{2}$ — $\surd 155\frac{1}{4}$. *Eademque GD duplicata exhibet totam ED* $\surd 621$, *cui FD quoque æquatur: sed & KD ipsi DC æqualis est, cum latera interioris trianguli* FIK *exterioris trianguli lateribus æquidistanter parallela sint, rhombus ergo est figura KDCP: quamobrem de* FD $\surd 621$ *subducta DC* $14\frac{1}{2}$ — $\surd 155\frac{1}{4}$ *exhibet latus quæsitum F*K $\surd 1397\frac{3}{4}$ — $14\frac{1}{2}$.

31 ZETEMA.

Diameter circuli circa datum triangulum ABC *circumscripti educta ab angulo A intersecutur à perpendicularibus à reliquis angulis in ejus crura demissis. quæruntur latera trianguli KLG ab intersegmentis diametri & perpendicularium comprehensi.*

Respõdeo latus KL valere $\surd 317\frac{23607}{67387}$, GK $\surd 495\frac{2311}{2657}$, LG $\surd 242\frac{1334}{1373}$, positis lateribus trianguli AB partium 40, BC 32, AC 28. jam Primum ab A perpendicularis AF demissa in basin BC per G cõmunem sectionem perpendicularium CE BD quoque transibit: harum trium perpendicularium magnitudo per antecedentia præcepta inveniatur, AF $\surd 773\frac{7}{36}$, DB $\surd 1010\frac{10}{45}$, EC $\surd 495$, & diametri AH $\surd 1621\frac{421}{457}$, radij NH $\surd 405\frac{229}{457}$, segmenti FC $3\frac{1}{4}$, FB $28\frac{3}{4}$.



Tumque ex N centro in basin BC demittatur perpendicularis NO, & in perpendicularem AF perpendicularis NM. Iam FC $3\frac{3}{4}$, subducito de OC 16 semisse inscriptæ BC, et reliquæ OF seu NM $12\frac{1}{4}$ quadratum subductum de quadrato NA relinquit quadratum AM, quare magnitudine datur AM $\surd 192\frac{3}{7}\frac{7}{5}\frac{6}{2}\frac{9}{6}$. dantur itaque trianguli rectanguli ANM latera omnia, cui simile est triangulum AFI; unde proportio, quemadmadum AM $\surd 192\frac{3}{7}\frac{7}{5}\frac{6}{2}\frac{9}{6}$ ad MN $12\frac{1}{4}$, sic AF $\surd 773\frac{1}{1}\frac{7}{6}$ ad FI $22\frac{4}{7}\frac{1}{7}\frac{6}{4}\frac{9}{1}$, huc addita FC $3\frac{3}{4}$ conflat totam CI $26\frac{1}{1}\frac{1}{7}\frac{1}{1}\frac{2}{7}$, quare reliqua BI quoque datur $5\frac{1}{1}\frac{3}{7}\frac{8}{1}\frac{1}{7}$. Et rursum ut AM $\surd 192\frac{3}{7}\frac{7}{5}\frac{6}{2}\frac{9}{6}$ ad AN $\surd 405\frac{2}{4}\frac{2}{5}\frac{9}{7}$, sic AF $\surd 773\frac{1}{1}\frac{7}{6}$ ad AI $\surd 1291\frac{1}{1}\frac{1}{5}\frac{2}{2}\frac{6}{7}\frac{2}{7}\frac{2}{1}\frac{1}{9}$: quare reliqua pars diametri IH erit $\surd 18\frac{8}{5}\frac{0}{5}\frac{2}{2}\frac{3}{2}\frac{5}{6}\frac{6}{5}\frac{5}{6}\frac{1}{5}\frac{4}{7}$. notato hos numeros symmetros esse, & per $\surd 495$ multiplicatos ad veros quadratos revocari. Porrò ad inventionem laterum trianguli KIG notato triangula ABF AHC esse similia per 21 & 31 propositionem lib. 3 *Euclid.* ideoque angulos FAB AHC inter se æquari: subducto angulo communi HAF reliquum BAH reliquo FAC æquari. atque ea propter triangula BAH, FAC similia esse, et utrique simile triangulum DBC, quia simile sit ipsi FAC ob angulum ad C communem, ad D & F rectos. sed per 21 prop. 3 lib. *Eucl.* BCH angulus angulo BAH æqualis est: atque hinc jam facile patet HIC, BIK, BIA triangula esse similia. per antecedentia autē dantur tria latera trianguli HIC. Et CI quidē $26\frac{1}{1}\frac{1}{7}\frac{1}{1}\frac{2}{7}$ HI $\surd 18\frac{8}{5}\frac{0}{5}\frac{2}{2}\frac{3}{2}\frac{5}{6}\frac{6}{5}\frac{5}{6}\frac{1}{5}\frac{4}{7}$ et HC $\surd 837\frac{4}{4}\frac{2}{5}\frac{1}{7}$, (cui æquatur ipsa BG) præterea cum minoris trianguli BKI latus BI jam sit cognitum $5\frac{1}{1}\frac{3}{7}\frac{8}{1}\frac{1}{7}$, per proportionem reliqua quoque cognoscentur Nam ut CI ad IH, sic BI ad IK $\surd \frac{1}{5}\frac{1}{3}\frac{2}{3}\frac{6}{2}\frac{6}{2}\frac{6}{2}\frac{5}{0}\frac{0}{3}\frac{6}{4}\frac{1}{1}\frac{1}{9}$. Et quemadmodum AD $24\frac{2}{7}$, ad AG $\surd 597\frac{4}{4}\frac{2}{5}\frac{1}{7}$, sic AE 17 ad lineam AL $\surd 292\frac{1}{1}\frac{1}{2}\frac{7}{1}\frac{1}{7}\frac{6}{7}$. ea subducta de AI relinquit segmentum LI $\surd 354\frac{1}{1}\frac{1}{3}\frac{7}{1}\frac{4}{0}\frac{0}{6}\frac{4}{6}\frac{8}{4}\frac{8}{1}\frac{3}{3}\frac{5}{7}\frac{4}{7}$, hinc rursum subducta IK reliquam facit LK $\surd 317\frac{1}{6}\frac{7}{7}\frac{1}{3}\frac{1}{1}\frac{1}{3}$, ut initio respondimus. in de cætera jā deceps sunt faciliora.

Vnicum illud quod autor levissime monuit, nescio an quia per numeros ita deprehenderat, demonstrare hic libet. Videlicet BG & HC rectas æquari. triangula enim CGB, CHB latus BC habent commune et angulum GBC angulo HCB æqualem, quia BD, HC eidem AC perpendiculares inter se parallelæ sunt, atque angulus GCB angulo HBC quoque æqualis est, quia HBC angulo HAC in eadem peripheria æquatur, et HAC ipsi BAF supra ab autore æqualis demonstratus est, et BAE ipsi BCE (quia hæc triangula similia) quare BCE seu BCG ipsi HBC æqualis est. Atque ideo cum HBC, GCB triangula ad communi crus BC, angulos æquales habeant erunt æquilatera. quare latera BG, HC æqualibus angulis subtensa sunt æqualia. Vsus hujus perinsignis est ut facili negotio BG, GA, GC segmenta perpendicularum inveniantur, quam ad rem theorema tale concipio.

Latus dati trianguli cum segmento perpendicularis ab angulo opposito ad communem perpendicularium sectionem æque possunt diametro circumscripti circuli.

Perpen-

Perpendiculares △*AF BD CE in eodem puncto se intersecare alias demonstratum est. Ajo itaque BA cum segmento CG æque posse diametro AH: quia BH GC æquantur, nam BGCH parallelogrammum esse demonstravimus. Et sic AC & BG segmentum perpendicularis in latus* △*AC demissæ æque possunt diametro AH, quia HC & BG æquari ante demonstravimus. Deniq; BC & AG itidem diametro æque posse ex B ducta diametro simillime ostendes.*

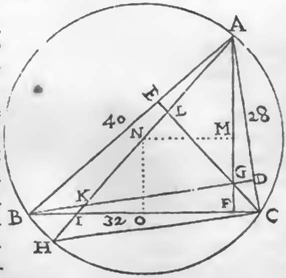
Non sum nescius idem ex similitudine hoc modo concludi potuisse, ut AF ad AC, sic AD ad AG, quia tamen hoc epichirema novum & scitulum & suo loco adhibitum compendij non nihil afferat, tanquam consectarium ex ista quam supra attulimus demonstratione, derivari non minus jucundum quam utile existimavi.

Cæterum scrupulosa & prolixa est autoris hæc via, & incurrit in numeros nimium vastos, quibus hoc Zetema interpolare minime opus fuit: quamobrem studioso lectori breviorem aliam subministrabo.

Si diameter circuli triangulo circumscripti ab angulo trianguli educta perpẽdiculares ab reliquis angulis in illius crura demissas intersecet diametri, segmẽtum inter ipsas & perpẽdicularium inter ipsam & mutuam sectionem comprehendent triangulum dato triangulo æquiangulum.

Triangulo △*ABC circumscribatur circulus, hujus diameter per angulum A sit AH, perpendiculares in ejusdem anguli crura à reliquis angulis demissæ BD EC, mutuò sese intersecent in G. ajo triangulum LGK ab intersegmento diametri KL & intersegmentis perpendicularium LG GK comprehensum dato ABC esse æquiangulum. angulus enim KLG angulo ACB æqualis est, quia æquantur eidem AHB, ille ob parallelismum perpendicularium HB, EC, hic quia sunt in eadem sectione. Et angulus LGK angulo BAC æqualis erit, quia æquantur eidem HBK ob causas simillimas: quare & tertius LKG æquatur tertio ABC. atque ideo KLG triangulum triangulo ABC æquiangulum & simile: quod demonstrandum fuit. Vnde ad laterum investigationem expedita jam erit via.*

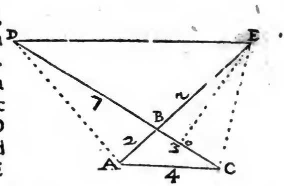
Inveniantur enim perpendiculares CE & AF, itemque segmenta AE & EB & FB quemadmodum ante edoctus es. hinc ob similitudinem triangulorum AEL △*ABH seu AFC fiat, ut AF* $\sqrt{773\frac{7}{16}}$ *ad FC* $3\frac{1}{4}$, *sic AE 17 ad EL* $\sqrt{\frac{4}{1}\frac{8}{2}\frac{8}{3}\frac{4}{7}\frac{1}{5}}$. *deinde subducto quadrato AB de quadrato diametri AH* $\sqrt{1621\frac{4}{4}\frac{8}{9}\frac{1}{3}}$ *relinquetur quadratũ BH, atque ideo exhibebit ipsam BH* $\sqrt{21\frac{4}{4}\frac{8}{9}\frac{1}{4}}$ *cui GC æquari demonstravimus. jam EL & GC additæ constant* $\sqrt{\,^{1}\frac{4}{1}\frac{9}{2}\frac{0}{3}\frac{8}{7}\frac{1}{7}}$, *quæ subducta de EC* $\sqrt{495}$ *relinquit LG* $\sqrt{242\frac{1}{1}\frac{1}{3}\frac{3}{7}\frac{4}{7}}$ *unum trianguli latus. Hinc ob similitudinẽ demonstratã proportio, ut AC 28 ad CB 32, sic LG* $\sqrt{242\frac{1}{1}\frac{1}{3}\frac{3}{7}\frac{4}{7}}$ *ad LK* $\sqrt{317\frac{2}{6}\frac{3}{7}\frac{3}{7}\frac{0}{7}\frac{1}{7}}$. *Et quemadmodum AC 28 ad AB 40, sic LG* $\sqrt{242\frac{1}{1}\frac{1}{3}\frac{3}{7}\frac{4}{7}}$ *ad KG* $\sqrt{495\frac{2}{2}\frac{3}{6}\frac{1}{3}\frac{4}{7}}$, *quod quærebatur.*



AE	17
BE	23
EC	$\sqrt{495}$
FC	$3\frac{1}{4}$
FB	$28\frac{3}{4}$
AF	$\sqrt{773\frac{7}{16}}$
AH	$\sqrt{1612\frac{4}{4}\frac{2}{9}\frac{6}{5}}$
GC	$\sqrt{\,^{1}\,^{0}\frac{8}{4}\frac{4}{5}\frac{6}{7}}$
EL	$\sqrt{\frac{4}{1}\frac{8}{3}\frac{8}{3}\frac{4}{7}\frac{1}{7}}$
GC&EL	$\sqrt{\,^{1}\frac{4}{1}\frac{9}{2}\frac{0}{3}\frac{8}{7}\frac{1}{7}}$
LG	$\sqrt{242\frac{1}{1}\frac{3}{3}\frac{3}{7}\frac{4}{7}}$
LK	$\sqrt{317\frac{2}{6}\frac{3}{7}\frac{3}{7}\frac{0}{7}\frac{1}{7}}$
KG	$\sqrt{495\frac{2}{2}\frac{3}{6}\frac{1}{3}\frac{1}{7}}$

32 ZETEMA.

Triangnli anguli verticalis duorum crurum continuatione & basi data. Aream trianguli ab istis comprehensi invenire.

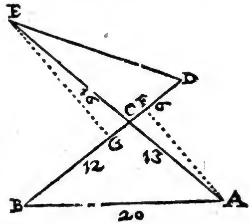


Trianguli ABC latus AB sit partium 2, BC 3, AC 4 continuatio BD 7, BE 5, hinc connexa DE, quæritur quanta sit area trianguli DBE. Respondeo $\sqrt{287\frac{7}{64}}$. Cum enim triangula æquealta sint ut bases, fiat ut BC 3 ad BD 7, sic area trianguli ABC $\sqrt{8\frac{7}{16}}$ ad aream ABD $\sqrt{45\frac{15}{16}}$. Et rursum quemadmodum AB 2 ad BE 7, sic triangulum ABD $\sqrt{45\frac{15}{16}}$ ad DBE $\sqrt{287\frac{7}{64}}$. si præterea rectam DE continuatarum terminos connectentem nouisse expetas, inventam aream $\sqrt{287\frac{7}{64}}$ per dimidiam BD $3\frac{1}{2}$ dividito, inde existet quantitas perpendicularis EO $\sqrt{23\frac{7}{16}}$. inde ex differentia quadratorum EO EB exhibetur BO $1\frac{1}{4}$, ideoque tota DO $8\frac{1}{4}$. denique é potentijs EO & OD dabitur recta quæsita DE $\sqrt{91\frac{1}{2}}$.

Idem aliter & facilius quam ab autore expediri potest, hoc modo cum parallelogramma & triangula æqua angulo habeãt rationem ex æqualium angulorum cruribus compositam, erit ut rectangulum sub AB in BC, ad rectangulum sub DB in BE comprehensum, ita triangulum ABC ad triangulum DBE. hoc modo ut 6 ad 35, sic $\sqrt{8\frac{7}{16}}$ ad $\sqrt{287\frac{7}{64}}$ pro area quæsita trianguli EBD.

33 ZETEMA.

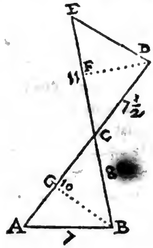
Trianguli ABC latera dantur AB 20, BC 12, AC 13, itemque continuationes EC 16 DC 6, quæritur connectens eas DE?



Respondeo DE esse $\sqrt{345\frac{7}{13}}$. duo sequentia zetemata similia sunt antecedenti sed paulo aliter soluta, demissis enim perpendicularibus AF & FG fiunt duo triangula AFC EGC similia: dantur autem per præmissa latera trianguli AFC, FC $3\frac{5}{8}$, AF $\sqrt{155\frac{55}{64}}$. unde proportio quemadmodum CA 13 ad AF $\sqrt{155\frac{55}{64}}$, sic EC 16 ad EG $\sqrt{236\frac{16}{169}}$. Et quemadmodũ AC 13 ad CF $3\frac{5}{8}$, sic EC 16 ad CG $4\frac{6}{13}$. jam CG & DG additæ æquantur ipsi DG $10\frac{6}{13}$. denique quadrata DG & GF constituunt quadratum ED $345\frac{7}{13}$, & ipsam ED longitudine $\sqrt{345\frac{7}{13}}$.

Esto

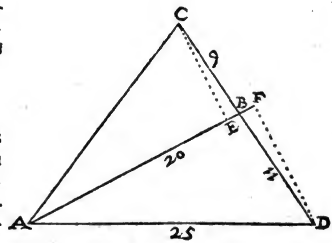
Esto exemplum secundum in triangulo ABC cujus latera dantur AB, 7 BC 8, AC 10, & continuationes EC 11, CD $7\frac{1}{2}$, quæritur connectens ED?



Respondeo ED esse $\sqrt{58\frac{2}{3}\frac{1}{2}}$. demissis enim perpendicularibus BG DF fiunt triangula similia DCF BCG, Et GB inveniatur $\sqrt{3015}$ GC $5\frac{3}{4}$ Vnde proportio quemadmodũ BC 8 ad GC $5\frac{3}{4}$, sic DC $7\frac{1}{2}$ ad CF $5\frac{2}{6}\frac{3}{4}$, ea subducta de EC 11 relinquit EF $5\frac{3}{6}\frac{9}{4}$. denique é quadratis linearum EF $^{1}\,^{2}\frac{8}{4}\frac{8}{8}\frac{8}{5}\frac{1}{6}$ & FD $^{1}\,^{1}\frac{1}{4}\frac{3}{8}\frac{7}{5}\frac{5}{6}$ datur quæsita ED $\sqrt{^{3}\,^{4}\frac{0}{4}\frac{2}{0}\frac{5}{5}\frac{6}{7}}$ hoc est terminis reductis $\sqrt{^{1}\,^{8}\frac{7}{3}\frac{7}{2}}$ vel $\sqrt{58\frac{2}{3}\frac{1}{2}}$.

34 ZETEMA.

Trianguli lateribus datis, dataque lateris unius continuatione quæritur ejus termini distantia ab angulo continuato lateri subtenso.

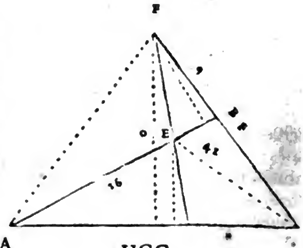


Datur AD 25 AB 20 BD 11, ejus continuatio BC 9, quæritur quantũ sit intervallũ AC. Respondeo $\sqrt{395\frac{1}{7}\frac{1}{7}}$. Nam ex D demissa perpendiculari DF, & ex C perpendiculari CE, fient duo triangula similia BDF BCE, & per præmissa datur DF $\sqrt{^{1}\frac{1}{1}\frac{4}{6}\frac{2}{6}}$. vnde proportio, quemadmodum BD 11 ad DF $\sqrt{114_{2}\frac{6}{7}}$, sic EB 9 ad CE $\sqrt{^{9}\frac{2}{1}\frac{3}{2}\frac{3}{1}\frac{4}{0}\frac{4}{6}}$, hinc CB datur $2\,\frac{7}{7}\frac{7}{7}$, & AE $17\frac{4}{7}\frac{8}{7}$: denique ex quadratis CE EA datur CA $\sqrt{395\frac{1}{7}\frac{0}{7}}$.

35 ZETEMA.

Datis lateribus AD 25, BD 11, & hujus continuatione BF 9, & segmentis tertij lateris AE 16, EB 4, quæritur si sit recta FC ab F continuationis vertice per E usque in basin AD quãtá sint segmenta basis AC CD.

Refpõdeo AC effe $16\frac{1}{14}$, CD $8\frac{13}{14}$ & ipfam FC $\sqrt{255\frac{89}{196}}$. hoc zetema paulo operofius quam fuperiora á doctiffimo & in his artibus verfatifsimo *Simone Stevino* olim anno 1582 propofitum, algebrice á me tum temporis folutum memini. Nunc vero fecundum expofitæ figurę lineamenta idem ita expedio. Inveniatur ex antecedentis zetematis analogia linea EF $\sqrt{\frac{3899}{49}}$ hoc eft $\sqrt{79\frac{28}{49}}$. tum inventa area trianguli A*B*D $\sqrt{11424}$ fiat, ut BD 11 ad DF 20, fic triãgulũ ABD $\sqrt{11424}$ ad AFD $\sqrt{\frac{4569600}{121}}$, tumque triangulo AFD per dimidiam bafin AD divifo datur perpendicularis FH $\sqrt{241\frac{2111}{3025}}$. dehinc inveniatur area trianguli *E*FB $\sqrt{\frac{11704}{121}}$, & fiat ut bafis E*B* 9 ad *B*D 11, fic EFD $\sqrt{\frac{25704}{121}}$ ad E*B*D $\sqrt{\frac{11424}{121}}$, hoc triangulum de triangulo ABD fubductum reliquum facit triangulũ AED $\sqrt{\frac{45696}{49}}$, idque per dimidiam bafin AD $12\frac{1}{2}$ divifum exibet perpendicularem EG $\sqrt{\frac{182784}{30625}}$, cui æqualis OH fubducta de FH relinquit FO $\sqrt{\frac{143302636}{3705625}}$. Vnde proportio, quemadmodum OF $\sqrt{\frac{143302636}{3705625}}$ ad EF $\sqrt{\frac{3899}{49}}$, fic HF $\sqrt{\frac{731136}{3025}}$, ad *FC* $\sqrt{\frac{50069}{196}}$. ab hujus quadrato deductum quadratum FH exhibet quadratum HC, atque ideo ipfam HC longitudine $3\frac{503}{776}$, quæ addita ad AH $12\frac{11}{27}$ dabit totam AC $16\frac{1}{4}$. reliquam CD $8\frac{13}{14}$. quemadmodum quærebatur. funt & aliæ ad hujus zetematis folutionem viæ.



HGC

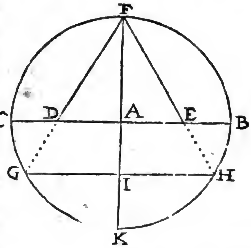
Poffunt verò hæc talia longè elegantifsimè & compendiofifsimè per rationum compofitionẽ abfque ulla vel perpendicularium, vel aliorum fegmentorum cognitione expediri atque unica proportione concludi, ut ita iftos, qui hic in autore occurrunt numeros furdos declinemus. Enimvero per ea quæ à Ptolomeo & ejus commentatore Theone libro primo μεγάλης συντάξεως demonftrata funt, ratio fegmentorum AC ad CD componitur è rationibus AE ad EB, & BF ad FD: hic AE 16, EB 4, BF 9, FD 20 magnitudine dantur, quare ratio quoque ab ipfis compofita datur hoc modo AE 16 BF 9, EB 4, FB 20, id eft $\frac{144}{80}$ *five 9 ad 5. ideoque AC ad CD fe habet ut 9 ad 5: datur autem AD 25: inde proportio, quemadmodum 9 et 5 fimul, hoc eft 14, ad 9, fic AD 25 ad AC* $16\frac{1}{14}$. *& quemadmodum 14 ad 5 fic AD 25 ad CD* $8\frac{13}{14}$.

36 ZETEMA.

Datur area trianguli æquilateri FDE 100, cujus bafis DE in diametro, vertex CF in medio femiperipheræ terminatur, quæritur circuli diameter.

Refpon-

Respondeo diametrum esse $\sqrt{}\sqrt{480000}$, & DE trianguli dati latus $\sqrt{}\sqrt{53333\frac{1}{3}}$. Inveniatur primum area cujuslibet triãguli æquilateri, ut assumpto latere uno $\sqrt{3}$ area erit $\sqrt{1\frac{1}{2}\frac{1}{8}}$. cumque figuræ similes sint inter se ut quadrata homologorum laterum fiat ut $\sqrt{1\frac{1}{2}\frac{1}{8}}$ ad 3, sic area FDE 100 ad $\sqrt{53333\frac{1}{3}}$ quadratũ lateris DE, quare ipsa DE datur magnitudine $\sqrt{}\sqrt{53333\frac{1}{3}}$. Deinceps vero ad inventionem diametri, inveniatur latus trianguli æquilateri super diametro nota descripti. esto diameter partium duarum, tum latus trianguli æquilateri erit $\sqrt{1\frac{1}{3}}$. tumque per proportionem



concludatur, quemadmodũ $\sqrt{1\frac{1}{3}}$ ad 2 sic DE $\sqrt{}\sqrt{53333\frac{1}{3}}$ ad diametrum CB $\sqrt{}\sqrt{480000}$. Denique si FG latus circulo inscriptum expetas, fiat quemadmodum 2 ad $\sqrt{3}$, sic $\sqrt{}\sqrt{480000}$ ad $\sqrt{}\sqrt{270000}$.

Operis veritatẽ ita experiri licebit, Inveniatur perpendicularis FA $\sqrt{}\sqrt{30000}$, ea per AE $\sqrt{3333\frac{1}{3}}$ dimidiam basin multiplicata reddet aream trianguli DFE $\sqrt{}\sqrt{100000000}$, hoc est 100 quemadmodum decuit.

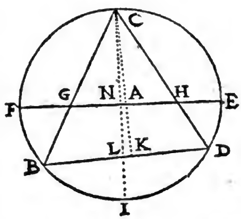
Non opus fuit secunda similitudine ad inventionem diametri: nam quia AF perpendicularis a vertice trianguli in centrum ex thesi cadat, ex latere FE $\sqrt{}\sqrt{53333\frac{1}{3}}$ datur perpendicularis FA $\sqrt{}\sqrt{30000}$ æqualis radio, hujus duplũ $\sqrt{}\sqrt{480000}$ pro quæsita diametro FK.

Idem aliter effinge triangulum æquilaterum à latere 2, ejus perpendicularis erit $\sqrt{3}$, duplum $\sqrt{12}$ pro diametro circuli, & area $\sqrt{3}$. hinc proportio. Quemadmodum area trianguli $\sqrt{3}$ ad 12 quadratum diametri, sic 100 area data ad $\sqrt{480000}$ diametri quadratum, inde ipsa datur $\sqrt{}\sqrt{480000}$.

37 ZETEMA.

Si à termino diametri CI rectæ duæ inscriptæ diametrum ei normalem intersecent, data diametro partium 18. & segmentis alterius diametri is FG $4\frac{2}{3}$, HE 4, quæruntur inscriptæ BC CD & connectens BD.

Cum FG detur partium $4\frac{2}{3}$, & FN 9, datur quoque GN $3\frac{1}{3}$. itaque ob crura anguli recti GN NC datur GC $\sqrt{99\frac{7}{9}}$: haud aliter invenietur CH $\sqrt{106}$. cumque triangulum GCN triangulo ICB, & HCN ipsi ICD simile sit per 21 & 31 propos. lib. 3. *Euc.* & præterea CDK triangulum simile triangulo CIB hoc est ipsi CGN, & CBK ipsi CNH, erit quemadmodum GC $\sqrt{99\frac{7}{9}}$ ad NC 9, sic IC 18 ad BC $\sqrt{263\frac{1}{4}\frac{1}{4}\frac{1}{9}}$. Et quemadmodum HC $\sqrt{106}$ ad CN 9, sic IC 18 ad CD $\sqrt{247\frac{3}{5}\frac{1}{3}}$. Et quemadmodũ GC $\sqrt{99\frac{7}{9}}$



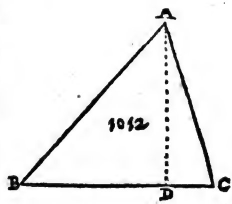
ad CN 9, sic CD $\sqrt{247\frac{3}{5}\frac{1}{3}}$ ad perpendicularem CK $\sqrt{200\frac{2}{2}\frac{3}{3}\frac{5}{7}\frac{6}{8}\frac{9}{7}}$. hujus quadratum de quadratis CD & CB subductũ relinquet quadrata DK & KB: inde & ipsæ longitudine dantur DK $46\frac{1}{2}\frac{4}{3}\frac{1}{7}\frac{4}{9}\frac{7}{7}$, KB $\sqrt{62\frac{1}{2}\frac{8}{3}\frac{1}{7}\frac{1}{3}\frac{1}{7}}$, quæ segmenta addita exhibent totã BD $\sqrt{216\frac{1}{2}\frac{3}{3}\frac{6}{7}\frac{7}{9}\frac{2}{5}}$.

Cum

Cum angulus CDB angulo CGH, & CBD ipsi CHG æquales sint triangula CGH DCB erunt similia: quare cognito latere CB reliqua per proportionem facilius potuerunt concludi hoc modo. ut CH $\sqrt{106}$ *ad CG* $\sqrt{99\frac{7}{5}}$, *sic CB* $\sqrt{263\frac{1}{4}\frac{1}{4}\frac{1}{5}}$ *ad CD* $\sqrt{247\frac{3}{5}\frac{2}{3}}$. *Et quemadmodum CH* $\sqrt{106}$ *ad HG* $9\frac{1}{3}$, *ita CB* $\sqrt{263\frac{1}{4}\frac{1}{4}\frac{1}{5}}$ BD $\sqrt{216\frac{3}{2}\frac{6}{3}\frac{2}{7}\frac{2}{5}\frac{}{7}}$.

38 ZETEMA.

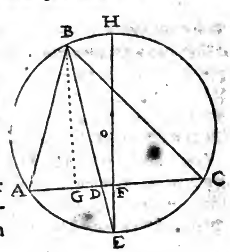
Data trianguli area et laterum ratione invenire triangulum.



Detur area trianguli decempedarū 1012, et ratione lateris BC ad AC ut 4 ad 3, et AC ad AB ut 2 ad 3. hanc rationem laterum in quatuor terminis disjunctam continuato in tribus, et qualium BC statuitur 16, talum erit AC 12, et AB 18: area trianguli ab his lateribus erit $\sqrt{8855}$: unde proportio, ut area hujus trianguli ad suorum laterum quadrata, BC 256, AC 144 AB 324; sic data area 1012 ad suorum laterum quadrata, atque inde latera ipsa longitudine dātur BC $\sqrt{\sqrt{7579706\frac{1}{5}\frac{8}{7}}}$, AC $\sqrt{\sqrt{2398266\frac{1}{5}\frac{8}{7}}}$, AB $\sqrt{\sqrt{12141224\frac{}{5}\frac{8}{5}}}$. calculum legittimé institutū inde cognsces si ex inventis trianguli lateribus area data 1012 redeat: idem erit si perpendicularem AD $\sqrt{\sqrt{2214061\frac{}{2}\frac{9}{5}\frac{9}{6}}}$ per dimidiam basin BC multiplices. factus erit area optata 1012. ubi illud monendus mihi es latus quadratum his esse bis extrahendum ob præfixam bis quadrati lateris notam. Zetema istud affine est 18.

39 ZETEMA.

Data recta angulum trianguli bisecante et basis segmentis, quæruntur crura.



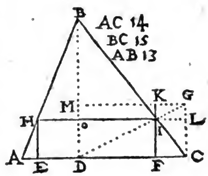
Datur BD $\sqrt{146\frac{1}{4}}$ bisecans angulum verticis B, dantur basis segmenta AD $6\frac{1}{2}$ DC $7\frac{1}{2}$, quæruntur crura AB, BC. Respondeo AB esse earundem partium 13, BC 15. Dato triangulo circulus circumscribatur & BD continuetur in E; quare AC peripheria bisecta est in E, cum ABE, CBE, anguli æquales sint ex thesi: ideo EH diameter basin AC perpendiculariter bisecat in F per 1 prop. lib. 3 *Eucl.* quare per 35 ejusdem rectangulum AD in DC æquatur rectangulo B D in

DE,

DE, datur autem AD $6\frac{1}{2}$, DC $7\frac{1}{2}$, factus $48\frac{3}{4}$ divisus per datam BD $\sqrt{146\frac{1}{4}}$, exhibet DE $\sqrt{16\frac{1}{4}}$. datur autem DF $\frac{1}{2}$ (differentia AD $6\frac{1}{2}$ et AF 7) quare perpendiculari BG facit duo triangula similia DBG, DEF; unde proportio, ut DE $\sqrt{16\frac{1}{4}}$ ad BD $\sqrt{146\frac{1}{4}}$ sic EF 4 ad BG 12: et sic DF $\frac{1}{2}$, ad DG $1\frac{1}{2}$ hæc subducta de AD $6\frac{1}{2}$ reliquam facit AG 5, & inde GC 8: jam é quadratis AG, GB, datur AB 13; ex CG & GB datur BC 15, quemadmodum quærebatur.

40 Zetema.

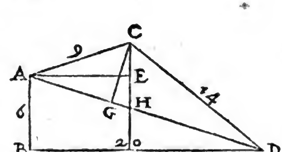
In datum triangulum parallelogrammum rectangulum sub data laterum ratione inscribere.



Idem est cum 25 sed pauso aliter solutum. Dantur latera trianguli AB 13, BC 15, AC 14, ratio longitudinis ad latitudinem inscripti parallelogrammi sit $2\frac{4}{5}$. quæruntur latera. Fiat ut $2\frac{4}{5}$ ad 1, vel ut 14 ad 5, sic AC ad CG extremæ basi perpendicularem, tumque à vertice trianguli in eandem sit perpendicularis BD, et connectatur DG intersecans latus BC in I, unde parallela sit IH contra basin AC, denique ab I et H perpendiculares in basin AC demissæ comprehendent parallelogrammum EFIH inscriptum sub laterum ratione data. Vt veró eadem hæc latera per numeros quoque inveniantur figuram suis lineamentis ita adornavi quemadmodum hic vides, ubi triangula BMN, NKI, IFC similia sunt: itemque KGI et DIF; jam CG 5 seu ipsi æquali DM subducta de perpendiculari BD 12 relinquit BM 7. erit itaque ut BD ad DC 9, sic BM 7 ad MN $5\frac{1}{4}$: quare reliqua NG $3\frac{3}{4}$: verum NG ad DC eandem habet rationem, quam KI IF: divisa itaque KF 5 secundum hanc rationem $3\frac{3}{4}$ ad 9, dabitur KI $1\frac{8}{17}$, & latitudo optati parallelogrammi $3\frac{9}{17}$. huic addita OM $1\frac{8}{17}$ ad BM 7 existit BO $8\frac{8}{17}$; inde rursū proportio, ut BD 12 ad AC 14, sic BO $8\frac{8}{17}$ ad HI $9\frac{15}{17}$ quæsiti parallelogrammi longitudinem cujus ad latitudinem ratio est data quæ 14 ad 5. hoc zetema varijs modis solvi posse tanta exemplorum varietate satis ostendimis.

41 Zetema.

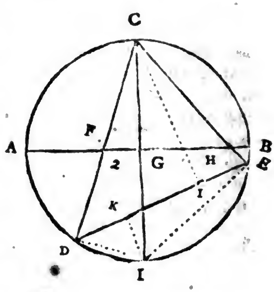
Dantur latera trapezij AB 6, BD 20, DC 14, CA 9 & angulus ad B rectus, ab angulo autem C demissa perpendicularis CF secet basin BD quæruntur segmenta basis BF, FD, & perpendicularis CE EF, itemque area trapezij ABCD.



Respondeo perpendicularem CF esse $\sqrt{20\frac{5095}{47524}}+3\frac{345}{436}$, unde subducta EF seu AB 6 relinquit C E $\sqrt{20\frac{5095}{47524}}-2\frac{91}{436}$: item FD $12\frac{139}{218}-\sqrt{1\frac{153911}{150096}}$, ea de BD 20 subducta relinquam tacit BF æqualem AE $7\frac{79}{218}+\sqrt{1\frac{153911}{150096}}$. deniq; area trapezij ABCD est $60+\sqrt{2388\frac{1}{16}}$. Solutionis via hæc est demissa perpendiculari CG efficiuntur triangula tria similia CGH, DFH, ADB. cumque AD $\sqrt{436}$ basis anguli recti ad B detur ob crura AB, BD data, dabitur igitur quoque perpendicularis CG $\sqrt{3\frac{8223}{1744}}$, quia tria latera trianguli ACD sint cognita. Inde proportio, ut BD 20 ad BA 6 sic, CG $\sqrt{3\frac{8223}{1744}}$ ad GH $\sqrt{\frac{144007}{174400}}$. cũq; AG & GD segmenta sint ipsius AD á perpendiculari CG dissectæ, addita AG ad GH datur tota AH $\sqrt{10\frac{3041}{1744}}+\sqrt{\frac{144007}{174400}}$, ergo reliqua HD erit $\sqrt{30\frac{3601}{1744}}-\sqrt{\frac{144007}{174400}}$. Porro autem quemadmodum AD $\sqrt{436}$ ad BD 20 sic HD $\sqrt{30\frac{3501}{1744}}-\sqrt{\frac{144007}{174400}}$ ad DF 12 $\frac{139}{218}-\sqrt{\frac{115391 1}{150096}}$. quare reliqua BF erit $7\frac{79}{218}+\sqrt{\frac{115391 1}{150096}}$. denique quemadmodum A D $\sqrt{436}$ ad A B 6 sic H D $\sqrt{30\frac{3601}{1744}}-\sqrt{\frac{144007}{174400}}$ ad H F $2\frac{91}{436}-\sqrt{\frac{1026061}{15009600}}$, quæ cum C H conflabit totam perpendicularem C F $\sqrt{20\frac{1095}{47524}}+2\frac{91}{436}$, unde FE seu ei æqualis AB subducta relinquit ipsam CE $\sqrt{20\frac{1095}{47524}}-2\frac{91}{436}$ ut supra.

42 ZETEMA.

Triangulo DCE circumscriptus est circulus à cujus vertice ducta diameter CI partium 12, et alia normalis AB, cujus segmentum inter crus CE et peripheriam datur HB 1, denique alterum inter centrum & crus reliquum FG 2, *quaeruntur latera trianguli.*



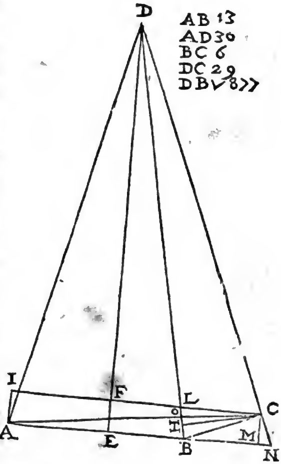
Respondeo DC $\sqrt{129\frac{3}{5}}$, DE $\sqrt{104\frac{32}{305}}$, CE $\sqrt{84\frac{60}{61}}$. Inquirito enim per 47 prop. 1 lib. *Euc.* longitudinem rectarũ CF $\sqrt{40}$ & CH $\sqrt{61}$, reliqua constructionis lineamẽta facile assequeris, perpendiculares enim sunt CI IK. hic FCG HCG triangulorũ latera cognita dantur, & triangulum DCI triangulo FCG, itemque triangulum CIE triangulo CHG simile est: Vnde proportio ut FC $\sqrt{40}$ ad CG 6, ita CI 12 ad DC $\sqrt{129\frac{3}{5}}$. Et quemadmodum CH $\sqrt{61}$ ad CG 6, ita CI 12 ad CE $\sqrt{84\frac{60}{61}}$. cumque triangula CIE & KDI, itemque CFG, KIE similia sint, deturque latus IE $\sqrt{59\frac{1}{61}}$, inde segmenta basis DK, KE invenire erit in proclivi, quæ simul conflant totam DE $\sqrt{104\frac{32}{305}}$. potuit etiam alijs modis expediri. Hoc zetema mihi ob latum est à viro harum artium perito.

Quam

Quam expeditissimam hujus Zetematis solutionem supra jam exbibui ad zetema 37, quam ob numerorum facilitatem hic iterabo. cum CF sit $\sqrt{40}$, & CG 6, CI 12, datur CD $\sqrt{129\frac{3}{5}}$. & rursum ut HCB $\sqrt{61}$ ad CG 6, sic CI 12 ad CE $\sqrt{84\frac{60}{61}}$. Sunt autem triangula HCF DCE similia, nam angulus ad C utrique est communis, & CHF angulo CDE aequalis est, quia aequantur eidem CIB, hic quia in eandē peripheriam insistit, ille ob similitudinem rectangulorum triangulorum CHG CIE: quare ut CF $\sqrt{40}$ ad FH 7, sic CE $\sqrt{84\frac{60}{61}}$ ad totā DE $\sqrt{104\frac{32}{305}}$.

43 ZETEMA.

Trapezij ABCD latera AB 13 AD 30 DC 29 BC 6 & angulus BCD rectus datur, quaeritur magnitudo diagonij AC, et quatuor triangulorū in quae ipsum à digonijs DB & AC dissecatur.



Quamvis hoc zetema varie solvi possit istum tamē quem ex constructione colligere haud est difficile modū hic usurpavimus. Principiò cū DC CB crura anguli recti dentur, etiam basis DB dabitur $\sqrt{877}$, hinc quia AD DB AB latera trianguli dantur basis segmenta AE EB in quae á perpendiculari ED dispescitur quoque dabuntur per 13 prop. 2. lib *Euc.* AE $7\frac{5}{13}$ EB $5\frac{8}{13}$, & perpendicularis CH per 8 prop. 6 lib *Eucl.* inveniatur $\sqrt{\frac{30276}{877}}$ seu quod idē est $\sqrt{34\frac{458}{877}}$, deinde HD $\sqrt{806\frac{419}{877}}$, HB $\sqrt{1\frac{419}{877}}$. Et quia triangulum HLC simile est triangulo DFL atque ideo quoque ipsi DEB erit quemadmodum DE $29\frac{1}{13}$ ad EB $5\frac{8}{13}$, sic HC $\sqrt{34\frac{458}{877}}$ ad HL $\sqrt{\frac{4481689}{3480813}}$, quae addita ad HB exhibet totam BL $\sqrt{5\frac{2080}{3969}}$.

Itemque ut DB $\sqrt{877}$ ad DE $29\frac{1}{13}$, sic LD $\sqrt{\frac{2950228}{3969}}$ ad DF $26\frac{10}{13}$, ea de ED $29\frac{1}{13}$ deducta relinquit EF hoc est ipsam AI $2\frac{4}{13}$. tum é differentia quadratorum datur FL $5\frac{139}{819}$. jam quemadmodum DE $29\frac{1}{13}$ ad DB $\sqrt{877}$, sic HC $\sqrt{34\frac{458}{877}}$ ad LC $5\frac{62}{63}$. porro cum EA & IF aequentur, é tribus lineis IF FL LC conflatur IC $18\frac{7}{13}$, hujus quadratum cum quadrato AI exhibet quadratum AC, ideoque ipsam AC longitudine $\sqrt{\frac{58981}{169}}$, vel quod idem sit $\sqrt{349}$. reliqua sunt faciliora. Triangulorū areae ita sunt, OCB $4\frac{67}{92}$. OAB $10\frac{25}{92}$, ODC $82\frac{25}{92}$ OAD $178\frac{67}{92}$. Et ipsius quadrilateri area, quatuor istis triangulis aequalis 276. cujus periculū est tibi facere licebit.

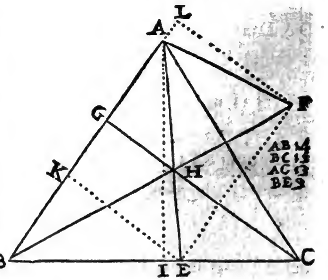
44 ZETE-

44 ZETEMA.

Ex angulo dati trianguli ABC linea educta, ex C recta CG perpendicularis in latus AB, ex A recta AE ut segmentum BE sit 9, EC 6, denique tertia ab angulo tertio B per H earum communem sectionem educta comprehendit triangulum BAF æquale absumpto triangulo BAE, quæruntur latera BF FA.

Idque non per numeros solum verum sed etiam per lineas Geometrica factione expediendum esto. Ducito EF lineam ab E puncto parallelam contra AB quæ occurrat rectæ infinitæ BH in F, & connectatur FA ea comprehendet triangulũ BAF æquale dato BAE. demonstratio é 37 propos. 1. lib. *Euc.* manifesta est. In numeris autem ita erit:

Exponantur latera AB partium 14, BC 15, AC 13; continuetur BA ut FL in eam perpendicularis cadat in L, itemque ab E perpendicularis EK, quæ æquabitur ipsi FL, & AI sit perpendicularis in BC. tumque inveniatur per præcepta exposita IB $8\frac{2}{5}$, AI $11\frac{1}{5}$, IC $6\frac{3}{5}$: cumque BE sit 9 reliqua IE erit $\frac{3}{5}$. jam ex AI & IE cruribus datur basis anguli recti AE $\sqrt{125\frac{4}{5}}$: inde per 13 prop. lib. 2 *Euc.* inveniatur BK $5\frac{2}{5}$, & AK $8\frac{3}{5}$, & perpendicularis KE $7\frac{1}{5}$, ea ob parallelismum BA EF æquatur ipsi LF.

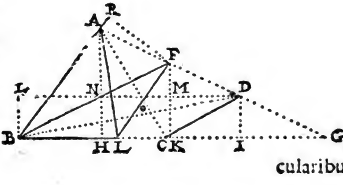


Sunt autem triangula KEA & AHG similia, itemque LFB & GHB: unde proportio, ut AK $8\frac{3}{5}$ ad KE $7\frac{1}{5}$, sic GA 5 ad GH $4\frac{1}{7}\frac{3}{5}$ itemque, ut GH $4\frac{1}{7}\frac{3}{5}$ ad GB 9, sic LF $7\frac{1}{5}$ ad LB $15\frac{1}{2}\frac{3}{5}$, inde deducta AB relinquit AL $1\frac{1}{2}\frac{1}{5}$, datur autem etiam LF, quare subtensa recto AF quoque dabitur $\sqrt{54\frac{1}{6}\frac{1}{2}\frac{1}{5}}$. denique é datis LF & LB dabitur ipsa FB $\sqrt{291\frac{2}{6}\frac{9}{2}\frac{4}{5}}$. quare latera trianguli AFB omnia dantur, ideoque etiã ipsa area $50\frac{2}{5}$: tantaque est area trianguli AEB. quamobrem factum est quod oportuit.

45 ZETEMA.

In quadrangulo ABCD dantur latera AB, 14 AD $\sqrt{250}$, DC 9, BC 15, & angulus ACD à diagonio & latere comprehensus rectus, unde triangulo ABE absumptum est triangulum æquale ABF, daturque BE 9, quæritur diagonius BD, itemque BF & FA.

Diagonius BD eo, quem supra 43 zetemate commonstravi modo inveniri potest, sed hic etiam alium ab isto non multum diversum usurpare libet. Inquirito segmenta BH $8\frac{2}{5}$, HC $6\frac{3}{5}$ & perpendicularem AH $11\frac{1}{5}$: & compleatur diagramma parallelis ac perpendi-



cularibus

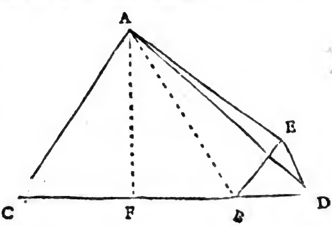
cularibus, quemadmodum hic vides. Sunt itaque CAH, OAN, COD triangula similia; atque ideo ut AH $11\frac{1}{7}$ ad HC $6\frac{1}{7}$, ita CD 9 ad CO $5\frac{1}{7}\frac{7}{8}$, ea subducta de AC 13 relinquit AO $7\frac{3}{7}\frac{9}{8}$. Et rursum, ut AH $11\frac{1}{7}$ ad AC 13, ita DC 9 ad DO $10\frac{2}{7}\frac{1}{8}$. Et quemadmodum AC $11\frac{1}{7}$ ad HC $6\frac{3}{7}$, sic AO $7\frac{3}{7}\frac{9}{8}$ ad ON $3\frac{3}{3}\frac{3}{6}\frac{0}{4}\frac{1}{8}$, quæ composita cum DO $10\frac{2}{7}\frac{1}{8}$, & LN seu BH $8\frac{1}{7}$ conflabit totam LD $22\frac{4}{6}\frac{9}{7}$. dehinc porro ut AC 13 ad AH $11\frac{1}{7}$, sic OA $7\frac{3}{7}\frac{9}{8}$ ad AN $6\frac{4}{6}\frac{1}{7}$, quæ deducta de AH $11\frac{1}{7}$ relinquet NH, sive DI vel LB $4\frac{3}{6}\frac{7}{7}$. jam é datis LD & LB dabitur diagonius BD $\sqrt{537\frac{3}{3}\frac{0}{6}\frac{3}{7}\frac{0}{8}\frac{1}{2}\frac{7}{7}}$. Porró autem ad investigationem linearum AF FB notato triangulum NAD cujus latera omnia jam cognita sunt, simile esse triangulis AHG, & DIG, atque horum latera AH DI quoq; dari: Ideoq; ut NA $6\frac{4}{6}\frac{1}{7}$, ad ND $13\frac{2}{6}\frac{2}{7}$ (est enim cōposita é duabus rectis NO & OD) sic DI $4\frac{3}{6}\frac{7}{7}$ ad IG $9\frac{2}{3}\frac{4}{3}\frac{0}{6}\frac{4}{6}\frac{6}{7}$, ad eam addita IH seu ND & HB dabit totam BG $32\frac{2}{4}\frac{7}{3}\frac{8}{7}$ Præterea ut AN $6\frac{4}{7}\frac{1}{6}$ ad AD $\sqrt{250}$, sic $4\frac{3}{6}\frac{7}{7}$ ad DG $\sqrt{118\frac{1}{7}\frac{3}{8}\frac{3}{7}\frac{4}{7}\frac{3}{6}\frac{2}{7}}$, huc addita AD $\sqrt{250}$ exhibet totam AG $\sqrt{713\frac{4}{7}\frac{3}{8}\frac{4}{7}\frac{0}{7}\frac{7}{6}\frac{7}{7}}$: notato hic AD & DG symmetros esse, & per communem mensuram $\sqrt{10}$) ad veros quadratos revocari. Porrò autem cum EF linea contra ipsam BA parallela sit, erit per 2 propos. 6 lib. *Euc.* quemadmodum BG $32\frac{2}{4}\frac{7}{3}\frac{8}{7}$ ad AG $\sqrt{713\frac{4}{7}\frac{8}{8}\frac{4}{7}\frac{0}{5}\frac{7}{6}\frac{7}{7}}$, sic BE 9 ad AF $\sqrt{54\frac{9}{7}\frac{3}{4}\frac{4}{3}\frac{}{9}}$: Et quemadmodum BG ad GA, sic EG ad GF $\sqrt{374\frac{1}{3}\frac{1}{3}\frac{9}{8}\frac{4}{8}\frac{3}{1}\frac{1}{7}\frac{1}{2}\frac{4}{9}}$. Itemque ut AG ad AH, sic FG ad FK $8\frac{1}{7}\frac{1}{6}\frac{8}{4}\frac{8}{7}$. tū subducta BH de BG reliqua est HG $24\frac{1}{7}\frac{1}{1}\frac{2}{7}\frac{8}{7}$. hinc jam ut AG ad GH, sic FG ad GK $17\frac{4}{7}\frac{0}{2}\frac{4}{3}\frac{8}{8}\frac{7}{2}\frac{9}{7}$, quæ subducta de BG relinquet KB $15\frac{1}{7}\frac{1}{6}\frac{4}{7}\frac{1}{7}$. sed KB & KF æque possunt ipsi BF: datur itaque BF $\sqrt{293\frac{7}{2}\frac{6}{2}\frac{3}{4}\frac{1}{4}\frac{1}{7}}$.

Eadem BF etiam expeditus invenietur hoc modo. continuetur BA ad perpendicularem FR. datur autem hæc perpendicularis, quia triangulum ABF triangulo ABE $50\frac{2}{7}$ æquale sit: divisa enim area ABF $50\frac{2}{7}$ per dimidiam basin AB 7 exhibet in quoto perpendicularem FR $7\frac{1}{7}$. latus differentiæ quadratorum FR & FA exhibet longitudine ipsam AR $1\frac{1}{7}\frac{8}{7}\frac{1}{7}$, hæc addita ad AB dat rectam BR $15\frac{1}{7}\frac{8}{7}\frac{1}{7}$. itaque latus summæ quadratorum BR & RF dabit quæsitam BF $\sqrt{293\frac{7}{2}\frac{6}{2}\frac{3}{4}\frac{1}{4}\frac{1}{7}}$, ut supra.

46 ZETEMA.

Datis lateribus quadranguli AC 14, CB 15, BE 5, AE 12 & angulo ad E recto, quærantur latera trianguli æqualis ADC in angulo C & latere communi AC.

Quæratur primum area dati trapezij, idque per partes, triangulum ABE quidē 30, & ABC 84, quare tota trapezij area erit 114: jam inveniatur perpendicularis AF $11\frac{1}{7}$, per hujus semissem $5\frac{3}{7}$ divisa area 114 dabit in quoto basin CD $20\frac{1}{7}\frac{1}{4}$, unde subducta CF reliquam facit FD $11\frac{6}{7}\frac{7}{8}$: denique latus summę quadratorum AF FD exhibebit quæsitam AD $\sqrt{268\frac{8}{7}\frac{1}{7}}$.

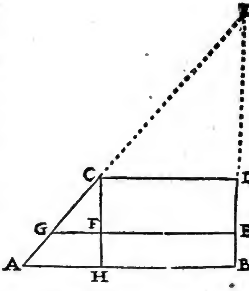


 47 ZETE-

47 ZETEMA.

Trapezij oppositis lateribus AB & CD parallelis angulus ad B rectus datur, latusque DB, 18 totaque AB 44, & segmentũ HB æquale ipsi CD 28, quæritur magnitudo parallelæ GE datum trapezium bisecantis.

Linearum ductum ad zetematis solutionem, quia is per se manifestus est, verbis explicare nihil attinet: calculi autem abacus talis est, quemadmodum AH 16 ad CH 18, ita CD 28 ad DI $31\frac{1}{2}$, unde tota BI datur, $49\frac{1}{2}$, & area trianguli rectanguli ABI 1089; trapezij autem ABDC area est 648, hujus semissis 424 de 1089 deductus, reliquum facit triangulum GIE 765. atqui cum figuræ similes eam inter se habeant rationem quã homologorum laterum quadrata, erit quẽadmodum 441 area trianguli CDI ad 784 quadratum lateris CD, sic est area trianguli GIE, ad 1360 quadratum lateris GE, itaque EG datur $\sqrt{1360}$. hujus periculum facere haud erit operosum.



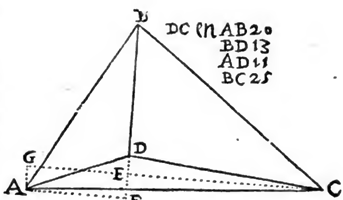
In sequentibus zetematis minus ero verbosus, & operis tantum ductum pressè sequar: harum enim non omnino rudis, é figurarum constructione, me ita literis & notis tantum loquentem haud difficulter assequetur. sicubi tamen res ipsa obscurior sit, non committam ut illic explicatio clarior desideretur.

48 ZETEMA.

Quadranguli ABCD *latera cum diagonio dantur*, AB 20, BC 25 CD 20, DA 11, BD 13, *quæritur recta vertices connectens* AC.

Respondeo *A*C esse $\sqrt{\cdot\, 502\frac{131}{169}} + \sqrt{163047\frac{22569}{28561}}$. figuræ lineamenta facile vides:

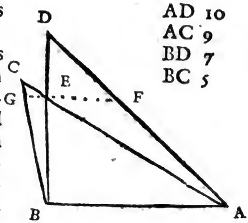
Investigatio per 12 prop. lib. 2 *Euc.* quantitatem linearum DE & D*F*, deinde perpendicularium AF EC, & dabitur AF quidem $10\frac{2}{13}$ cui æquantur GE, & EC $\sqrt{395\frac{61}{169}}$. unde cõflatur tota CG $\sqrt{395\frac{61}{169}} + 10\frac{2}{13}$. hinc subducta DE de DF relinquitur EF vel GA $2\frac{1}{13}$. denique latus summæ quadratorum AG & GC dabit quæsitam AC $\sqrt{\cdot\, 502\frac{131}{169}} + \sqrt{163047\frac{22569}{28561}}$, ut supra.



49 ZETEMA.

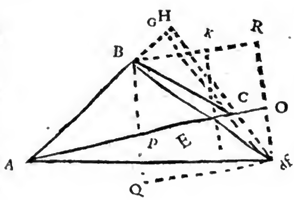
Triangulorum duorum super eadem basi latera quemadmodum hic notata vides omnia dantur, quæruntur segmenta CE ED, & recta GF per E communem sectionem contra basin BA parallela.

AB 6
AD 10
AC 9
BD 7
BC 5



Et hoc ipsum zetema, & quod jam proxime antecessit, mihi solvẽdum proposuit industrius Geodæta *Adrianus Ockerus*.

Vt quæsito satisfiat figuram suis lineamentis adornavimus, quemadmodum hic vides. jam anquirito continuationes BG BH & earum differentiam GH ipsasque perpendiculares CH DG, hinc vides CF æquari differentiæ rectarum HC DG, itemque FD ipsi HG æqualem & parallelam, harum quantitatem infra ordine adscripsimus. Præterea cum CFD triangulum ad D sit rectangulum dabitur ipsa CD $\sqrt{69\frac{1}{4}} - \sqrt{4216\frac{2}{3}}$. Quamobrem hoc zetema eo deductum est, Datis quadranguli ABCD quatuor lateribus cum diagonio utraque AC BD ipsarum segmenta invenire. id autem proximo zetemate jam demonstratum est.



Idem quoque hoc modo expediri potest. Quia trianguli ADC latera cognita sunt, dabuntur quoque CO $\sqrt{6325} - 305$, & perpendicularis OD $\sqrt{401511} - \sqrt{21025}$. datur autem quoque perpẽdicularis BP $\sqrt{810}$, & segmentum PC $3\,8$. dehinc ob parallelismum BP & RO, itemque PQ & DO æquales sunt, quare tota DR seu BQ datur $\sqrt{113837} - \sqrt{625}$. datur autem PO (composita é segmentis PC & CO) hoc est ei æquales BR vel DQ $\sqrt{6325} + 1\,07$. hinc propter similitudinem triangulorum BQD & BPC, quemadmodum BQ $\sqrt{113837} - \sqrt{625}$ ad BD 7, sic BP $\sqrt{800}$ ad BE $\sqrt{2265449292800} + 168000$, ea subducta de BD 7 relinquet ED, quæ eadem per proportionem quoque concludetur hoc modo ut RD $\sqrt{113837} - \sqrt{625}$ ad BD 7, sic DO $\sqrt{401511} - \sqrt{21025}$ ad DE $2607577 - \sqrt{2266449292800}$. Et quemadmodum BQ ad QD, sic BP ad PE $7212800 + \sqrt{99595980000}$, huc addita PA dabit totam AE $2828034 + \sqrt{99595980000}$, hęc subducta de AC relinquet segmentũ EC: his cognitis etiam parallela GF inventu haud est difficilis. Linearum quæsitarum magnitudinem hic ordine subjeci.

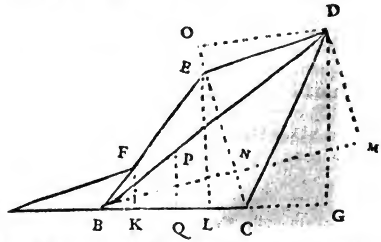
GF

GF	$9\frac{3}{3}\frac{4}{5}\frac{9}{6}\frac{7}{5}\frac{1}{1}\frac{0}{8} - \sqrt{14\frac{5}{1}\frac{1}{5}\frac{3}{7}\frac{0}{2}\frac{0}{2}\frac{2}{6}\frac{0}{9}\frac{3}{7}\frac{1}{3}\frac{0}{1}\frac{6}{2}\frac{6}{6}}$
DE	$6\frac{8}{3}\frac{8}{5}\frac{8}{6}\frac{5}{5}\frac{1}{8}\frac{1}{1} - \sqrt{14\frac{6}{1}\frac{5}{5}\frac{3}{7}\frac{5}{2}\frac{5}{2}\frac{6}{6}\frac{6}{9}\frac{9}{7}\frac{1}{3}\frac{0}{1}\frac{6}{2}\frac{6}{8}}$
EB	$\sqrt{14\frac{5}{1}\frac{5}{5}\frac{3}{7}\frac{5}{2}\frac{5}{2}\frac{6}{6}\frac{6}{9}\frac{9}{7}\frac{1}{3}\frac{0}{1}\frac{6}{2}} + \frac{1}{3}\frac{6}{9}\frac{8}{8}\frac{0}{5}\frac{0}{1}\frac{0}{1}$
AE	$7\frac{3}{3}\frac{2}{5}\frac{4}{6}\frac{5}{5}\frac{7}{1}\frac{1}{1} + \sqrt{\frac{9}{1}\frac{9}{5}\frac{5}{7}\frac{9}{2}\frac{5}{2}\frac{9}{6}\frac{8}{9}\frac{0}{7}\frac{0}{3}\frac{3}{8}\frac{0}{2}\frac{0}{8}}$
EC	$1\frac{3}{3}\frac{4}{5}\frac{4}{6}\frac{0}{5}\frac{5}{1}\frac{4}{5} - \sqrt{\frac{9}{1}\frac{9}{5}\frac{5}{7}\frac{9}{2}\frac{3}{2}\frac{9}{6}\frac{8}{9}\frac{0}{7}\frac{0}{3}\frac{0}{1}\frac{0}{2}\frac{0}{2}}$

50 ZETEMA.

Dantur in exposito diagrammate latera trianguli ⌒ABF & trapezij contigui BCDE, AB 7, AF 10, BF 4, BC 10, CD 14, DE 9, EB 12, quæritur diagonius DB.

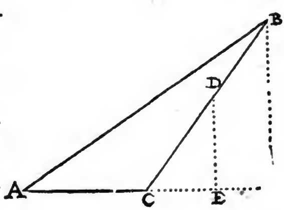
Respondeo DB esse $\sqrt{.240\frac{7}{5}\frac{9}{4} + \sqrt{29807\frac{6}{8}\frac{9}{8}\frac{7}{3}\frac{3}{6}}}$. Lineamentorum apparatum industrius lector facile intelliget, qui fere perpendicularibus & parallelis constat. Inveniatur itaque primo continuatio BK $2\frac{1}{2}$, & perpẽdicularis FK $\sqrt{9\frac{3}{4}}$. jam ut BF 4 ad BK $2\frac{1}{2}$, sic BE 12 ad BL $7\frac{1}{2}$. & reliqua LC erit $2\frac{1}{2}$. itemq; ut BF 4 ad FK $\sqrt{9\frac{3}{4}}$, sic BE 12 ad EL $\sqrt{87\frac{3}{4}}$. itaq; cognitis recti anguli cruribus EL & LC dabitur basis EC $\sqrt{94}$, hinc in EC continuatam cadat perpẽdicularis DO, itemq; BN, & inveniantur segmẽnta NC $\sqrt{6\frac{6}{5}\frac{1}{4}}$, NE $\sqrt{50\frac{6}{5}\frac{1}{4}}$, EO $\sqrt{1\frac{}{3}\frac{6}{7}\frac{5}{6}}$, & perpendicularis BN $\sqrt{93\frac{3}{5}\frac{2}{4}}$, DO seu NM $\sqrt{79\frac{3}{3}\frac{1}{7}\frac{1}{6}}$. itaque datur tota BM $\sqrt{93\frac{3}{5}\frac{2}{4}} + \sqrt{.79\frac{3}{3}\frac{1}{7}\frac{1}{6}}$. itemque é segmentis NE $\sqrt{50\frac{6}{5}\frac{1}{4}}$ & OE $\sqrt{1\frac{}{3}\frac{6}{7}\frac{5}{6}}$ datur NO vel DM $\sqrt{67\frac{}{3}\frac{8}{7}\frac{2}{6}}$, atque ideo datis cruribus anguli recti BM MD dabitur BD $\sqrt{.240\frac{7}{5}\frac{9}{4} + \sqrt{29807\frac{6}{8}\frac{9}{8}\frac{7}{3}\frac{5}{6}}}$ quæsita diagonius. Infra etiam secundo zetemati respondebitur, videlicet quanta sit area trianguli BFP.



51 ZETEMA.

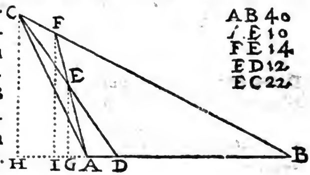
Datur AC 12 crus anguli ⌒ACB, & segmenta reliqui cruris CD 12, DB 10, item ⌒AD 22 quæritur basis AB.

Demittito perpendiculares DE BF, tum per 12 prop. lib. 2 & 2 prop. lib. 6. *Euc.* Investigato segmenta CE $8\frac{1}{6}$ & EF $6\frac{2}{3}\frac{9}{6}$ quare tota CF erit $14\frac{3}{3}\frac{5}{6}$. unde ipsa perpendicularis BF datur $\sqrt{259\frac{1}{1}\frac{0}{1}\frac{7}{3}\frac{9}{6}}$. jam cũ AC & CF dẽtur, dabitur quoque tota AF $26\frac{3}{3}\frac{5}{6}$. quamobrẽ è cruribus anguli recti AF & FB dabitur optata basis AB $\sqrt{987\frac{1}{3}}$.



52 ZETEMA.

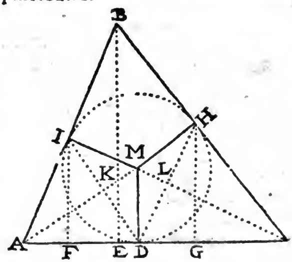
Ex angulo trianguli C educta CD occurrat baſi in D, & ex angulo A eidem baſi perpendicularis interſecet priorem CD in E & producta occurrat lateri BC in F, data baſi AB 40, & ſegmẽtis eductarũ AE 10, EF 14, ED 12, EC 22 quærũtur reliqua trianguli latera AC & CB.



AB 40
AE 10
FE 14
ED 12
EC 22

Demittantur in baſin AB tres perpẽdiculares EG, FI, & CH. hinc multiplicato CD in EF fiunt 476, itemque AF in CE & fiunt 528. hinc jam quemadmodũ 28 ad 476 ſic eſt tota AB 40 ad DB $36\frac{2}{33}$, igitur reliqua AD $3\frac{31}{33}$. dãtur itaque latera trianguli DEA, quare & ipſa AG $3\frac{1638}{2664}$, & GD $7\frac{1189}{2145}$, & perpendicularis EG $\sqrt{\frac{399324884}{4601025}}$. Vnde proportio, quemadmodum DE ad EG, ita DC ad CH $\sqrt{\frac{28851222869}{41409225}}$. Et quemadmodum ED ad DG, ſic CD ad DH $21\frac{2599}{6435}$, cum qua addita DB dabit totam HB $57\frac{2989}{6435}$ quare ex CH & HB cognitis dabitur CB $\sqrt{4000\frac{16918}{331275}}$. Denique ſubducta AD de HD relinquetur HA $17\frac{2989}{6435}$, itaque ex cognitis CH & HA dabitur CA $\sqrt{1002\frac{1461174}{1873165}}$. quemadmodum quærebatur.

53 ZETEMA.



Circulus in triangulum ABC inſcriptus latera ejus contingat in punctis D, H, I, dataque diametro ejus partium 8, & ſegmentis baſis AD 6, DC 8, quæruntur latera reliqua AB & BC.

Linearum ductum oculis & animo lector induſtrius haud difficulter aſſequetur. ipſarum vero magnitudo eodem plane modo quem 13 zetemate docuimus erit anquirenda, omnes autem hic á latere in tabella conſpiciendas exhibemus.

Notato autem triangula AIF & ABE, itemque BEC & HGC ſimilia eſſe, atque ideo quemadmodum IF $5\frac{7}{13}$ ad AF $2\frac{4}{13}$ hoc eſt ut 1 ad $\frac{5}{12}$ ita quoque eſſe BE ad EA. & HG $6\frac{2}{5}$ ad GC $4\frac{4}{5}$, hoc eſt ut 1 ad $\frac{3}{4}$, ita quoque eſſe BE ad EC : quare AE ad EC habet eam rationem quam $\frac{5}{12}$ ad $\frac{3}{4}$. quamobrem AC ſecundũ iſtam rationem diviſa dabit ſegmenta AE 5, EC 9.

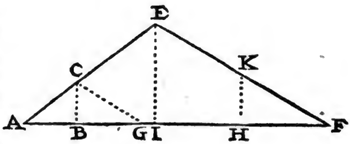
Inde proportio, quemadmodum AF $2\frac{4}{13}$ ad FI $5\frac{7}{13}$, ſic AE 5 ad EB 12. Hinc jam cognita perpendiculari BE & ſegmentis AE & EC dabuntur latera AB 13, CB 15. Vel ſic, ut FA $2\frac{4}{13}$ ad AI 6, ſic AE 5 ad AB 13. & quemadmodum GC $4\frac{4}{5}$ ad CH 8, ſic EC 9 ad BC 15. ſupra etiam aliam ſolutionis viam commonſtravimus. Eſt quæſtio 57 in geometricis problematis *Simonis Iacobi*.

MA	$\sqrt{52}$
MC	$\sqrt{80}$
DC	$\sqrt{51\frac{1}{5}}$
HL	$\sqrt{12\frac{4}{5}}$
HD	$\sqrt{51\frac{1}{5}}$
KA	$\sqrt{24\frac{12}{13}}$
KI	$\sqrt{11\frac{1}{13}}$
ID	$\sqrt{44\frac{4}{13}}$
AF	$2\frac{4}{13}$
GC	$4\frac{4}{5}$
GH	$6\frac{2}{5}$

54 ZETE-

54 ZETEMA.

Data basi AF 32 & segmentis AB 5, HF 9, unde perpendiculares excitatæ reliquis lateribus occurrunt BC $2\frac{14}{23}$, HK 4, quæritur area trianguli AEF.

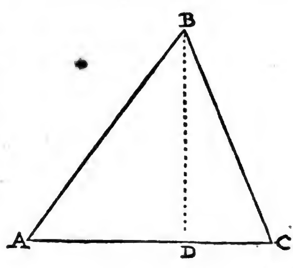


Solutio hujus zetematis antecedenti affinis est: sed & aliam faciliorem commonstrabo. sit CG parallela contra latus erit EF, itaq; CBG triangulum simile triangulo KHF, totumque triangulum ACG toti AEF. unde proportio, quemadmodum KH 4 ad HF 9, sic CB $2\frac{14}{23}$ ad BG $5\frac{20}{23}$. datur itaque tota AG $10\frac{20}{23}$. quemadmodum igitur AG $10\frac{20}{23}$ ad BC $2\frac{14}{23}$, sic AF 32 ad perpendicularem IE $7\frac{17}{25}$, ea per dimidiam basin AF 16 multiplicata dabit aream optatam $122\frac{22}{25}$.

55 ZETEMA.

Datis summis basis & cruris unius duorum triangulorum rectangulorum æquealtorum ipsa latera invenire.

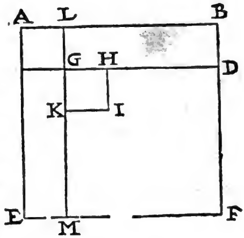
Exemplum tale esto, ex angulo B trianguli acutanguli ABC perpendicularis esto BD, á qua ipsum in duo rectangula triangula dispescitur, hic jam data summma linearum AB & AD 24, itemque BC & CD 18, quæruntur latera AB BC, & tota AC. hoc zetema est vagum, infinita enim dari possunt latera ad quæsiti præscriptum, libet tamen hoc theorema proponere, ut talium solutio solos explicabiles exhibeat.



Sum-

ſummam baſis & cruris AB & AD, vel BC & DC duplicato, ut duplum BC CD 18 eſt 36, qui numerus per quadratũ diviſus dabit quotũ quadratũ, hic de 9 dimidio ſubductus relinquet pro crure DC 5, ad eundẽ additus dabit 13 pro baſi BC, deniq; latus differentiæ quadratorum á BC & CD erit perpendicularis BD 12. jã porro dividatur linea compoſita ex BA & AD 24 in duo ſegmenta, ut differentia quadratorum á majore & minore ſegmento æquetur quadrato datæ lineæ BD. Id autem ſupra jam demonſtratum eſt: ad quam rem etiam præceptum tale concipi poteſt.

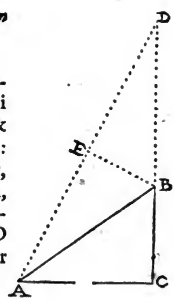
A data ſumma 24 quadrato 576 quadratum perpendicularis BD 144 ſubducito, reliquũ per 48 ejuſdem data ſumma duplum dividito, quotus 9 exhibet AD anguli recti crus reliquum, idque de ſumma 24 ſubductum dabit baſin AB 15.



Hujus demonſtratio iſta eſt. Quia Si recta ſecta ſit in duo inæqualia ſegmenta quadratum totius excedit differentiam quadratorum á ſegmentis duplici rectangulo ẽ tota & minore ſegmento. Id enim in appoſito diagrammate clarum eſt. Eſto AB inæqualiter ſecta in L ſitque AL minus LB ſegmentum majus. quadratum totius ABEF, quadratum minoris ſegmenti ACGL, majoris GMFD, differentia horum eſt gnomo HIKMFD. jam gnomo alter ECALB-DGM cum quadrato GHIK, æquatur duobus rectangulis ex tota AB & minore ſegmẽto AL.

56 ZETEMA.

Datis trianguli rectanguli crure AC 8, & ſumma reliquorum laterum AB BC 20, quæruntur ſingula.

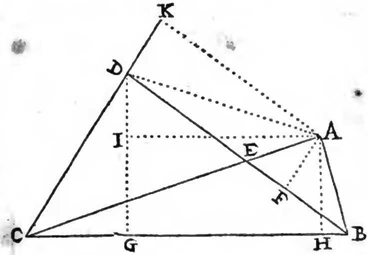


Reſpõdeo *AB* eſſe partium $11\frac{2}{5}$. *BC* $8\frac{3}{5}$. Solutio ex antecedente præcepto manifeſta eſt, verũ hoc modo quoq; ſolvi poteſt continuetur enim CB in D, ut *BA* BD æquentur & cõnectatur AD: erit itaque *ABD* triãgulum æquicrurum: hinc ex *B* demiſſa perpẽdicularis *B*E biſecabit ipſam AD, dantur autẽ *A*C 8, CD 20, dabitur itaque baſis *A*D $\sqrt{464}$, dimidium ED $\sqrt{116}$. hinc propter ſimilitudinem triangulorum DE*B* DCA, ut CD 20 ad D*A* $\sqrt{464}$, ita *E*D $\sqrt{116}$ ad D*B* vel *BA* $12\frac{2}{5}$, quare reliqua BC quoque datur $8\frac{3}{5}$, quod feciſſe oportuit.

57 ZETEMA.

Duorum rectangulorum triangulorum CD‿A C‿AB data basi CB $\sqrt{185}$, & crure CD 8, ‿AB 4. quaeruntur laterum reliquorum intersegmenta CE, E‿A, DE, EB.

Potuit etiã alia via hoc problema solvi. Ego istam nunc elegi. Principio inquirantur reliqua crura BD 11 CA 13, deinde AH, HB, HC, HG, AF, FB, ID



CA 13
DB 11
GB $\sqrt{79\frac{26}{185}}$
CG $\sqrt{22\frac{26}{185}}$
DG $\sqrt{41\frac{159}{185}}$
IG vel AH $\sqrt{14\frac{114}{185}}$
DI $\sqrt{7\frac{1}{185}}$
HB $\sqrt{1\frac{71}{185}}$
GH $\sqrt{59\frac{110}{185}}$.
DA $\sqrt{66\frac{111}{185}}$
FB $3\frac{1}{5}$
FD $7\frac{4}{5}$
DK vel AF $2\frac{2}{5}$
KC $10\frac{2}{5}$.

per 47 prop. lib. 1. & 13 propos. lib. 2 *Euc.* quarum quantitatem hic adscriptam vides. tum propter ID & IA datas dabitur ipsa DA $\sqrt{66\frac{111}{185}}$. Datis itaque lateribus trianguli DAB dabuntur quoque per 13 prop. lib. 2. *Euc.* DF FB & perpendicularis FA. jam ob similitudinem triangulorum CKA, CDE erit, quemadmodũ CK $10\frac{2}{5}$ ad KA $7\frac{4}{5}$, ita CD 8 ad DE 6. & quemadmodum CK $10\frac{2}{5}$ ad CA 13, ita CD 8 ad CE 10. denique DE 6 subducta de DB 11 reliquam facit EB 5; itemque CE 10 de CA 13 relinquit EA 3.

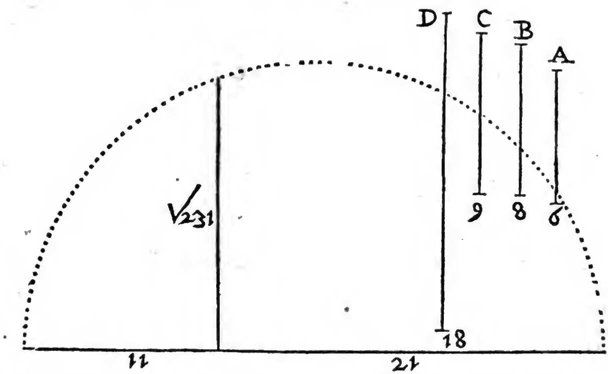
Ut hunc librum non sine auctariolo concludam, expeditissimam solvendi hujus zetematis viam proponam, in qua nullis novis perpendicularibus alijsve lineis nobis opus sit. Quia triangula CDE BAE similia sunt (anguli enim ad D & A recti, & ad E sunt ad verticem) ideo latera homologa erunt proportionalia, ut CD ad DE, sic BA ad AE: ut CD ad CE, ita BA ad BE: quare composite, ut CD 8 & BA 4, ad DE & EC & EA & EB hoc est ad CA 13 & DB 11, ita CD 8 ad DE & EC 16. datur itaq; CD 8 crus anguli recti & DE EC simul basis cũ crure reliquo 16, quaerantur singula latera CE ED per antepenultimum zetema, hoc modo. quadratum summae 16 videlicet 256 multetur quadrato cruris CD 64, reliqui 192 dimidium 96 per eandẽ summam 16 divisum dabit in quoto 6 pro crure DE, idque de 16 subductum relinquet 10 pro CE: unde & reliqua dabuntur EA 3, EB 5. Atque ita hic quarti libri finis esto.

LVDOLPHI à CEVLEN HILDESHEIMENSIS

Problematum miſcellaneorum liber quartus, quæ hic vel Geometricè per ſolas lineas, vel per canonem triangulorum, aut denique per Algebricas poſitiones ſolvuntur.

PROBLEMA I.

E datis quatuor lineis quadrangulum conſtruere quod ſit in circulo.



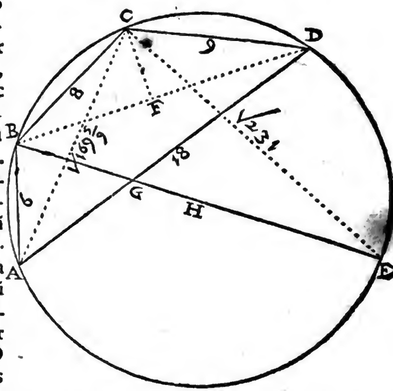
Annus hic agitur viceſimus ex quo vir harum artium ſtudioſiſsimis *Ioannes Pauli* problema iſtud mihi propoſuit, quod quidem tum tēporis per algebricas poſitiones ſolvi, poſtmodum vero etiam, ut algebræ ignaris inſervirem, theorema tale excogitavi, quo per numeros idem expediri commode poſſet. Exponantur itaque quatuor lineæ é quibus deformandum ſit quadrangulum dictum, ita ut & A ipſi D, & C ipſi B opponatur.

Præceptum.

Magnitudinem datam lineæ B divídito per lineam D, in iſto exemplo quotus erit $\frac{4}{9}$: itemque A per C, quotus erit $\frac{2}{3}$, hi quoti additi conſlant $1\frac{1}{9}$: deinde primum quotum multiplicato per lineam *A* 6, factum dividito per C 9 quotoque addito unitatem habebis $1\frac{8}{27}$, hic numerus per priorem ſummam multiplicatus dabit $\frac{350}{243}$.

Deinde

Deinde multiplicato A 6 cum C 9, & B 8 per D 18, factorum summâ 198 divisâ per inventum numerum $\frac{3}{8}\frac{5}{4}\frac{0}{5}$, quoti latus erit $\sqrt{2\,4\,\frac{0}{8}\frac{7.7}{7\,7}}$, hic numerus denique per $1\frac{8}{2\,7}$ multiplicatus dabit diagoniū infra designatum BD $\sqrt{231}$. Porrò ut idem per lineas pręstes inquirito lineam quę secundum expositam mensurã sit $\sqrt{231}$, hoc modo: continuato duas lineas in exposita mensura quarum rectangulū sit 231, ut in præmisso diagrãmate A*B* 11 BC 21 linea inter has proportione media *B*D erit optatæ magnitudinis $\sqrt{231}$. Inde ex lineis *B* & C atque ista basi construatur triangulum eique circulus circumscribatnr, in quẽ duæ reliquę lineæ inscribantur & habebis quadrangulum ABCD é datis lateribus & circulo inscriptum.

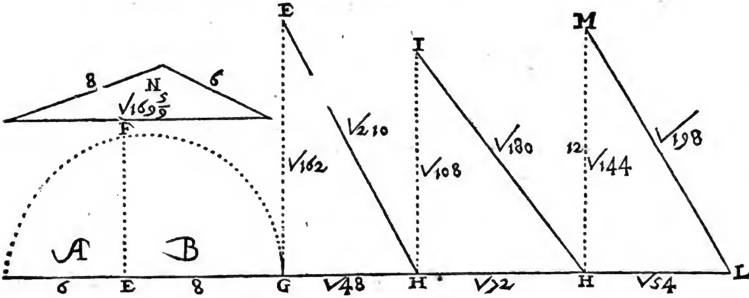


Constructionis causa dependet é 15 prop. et 21 lib.6. *Euc.* Diameter circuli cui id quadrilaterum inscribitur facile invenietur ob data latera trianguli *B*CD. demittatur enim perpendicularis CF, & agatur diameter *B*E atque connectatur CE, sunt itaque triangula CDF BCE similia, erit ita quemadmodum perpendicularis CF $\sqrt{7\frac{3}{8}\frac{3}{7}\frac{5}{1}}$ ad CD 9, ita *B*C 8 ad diametrum BE $\sqrt{359\frac{2}{7}\frac{3}{7}\frac{9}{7}}$. Hoc inventum meum est: verum juvenis ingeniosus *Cornelius Petrejus* Alcmarianus ante quadriennium mihi confessus est, se istud problema Geometrice solvisse, quod reipsa postmodum docuit. Fabrica autem ita habet.

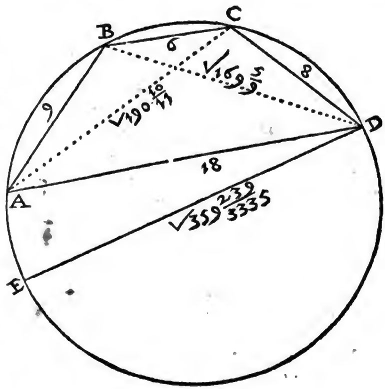
PROBLEMA 2

*Exponantur quatuor lineæ A*BC*D ut supra, è quibus construendum sit quadrangulum circulo inscriptibile,*

Itaque

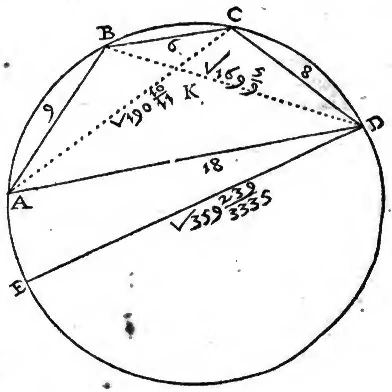


Itaque inter A & B inveniatur media proportionalis EF, & GE inter *C* & D, tũque iſtæ mediæ ſint crura anguli recti baſis HE: deinde iterum IH HH mediæ proportionales inter A & D, B & C comprehendant angulum rectum cujus baſis IH: tertio mediæ proportionales inter A & C, B & D comprehendant angulum rectum cujus baſis LM: tũque ſubtenſis HE HI LM, inveniatur quarta proportionalis N. Denique triangulo é cruribus D & C atq; baſi N circumſcribatur circulus in quem reliquæ A & *B* inſcriptæ & inter ſe & cum terminis baſis N in peripheria conterminæ inſcribent circulo quadrangulum optatum ABCD, ut infra hic vides. Deinde etiam notato, ſi fiat ut IH √180 ad EH √210, ſic LM √198 ad quartã N √231, atque triangulo é cruribus *B* C ac baſi N circulus circumſcribatur, ita quoque quadrangulum in circulum inſcribi. & quamvis hæc diagonius á priore diverſa ſit, circulum tamen eundem, & quadrangulum eadem magnitudine exhiberi.



Problema insigni pseudographemate Iosephi Scaligeri *noblitatum, qui in Cyclometricis suis mirabili paralogismo bene à se id præstitum alijs persuadere conatus est : atque eo ipso alios ad hujus solutionem inuitavit : qua occasione subtilissimus* Franciscus Vieta *hoc problema construxit, et Clarissimus* Ioachimus Prætorius *harum artium scientia nulli secundus, de quatuor lineis in circulo integrũ librũ publicavit, in quo multis modis ingeniose sane et acute hoc idem problema effici posse demonstravit. Et nos quoque à* Ludolpho *olim incitati idem præstitimus cujus analysin , quia eadem est cum ista fabrica quam ἀναπόδεικτον hic ponit, ut isti loco facem alluceam hic adscribere non abs re fore Iudicavi.*

Quamobrem factum id jam sit. Et quadrilaterum ABCD *sit in circulo , et agantur diagonij AC, BD sese intersecantes in K ; erunt itaque AKB & DKC, itemque BKC & AKD triangula similia. Ideoque ratio* DK *ad KA eadem quæ DC ad AB, id est sumpta communi altitudide* DA *in* A B *: Et rursum AK ad KB, ut A*D *ad* BC *, hoc est sumpta communi altitudine AB, ut* AD *quæ D C in D A ad* D A *in* AB *ad* AB *in BC: Denique etiã BK ad KC, ut AB ad CD, hoc est sumpta communi altitudine BC ut* AB *in* BC *ad BC in* CD. Datur *itaque ratio segmentorum utriusque diagonij* DK, KA BK, KC, *eadem quæ rectangulorum* DC *in* DA, DA *in* AB, AB *in BC*, BC *in* CD. *quamobrem* DK *et* KB, *ad* KA *&* KC *hoc est diagonius* DB *ad diagonium AC habebit rationem eandem, quam duo parallelogramma* DC *in* DA *plus* AB *in* BC, *ad* DA *in* AB *plus* BC *in* CD : *itaque inventis medijs proportionalibus inter horum parallelogrammorum latera habebis quadrata ipsis æqualia: itaque basis anguli recti cujus ista sunt crura ipsis æque poterit , quare pro* D C *in* D A *plus* A B *in* BC *ad sumes quadratum ab ML ; et pro* DA *in* AB *plus* BC *in* CD , *quadratum ab EH. est itaque ratio diagoniorum* BD *ad* AC, *quæ quadrati ab ML ad quadratum ab EH. Erit itaque ob similitudinem ratio facti á* BD *in* AC *ad quadratum á* BD, *quemadmodum plano-planum sub quadrato ML in quadratum EH ad quadrato-quadratum ab ML, hoc est de pressis magnitudinibus, quæ quadrati ab EH ad quadratum ML, datur autem rectangulum diagoniorum* BD *in* AC, *quod, per ea quæ Ptolomæus demonstravit cap. 9 lib. 1 magni operis, æquale est rectangulis oppositorum laterum* AD *in* BC *et* AB *in* DC, *cui æquale positum est quadratum HI Erit itaque ut quadratum HE ad quadratum ML sic quadratum HI ad quadratum diagonij* B D. *atque ideo ut linea HE ad ML , sic linea HI ad quæsitam* BD.



Quare

Quare ista est propositi problematis analysis, unde ad synthesin facilis est via.

Constructio problematis si lineas spectes satis operosa , in numerorum autem pragmatia valde expedita, quare ad diagonij inventionem δεδομένον tale concipio.

δεδομένον.

Quemadmodum duo rectangula ab oppositorum angulorum cruribus ad duo rectanguli sub cruribus reliquorum angulorum, ita duo rectangula oppositorum laterum ad quadratum diagoniæ primis angulis subtensæ.

Vt in exemplo, quæratur diagonius AC, rectangulum ab AD & DC 144 additum ad rectangulum ex AB in BC 54 conflat 198, rectangula AB in AD 162 & BC in CD 48 sunt 210; denique rectangula oppositorum laterum AD in BC 108 & AB in DC 72 sunt 180: hinc jam erit ut 198 ad 210, sic 180 ad 190 $\frac{1}{1}\frac{0}{1}$ quadratũ diagonij AC angulis B & D quorum crurum facti primum locum obtinent subtensi. hujus veritas ex analysi supra allata est manifesta.

Potuit autem ex eadem analysi alia etiam longe concinnior geometrica fabrica derivari in qua illam planoplanorum stereometriam in demonstratione declinemus: & quamvis diagrammate ad hanc rem idoneo destituamur, dicam tamen quam potero clarissime, iterata editione diagrammata majore sedulitate procuraturi.

Tria rectangula sub AD in DC, BC in CD, BC in AD applicentur ad quartam datarum AB: fiat deinde ut BC plus latitudine ex applicatione prima existente, ad AD plus latudine ortiva secundæ, sic CD plus latitudine ortiva tertia, ad quartam proportionalem, linea inter hanc & ipsam AB ad quam rectangula applicata sunt proportione media erit optata diagonius BD, subtensa angulo sub cruribus primæ *applicationis comprehenso. demonstratio ex analysi superiore repetenda est, quam totam exhiberem si diagramma esset idoneum.*

Hoc loco monendus mihi est lector istud theorema tanquam jam demonstratum ab omnibus assumi, videlicet Si rectangulum sub diagonijs æquatur rectangulis duobus ex oppositis quadrilateri lateribus, tale quadrilaterum esse in circulo.

Cum id à nemine hactenus unquam fuerit geometricis demonstrationum nominibus firmatum, conversum autem tantum Ptolemæo libro primo magni operis sit demonstratum: videlicet,

Si quadrilaterum sit in circulo rectangula oppositorum laterum rectangulo diagoniorum æquari.

Porrò ad investigationem areæ quadrilateri in circulum inscripti theorema hujusmodi habemus valde scitum.

Si de dimidio collectorum laterum dati quadranguli in circulum inscripti latera sigillatim subducantur, latus continue à quatuor differentijs facti erit area.

Dentur latera AB 9 BC 6 CD 8 DA 18 ut prius, & quæratur area. summa laterum omnium 41, dimidium 20 $\frac{1}{2}$ differentiæ laterũ omniũ 11 $\frac{1}{2}$ 14 $\frac{1}{2}$ 12 $\frac{1}{2}$ 2 $\frac{1}{2}$ numerus ab his continua multiplicatione factus 5210 $\frac{1}{1}\frac{5}{6}$, cujus latus $\sqrt{5210\frac{1}{1}\frac{5}{6}}$ est area dati quadranguli. ut hujus periculum facia addito areas duorum triangulorum BCD BAD. namque area trianguli BCD est $\sqrt{1334\frac{4}{4}\frac{0}{9}}$ seu $\sqrt{272\frac{1}{4}\frac{2}{9}}$ & BAD $\sqrt{24312\frac{2}{7}\frac{1}{8}\frac{5}{4}}$ seu $\sqrt{3101\frac{}{7}\frac{3}{8}\frac{1}{4}}$. areæ additæ constabunt aream quadrilateri ABCD $\sqrt{5210\frac{1}{1}\frac{5}{6}}$, ut prius.

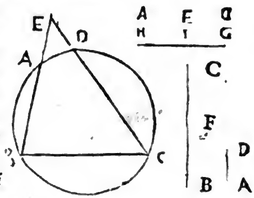
Namque √ 2 4 3 1 $\frac{3}{7}\frac{1}{8}\frac{5}{4}$ *et* √ 1 3 4 $\frac{4}{4}\frac{0}{9}$, *hoc est facta ad commune nomẽ reductione* √ 1 1 3 $\frac{4}{7}\frac{4}{8}\frac{0}{4}$ *sunt numeri symmetri et revocantur ad vere quadratos per* √ 1 $\frac{3}{7}\frac{3}{8}\frac{5}{4}$ *ille ad* √729 *hic ad* √64. *Inde jam cætera sunt faciliora: quamobrem quantò operosior est hæc uulgata ad investigandam aream via, tanto gratius novum hoc nostrum theoremation benevolo lectori futurum speramus.*

Secunda ista quam exhibuimus fabrica potuit ad Geometricam hujus problematis solutionem satis esse oportuna, quia tamen olim ab amicis, ipsoque adeó Ioanne Prætorio, *cum eum* Altorfij *salutassem (simul cum clarissimo* Adriano Romano Pragra *rsdiens) ad hoc argumentum invitarer, tres alias hujus problematis factiones excogitavimus, quarum facillimam hic adscribam.*

E datis quatuor rectis quadrilaterum cui circulus circumscribi possit construere.

Oportet autem tres quaslibet simul sumptas esse majores reliqua.

Exponantur magnitudine quatuor rectæ A B, *BC, CD, DA, é quibus quadrangulum construendum esto quod sit in circulo. Hic commoditatis gratia assumantur opponi maxima BC et minima AD, quarum differentia sit FC. fiat itaque ut FC differentia ad oppositam minorem DA, sic ambæ intermediæ AB & CD ad quartam GH: deinde quemadmodum BC et DA maxima et minima ad inventam GH plus alterutra intermediarũ AB, sic minima AD ad IG seu ED continuationem reliqui intermadij CD. Si itaque triangulum ex AD, GI & IH segmentis rectæ HG construatur, sitque IG ipsi ED, et IH æqualis ipsi AE; et ED continuetur inter mediâ DC, & AE ipsa A*B, *(videlicet minori segmẽto major majori minor, intermedia) et connectatur BC, ajo quadrilaterum ABCD et é datis lineis constructum et in circulo esse. Cum enim sit per fabricam ut exposita BC et AD ad AD, sic* BE *et ED ad ED; erit quoque dividendo, ut data* BC *ad AD, sic* BE *ad ED. Et rursum, quia est ex fabrica ut CF ad DA sic AB &* DC *ad* DE *& EA: erit componendo ut CB ad* DA *sic* BE *& EC ad* DE *& EA, atque ita jam demonstravimus esse BE ad* ED: *quare ex æquo, ut BEC ad* DEA, *sic BE ad ED: & dividendo ut CE ad* EA, *sic* BE *ad ED: quare cum crura ejusdem anguli proportionalia sint triangula BEC,* DEA *erunt similia, et anguli homologis lateribus subtensi æquales. quare* EAD *angulo* BCD *æqualis erit: ideoque* BAD, *BCD duobus rectis æquales, atque ideo quatuor pũcta* ABCD *erunt in circulo. Sed et basis exposita BC æqualis est, quia per fabricam* DEA *ad* BEC *est ut* AD *ad datam* BC: *atque per similitudinem* AED *ad* BEC *quoque est ut* AD *ad hanc* BC: *quare ista inscripta datæ æqualis est. Atque ideo quadrilaterũ è datis lateribus construximus et circulo complexi sumus: quemadmodum imperabatur.*

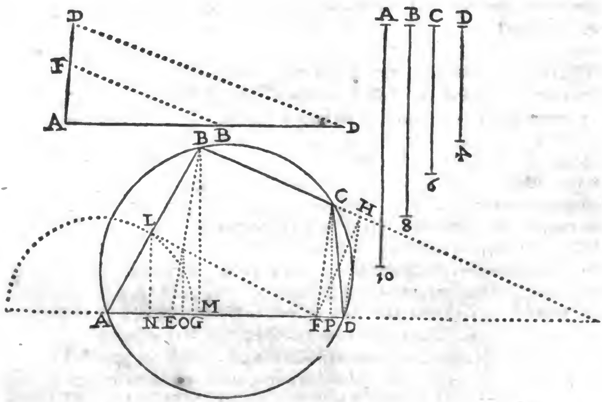


Problema 3

Idem aliter ex invento Vietæo.

Cum jam olim ante complures annos illud haud difficulter é 22 propos. 3 lib. *Eucl.* obſervaſſem angulos oppoſitos quadrilateri in circulum inſcripti duobus rectis æquari, atque hic me multum excruciaſſem modo hanc modo iſtam viam tentando, ut aliquo ſaltem modo propoſitum problema ſolverem, fateor me irrito conatu omnia expertum, neque quidquam profeciſſe. Idem tamen toties à me fruſtra tentatum clariſſimus et ſubtiliſſimus ille *Franciſcus Vieta* Regius quondam conſilarius et libellorum ſupplicum in curia Pariſienſi magiſter ſolvit, cujus demonſtrationem ex ipſius libro, quem anno 1595 Pariſijs publicavit hic adſcribam, et numeris deinceps comprobabo.

Exponantur quatuor lineæ rectæ ABCD é quibus quadrilaterum in circulo conſtruendum eſto: ordo autem laterum hic ſit ut A & C, B et D latera interſe opponantur.



Expoſitæ A ſtatuatur æqualis AD: tumque differentia inter oppoſitas A et C ſecetur ſecundum rationem reliquarum B et D, ſitque ſegmentum minus FD & majus AE utrinque abſumptum à terminis rectæ AD; quare interſegmentum EF relinquitur æquale ipſi *C*: Iam centro A intervallo æquali differentiæ reliquarum duarum B & D deſcribatur ſemicirculus ML, deinde agatur ipſi ED æqualis recta FL a puncto F ad peripheriam in L pertingens, inde ex A centro per L

agatur *AB* æqualis ipsi *B*: sitque *BC* datæ C æqualis parallela contra LF & connectatur *CD*: Ajo CD quartæ datarum D æquari & quadrilaterum A*B*CD esse in circulo. Continuentur enim *A*D *B*C & quia annuunt concurrant in I: & ab F sit FH parallela contra A*B*, itaque oppositæ BH & FL hoc est ipsa *E*D æquabuntur: atqui *B*C & EF æquales demonstratæ sunt: quare CH & FD quoque æquales erunt: facta autem est AE ad FD, ut A*B* ad FH, & anguli *BA*E HFD ob parallelismum linearũ *AB* FH æquales, atque ideo ipsa triangula *B*AE FHD æquiangula & similia: ideoque BE quoque parallela contra HD: ideoque BH ED, HI ID proportionalia, & cum prima secundæ æqualis sit, tertia HI quartæ DI quoque æqualis erit, cumq; CH FD æquentur triangula CID *F*HI erunt æquilatera, ideoq; CD & FH, hoc est ex demonstratis LB æqualia: quare CD est datæ D æqualis. cumque ob parallelismum laterum FH & *A*B triangula A*B*I & FHI similia sint; FHI autem & CDI æquilatera, erunt quoque AIB CID similia, ideoque DCI ipsi BAI æqualis: quare *B*AI & *B*CD duobus rectis æquales comprehẽdũtur á quatuor lineis datis AB*C*D circa quas circulus describi possit, sicut oportebat.

Demonstratio quam pro Vietana hic autor supposuerat non satis erat idonea: quare eã ad Vietæ normam reformare nobis fuit necesse.

Vt fabricæ istius ductum numeris exprimamus demittantur perpendiculares LN KO BM CP. itaque latera trianguli rectanguli *A*LN, quę dantur ob data latera AL*F* per 13 propos. lib. 2 *Euc.* *E*st autẽ ALN triangulum simile triangulo A*B*M, daturque A*B* 8, quare AM $4\frac{4}{119}$ & BM $\surd 45\frac{79}{119}$ quoq; dabũtur, quarũ omnium magnitudinẽ hic á latere expressam vides. tumque AM subducta de AD *1*0 reliquam facit, MD $5\frac{9}{119}$. itaque cum BM & MD æque possint subtensæ *B*D, dabitur ipsa diagonius BD $\surd 77\frac{11}{119}$. reliquam autem ad diagonium AC ista assequeris.

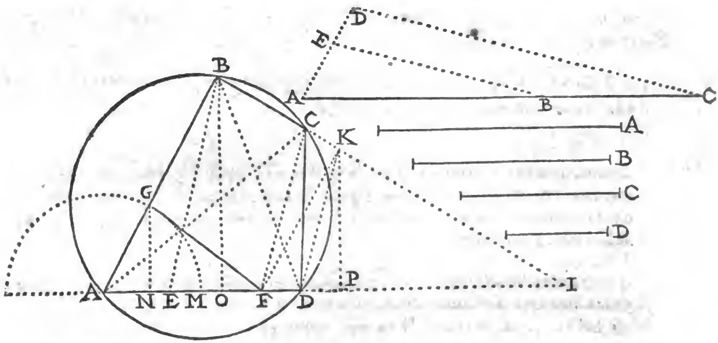
Cũ triangula LNF KOF similia sint, erit ut LF $7\frac{1}{7}$ ad LN $\surd 11\frac{60}{119}$, sic K*F* seu EF 6 ad KO $\surd 7\frac{12377}{16475}$. & crus recti EO $\frac{96}{119}$. est quare basis recti KE $\surd 8\frac{8}{149}$. & EM $1\frac{25}{35}$. datis autem EM & BM dabitur quoq; EB $\surd 48\frac{16}{117}$, unde deducta EK $\surd 8\frac{8}{149}$ relinquetur BK seu FC $\surd 16\frac{30}{35}$. datis itaq; lateribus trianguli *F*CD, dabitur quoque perpendicularis CP $\surd 15\frac{11}{117}$ & segmentum FP $\frac{32}{35}$, PD $\frac{4}{35}$. quare AP erit $9\frac{2}{17}$. quare datis cruribus anguli recti APC dabitur quoque basis A*C* $\surd 108\frac{1}{17}$. Continuationes laterum CI DI invenientur hoc modo, ut KO $\surd 7\frac{12377}{16475}$ ad K*F* 6, ita BM $\surd 45\frac{79}{119}$ ad BI $14\frac{2}{7}$, inde subducta BC relinquet

AE	$2\frac{4}{7}$
FD	$1\frac{1}{7}$
AF	$8\frac{2}{7}$
KL	$7\frac{1}{7}$
DE	$7\frac{1}{7}$
AL	4
AN	$2\frac{4}{119}$
NF	$6\frac{30}{119}$
LN	$\surd 11\frac{60}{119}$
BM	$\surd 45\frac{79}{119}$
AM	$4\frac{4}{119}$
MD	$5\frac{9}{119}$
BD	$\surd 77\frac{11}{119}$
OF	$5\frac{47}{149}$
FE	6
EO	$\frac{96}{119}$
EM	$8\frac{25}{35}$
KE	$\surd 1\frac{8}{149}$
KO	$\surd 7\frac{12377}{16475}$
EB	$\surd 48\frac{16}{117}$
AB	$\surd 16\frac{30}{35}$
FP	$\frac{32}{35}$
PD	$\frac{4}{35}$

quet CI $8\frac{2}{3}$. atque tanta est quoque ipsa FI. ejus veritas comprobatur hoc modo, ut KO $\sqrt{7\frac{1}{2}\frac{2}{6}\frac{3}{4}\frac{7}{4}\frac{7}{9}}$ ad OF $5\frac{4}{7}\frac{7}{4}\frac{7}{3}$, sed BM $\sqrt{48\frac{7}{7}\frac{7}{6}\frac{7}{9}}$ ad MI $13\frac{1}{3}\frac{1}{9}$. unde subducta FM $4\frac{1}{3}\frac{4}{9}$ relinquet FI $8\frac{2}{3}$ ut supra: hinc porró subducta FD $1\frac{1}{3}$ dabit reliquam DI $7\frac{1}{3}$, tanta igitur quoque erit ipsa HI, & CH item $1\frac{1}{3}$.

Est autem triangulum *C*DH æquilaterum triangulo DFH, & simile ipsi AE*B*, ut jam supra ostensum fuit, atque anguli BAE DCH ęquales. itaque ut *C*H ad CD, sic AE ad A*B*. Sunt autem quoque FH CD L*B* lineæ æquales. hinc autem causa plana est cur differentia laterum oppositorum quales assumuntur A & C divisa sit secundum rationem reliquarum *B* & D. Et quam ob causam radius semicirculi AL æqualis assumatur differentiæ reliquorum laterum *B* & C.

CP	$\sqrt{15\frac{1}{3}\frac{2}{2}\frac{2}{7}}$
AP	$2\frac{7}{7}\frac{7}{1}$
AC	$\sqrt{108\frac{8}{3}\frac{8}{6}}$
BI	$14\frac{2}{3}$
CI	$8\frac{2}{3}$
MF	$4\frac{1}{3}\frac{4}{9}$
FI	$8\frac{2}{3}$
HI. DI	$7\frac{1}{3}$



Aliud ejusdem generis. Exponantur eędem quatuor rectæ lineæ, & secunda *B* assumatur pro basi, cui opponatur quarta D. earumque differentia sit secta secundum rationem reliquarum A & C, & segmentum majus sit AE minus FD: deinde figura tota construatur ut prius. Hic autem *B*I & AI etiam aliter invenire licebit. Nam cum triangula *A*GF *AB*I similia sint, erit ut G*A* 4 ad *A*F $6\frac{1}{2}$, sic A*B* 10 ad *A*I $16\frac{1}{4}$. Et quemadmodum AG 4 ad GF $5\frac{1}{2}$ sic *A*B 10 ad *B*I $13\frac{3}{4}$. itaque CI $9\frac{3}{4}$, DI $8\frac{1}{4}$. datis itaque lateribus trianguli ABI dabitur per 13 prop. lib. 2. *Eucl.* OI $10\frac{2}{7}\frac{3}{2}$ & *A*O $5\frac{5}{7}\frac{5}{3}$. datur itaque perpendicularis *B*O $\sqrt{71\frac{1}{3}\frac{1}{6}\frac{1}{9}}$. & *C*P $\sqrt{35\frac{8}{3}\frac{5}{2}\frac{5}{3}}$. jam per 47 propos. 1 lib.

AE	$2\frac{1}{2}$
FD	$1\frac{1}{2}$
AF	$6\frac{1}{2}$
DE	$5\frac{1}{2}$
GF	$5\frac{1}{2}$
FH. FE.	4
AO	$5\frac{5}{7}\frac{5}{3}$
OD	$2\frac{8}{7}\frac{8}{1}$
GN	$11\frac{6}{7}\frac{6}{6}\frac{1}{9}$
BO	$\sqrt{71\frac{1}{3}\frac{1}{6}\frac{1}{9}}$
BD	$\sqrt{77\frac{1}{3}\frac{1}{3}}$
CP	$\sqrt{35\frac{8}{3}\frac{5}{2}\frac{5}{3}}$
AC	$\sqrt{108\frac{8}{3}\frac{8}{3}}$

lib. *Eucl.* facillime invenietur diagonius BD $\sqrt{77\frac{11}{13}}$. itemque AC $\sqrt{108\frac{8}{11}}$. Vt aream dati quadranguli invenias investigato primum aream trianguli totius ABI multiplicata perpendiculari *B*O in dimidiam basin *A*I factus erit $\sqrt{4687\frac{1}{2}}$. deinde multiplical dimidiam perpendicularē CP $\sqrt{35\frac{85}{121}}$ per basin DI dabitur area trianguli CDI $\sqrt{607\frac{1}{2}}$ quæ de ABI subducta relinquet $\sqrt{1920}$ aream quadranguli *AB* *C*D.

AI $16\frac{1}{4}$
BI $13\frac{3}{4}$
DI $8\frac{1}{4}$
CI $9\frac{3}{4}$
OI $10\frac{45}{52}$
OD $2\frac{8}{13}$
PI $7\frac{31}{44}$. P*A* $8\frac{6}{11}$: circumscripti circuli diameter $\sqrt{109\frac{19}{30}}$

Antequam hic manum de tabula tollam monendus est mihi lector, Ad diagoniorum in numeris inventionem non esse expeditiorem aliam ullam rationem ea, quam supra primo loco in nostris commentarijs ostendimus, cujus epilogismum hic iterum exhibeo. rectangulum AB 10 in BC 4 est 40. BC 4 in CD 6, 20: CD 6 in DA 8, 48: DA 8 in AB 10. 80: denique rectangula oppositorum laterum BC 4 in AD 8, 32: AB 10 in CD 6, 60:

Itaque ut 40 & 48 ad 24 & 80, sic 32 & 60 ad quadratum diagonij hoc est ut 88 ad 140, sic 92 ad $108\frac{8}{11}$ *cujus latus* $\sqrt{108\frac{8}{11}}$ *ipsa diagonius AC. Et sic ex ijsdem terminis quoque dabitur BD hoc modo Vt 104 ad 88 sic 92 ad* $77\frac{11}{13}$ *quadratum diagonij BD, quare ipsa BD erit* $\sqrt{77\frac{11}{13}}$.

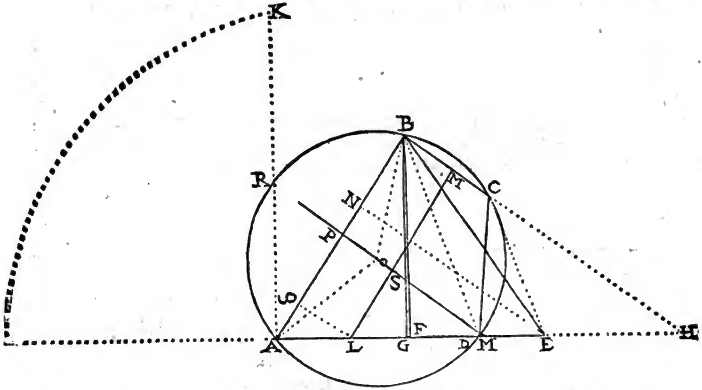
Secundo ad investigationem areæ quadranguli in circulo nihil parabilius fieri potest isto quem exhibuimus modo, Vt hic summa omnium laterum 28, dimidiū 14, quatuor laterū ab ex differentiæ 46810, numerus ab his differentijs continua multiplicatione factus 1920 cujus latus $\sqrt{1920}$ *est optata qua-dranguli area.*

Tertiò ad inveniendas laterum concurrentium continuationes facilem suppeditat viam secunda nostra hujus problematis solutio. namque ut differentia AB & CD 4 ad summam BC & AD 12, sic DC 6 ad simulutramque continuationem DI & IC 18. Et rursum ut AB & CD 16 ad quartum 18 & alterutrum continuatorum ut CD 4, hoc est ad 22, sic DC 6 ad reliqui lateris continuationem DI $8\frac{1}{4}$ *quæ de 18 subducta facit CI reliquam* $9\frac{3}{4}$.

PROBLEMA 4.

Quadrilaterum é datis quatuor lineis circulo inscriptum linea contra datum latus parallela secare ratione data.

Dantur latera eadem quæ in novissimo exemplo problematis antecedentis eodemque ordine, imperatur linea LM parallela contra AB ita educi, ut dictum



dictum quadrilaterum ab ea bisecetur, hæc Geometrice; tum etiam in numeris quanta sit bisecans LM, quantaque ejusdem a centro circuli circumscripti distantia sit quæritur. Revocetur itaque quadrangulum ABCD ad triangulum æquealtum & æquale ABE, cujus basin AE bisecet linea BG. deinde inter totam AH et segmentum HG sit AK sive ipsi æqualis HL media proportionalis: inde LM ab L termino contra AB parallela bisecabit datum quadrangulum, cujus bisegmenta erunt ABML, MCDL: demonstratio é 16 problemate libri 3 est manifesta. Numerorum abacus ita habet. Aream trianguli ABE dicto quadrangulo æqualis $\sqrt{1920}$ per totam perpendicularem BF $\sqrt{71\frac{1}{117}}$ dividito quotus erit AG $5\frac{1}{7}$ dimidium basis AE: datur autem AF, quare et reliqua FE quoque dabitur, inde ex notis BF & FE dabitur basis recti, BE, quarum omnium magnitudinem hic contra notatam vides. quærantur deinde BN NA segmenta, et perpendicularis EN. Porrò latus facti AH in HG dabit mediam proportionalem LH $\sqrt{179\frac{9}{16}}$, quæ subducta de AH $16\frac{1}{4}$ relinquit AL $16\frac{1}{4}-\sqrt{179\frac{9}{16}}$. jam ut HA $16\frac{1}{4}$ ad AB 10, sic HL $\sqrt{179\frac{9}{16}}$ ad LM $\sqrt{68}$. Hinc diameter circuli dicto quadranguli circumscripti inventa esto $\sqrt{109\frac{49}{136}}$, ideoq; radius BO vel AO $\sqrt{27\frac{49}{136}}$. quare PO bisecans inscriptam AB erit $\sqrt{2\frac{49}{136}}$.

diameter circuli	$\sqrt{109\frac{49}{136}}$
RO $\sqrt{27\frac{49}{136}}$	BF $\sqrt{71\frac{1}{117}}$
AG	$5\frac{1}{7}$
AE	$10\frac{2}{7}$
AF	$5\frac{6}{17}$
GF	$\frac{12}{67}$
FH	$10\frac{45}{77}$
GH	$11\frac{1}{116}$
FE	$5\frac{1}{17}$
AK seu HL	$\sqrt{179\frac{9}{16}}$
BE	$\sqrt{96\frac{9}{37}}$
NB	$4\frac{2}{7}$
AN	$5\frac{3}{7}$
NB	$4\frac{1}{7}$
AN	$5\frac{3}{7}$
perpend. NE	$\sqrt{76\frac{4}{7}}$
AH	$16\frac{1}{4}$
AL	$16\frac{1}{4}-\sqrt{179,}$
LM	$\sqrt{68}$
PE	$\sqrt{2\frac{49}{136}}$
PS	$\sqrt{187\frac{1}{2}}-\sqrt{127\frac{1}{2}}$.
perpend. ab M in LH demissa	$\sqrt{48\frac{1}{167}}$.

 Sunt

Sunt autem triangula ANE AQL ſimilia, itaque ut AE $10\frac{2}{7}$ ad NE $\sqrt{76\frac{4}{7}}$, ſic AL $16\frac{1}{4}$ -- $\sqrt{179\frac{9}{16}}$ ad QL ſeu PS $\sqrt{187\frac{1}{2}}$ -- $\sqrt{127\frac{1}{2}}$ — unde ſubducta PO relinquit OS $\sqrt{147\frac{46}{117}}$ — $\sqrt{127\frac{1}{2}}$ quęſitam parallelæ LM á centro diſtantiam. Vt autem conſtet ſit ne ABNL dimidium dati quadranguli ABCD, concipito perpendicularem ab M in baſin AH demiſſam, cujus quantitatem proportione concludes, hoc modo, ut AB ad FB, ſic LM ad perpendicularem dictam $\sqrt{48\frac{1}{11}}$, quæ per dimidiam LH multiplicata dabit aream trianguli MLH $\sqrt{2167\frac{1}{2}}$: ſupra autem jam inventa eſt area trianguli ABC $\sqrt{4687\frac{1}{2}}$, quare illa de hac deducta reliquum facit triangulū ABM $\sqrt{480}$. quod dimidium eſt quadranguli ABCD $\sqrt{1920}$. quare factum eſt quod oportuit.

Si velis experiri an BMLA ſit dimidium totius quadranguli id poterit commodiſsime quoque fieri hoc modo. quia PS perpendicularis jam inventa eſt $\sqrt{187\frac{1}{2}}$ — $\sqrt{127\frac{1}{2}}$, & BA LM parallelæ dantur, multiplicata perpendiculari PS per dimidium ſimul utriuſque parallelæ BA & LM dabitur area BMLA. eſt autem utraque ſimul $10 + \sqrt{68}$, dimidiū $5 + \sqrt{17}$ multiplicatio talis erit.

$$\sqrt{187\tfrac{1}{2}} - \sqrt{127\tfrac{1}{2}}$$
$$5 + \sqrt{17}$$

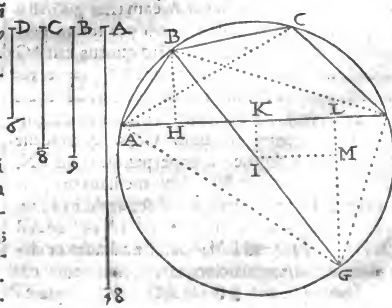
$$+\sqrt{3187\tfrac{1}{2}} - \sqrt{2167\tfrac{1}{2}}$$
$$\sqrt{4687\tfrac{1}{2}} - \sqrt{3187\tfrac{1}{2}}$$

factus $\sqrt{480}$.

Namque $+\sqrt{4687\frac{1}{2}}$ & $-\sqrt{216\frac{1}{2}}$, reducantur per $\sqrt{7\frac{1}{2}}$ ad vere quadratos: in diverſis autem ſignis additi eſt ſubductio quare univerſus factus ut ſupra erit $\sqrt{480}$ dimidium dati quadrilateri $\sqrt{1920}$. quemadmodum imperabatur.

PROBLEMA. 5

Si dati quadrilateri in circulū inſcripti diameter ab ejus angulo aliquo educta latus oppoſitum interſecet, quærantur & lateris & diametri mutua ſegmenta.



Ab angulo B quadrilateri ABCD cujus latera ſunt data AB 6, BC 8, CD 9, DA 18, educatur diameter BG eaq; interſecet A in F, quærantur ſegmenta AF, FD, BF, FG. jā ex præmiſsis dabitur diagonius

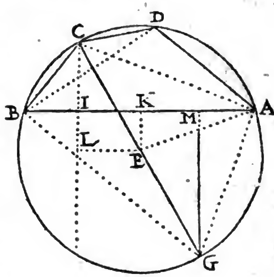
gonius BD $\sqrt{231}$, ideoque etiam é lateribus ABD perpendicularis BH & segmenta AH, HD. sit jā IK perpendicularis é centro in AD ea itaq; erit uoq; data quia AK dimidium est ipsius AD, & AI radius detur: cum sit ut BH ad BD, sic BA ad diametrum BG $\sqrt{359}$ [illegible]. Præterea AG GD quoque dantur, quia anguli BAG BDG in semicirculo recti sunt: dabitur itaque perpendicularis LG, & segmenta lateris AL LD; subductaque LD de KD 9 reliqua KL sive ei æqualis IM quoque erit data: itaque data IG basi & MI crure recti dabitur quoque crus MG. est itaque IMG triangulum datorum laterum simile triangulis FLG, BHF, IKF. ideoque eorum latera proportionalia, ut MG ad FL, sic IG ad GF $\sqrt{171}$ [illegible]. itemque ut MG ad GI, sic BH ad BF $\sqrt{34}$ [illegible]. tanta igitur sunt segmenta diametri quæsita. quæ addita rursum totam diametrum conflabunt $\sqrt{358}$ [illegible]. Porró quemadmodum MG ad MI, sic GL ad LF 7 [illegible], ea addita ad LD dabit DF 11 [illegible]. denique quemadmodum GM ad MI, sic BH ad HF 3 [illegible], quæ cum AH composita conflat totam AF 6 [illegible]. Fac periculum & nihil in opere commissum deprehendes.

AC	$\sqrt{169}$ [illegible]
BD	$\sqrt{231}$.
AH	3 [illegible]
HD	14 [illegible]
BH	$\sqrt{23}$ [illegible]
BG	$\sqrt{359}$ [illegible]
LG	$\sqrt{89}$ [illegible]
AG	$\sqrt{323}$ [illegible]
DG	$\sqrt{128}$ [illegible]
HL	14 [illegible]
LD	3 [illegible]
IG	$\sqrt{115}$ [illegible]
IM, LK	5 [illegible]
MG	$\sqrt{60}$ [illegible]
FL	7 [illegible]
FD	11 [illegible]
AF	6 [illegible]
HF	3 [illegible]

Videtur autor dedita opera tanta circuitione usus: certe longe facilius istoc opere defungi potuit, hoc modo. quia diagonius quadrilateri DB datur, dantur jam tria latera trianguli ABD, quare segmenta basis AH HD & perpendicularis BH quoque dabuntur. atque ideo ut BH ad BA, sic BD ad diametrum BG, quare & radius AI datur, & quia IK perpendicularis in inscriptam AD ipsam bisecat dabitur bisegmentum AK, cum itaque in rectangulo triangulo AKI basis anguli recti AI & crus AK dentur, dabitur quoque perpendicicularis IK: Sunt autem triangula BHF, FIK similia, ob angulum verticalem ad F, & rectos ad K & H. itaq; ut BH $\sqrt{}$ [illegible] *& KI* $\sqrt{}$ [illegible] *hoc est* $\sqrt{}$ [illegible] *ad radium BI* $\sqrt{}$ [illegible] *sic KI* $\sqrt{}$ [illegible] *ad FI* $\sqrt{}$ [illegible]. *jam FI segmentum additum ad radium GI* $\sqrt{}$ [illegible] *hoc est si ad commune nomen reducatur* $\sqrt{}$ [illegible], *(divisor autem communis maximus per quem ad vere quadratos reducuntur hic est* $\sqrt{}$ [illegible]*) conflabunt GF* $\sqrt{171}$ [illegible]. *Eadem FI de eodem radio subducta reliquam faciet BF* $\sqrt{34}$ [illegible]. *Deinde quoque fiat ut simulutraque BH & KI* $\sqrt{}$ [illegible] *ad HK* [illegible], *ita KI* $\sqrt{}$ [illegible] *ad FK 2* [illegible] *quæ addita & subducta de dimidia AK 9, dabit segmenta DF 11* [illegible] *& AF 6* [illegible]. *quemadmodum imperabatur.*

PROBLEMA 5.

Si dati quadrilateri in circulum inscripti diameter ab ejus angulo aliquo educta ipsam secet quæritur area segmentorum.



Dantur latera *A*B 24, *B*C 11, *C*D 8, DA 14, ab angulo C sit diameter CG secans basin AB in F dividat ipsum quadrilaterum in triangulum BCF & trapezium FCDA, quærantur primum diagonij BD AC diameter CG, quarum magnitudinem hic adscriptam vides, deinde ipsum diagramma suis lineamentis adornato, quemadmodũ superiore problemate edoctus es, ut hic vides: inde inveniantur segmenta *B*I IA & perpendicularis IC: hinc differentia quadratorum radij AE & segmenti *A*K relinquet quadratum EK, erit itaque EK $\sqrt{}$ 3 [illegible]. quæ addita ad CI dabit totam *C*L $\sqrt{}$ 115 [illegible], unde proportio ut CL ad L*E* 5 [illegible], ita KE ad KF [illegible], quæ ad BK dimidium ipsius AB addita dabit *FA*, & indidem subducta reliquam faciet *B*F. jam dimidia BF multiplicata cum perpendiculari CI dabit aream trianguli BCF $\sqrt{}$ 2449 [illegible]. Porrò autem duo triangula BCA $\sqrt{}$ [illegible], & CDA $\sqrt{}$ [illegible] addita conflabunt aream dati quadranguli ABCD $\sqrt{}$ 23408 [illegible]. hinc subductum triangulum BCF relinquet segmẽtum quæsitum FCDA $\sqrt{}$ 10713 [illegible]. *C*ujus periculum ita fieri poterit si aream trianguli CFA (quæ invenitur multiplicata perpendiculari *C*I in dimidiam basin *FA*) addas ad triangulum CDA, idem enim numerus redibit pro area quadranguli BCDA.

PROBLEMA. 7

Datum triangulum à dato extra puncto secare ratione data.

Problema valde difficile & arduum, á multis irrito conatu quæsitum. Anno á nativitate Salvatoris Domini nostri Iesu Christi *1582*, Clarissimus & recõditæ eruditionis Mathematicus *Simon Stevinus* alijs hujus generis quæstionibus, quas ad me tum misit, hoc ipsum problema quoque tanquam singulare ultimo loco subjunxit tali encomio & inscriptione ornatum.

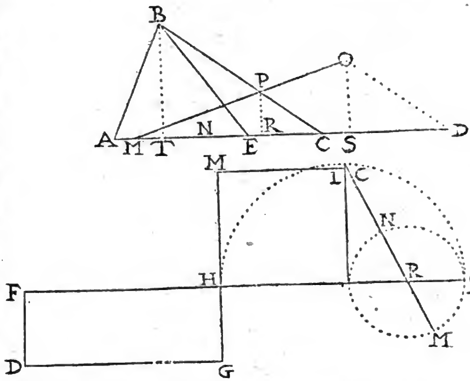
Antecedentia quidem problemata omnia solvi, hoc solo excepto, quod á quibusdam Geometris multum commendatũ, quamvis non admodum difficile videatur, á nemine tamen hactenus solutum est ideoque quisquis hoc ipsum demonstraverit solveritque, haud dubie hoc ipso nomini suo claritatem additurus.

His territus labori supersedi, quia me imparem isti negotio facile sentiebam, nedum ut alijs, qui in istis artibus me magis lyncei sunt plus sapere aut intelligere velle viderer. Postmodum vero etiam *Ioannes Pauli* non ignobilis Gæodæta & harum studiosissimus, quæstiunculam ad me misit de triangulo recta per datum

intra

intra ipſum puncto educta bifariam ſecando, quam non geometrico linearum ductu demonſtrari, ſed numerorum ſubſidio ſecundum datam menſuram ſoliv poſtulabat: cui tum Algebricarum poſitionum abaco á nobis factum eſt ſatis, neque ante deſtiti quam idem quoque Geometrice expediviſſem. Interea temporis tot jam annis quaſi obliterata erat apud me illius de dato extra puncto problematis memoria, donec tandem labente hoc anno 1599 jam ſupra nobis nominatus Iuvenis *Cornelius Petrejus Alcmarianus*, ſcripto mihi ſignivicavit inventam á ſe viam triangulum á dato extra puncto data ratione ſecandi. Paucis deinde poſt diebus ejuſdem ſolutionem á me rogatus *Willebrordus Snellius* RF, exhibuit mihi librum *Ioannis Baptiſtæ de Benedictis* de quæſitis per epiſtolam editū Venetijs anno 1585, in quo acutiſsimus & doctiſsimus ille Mathematicus & ingenioſe & acute propoſitum problema demonſtrat, cujus factio præcipuę é 28 propoſitione libri 6 *Euc*. ſubtiliſſime derivatur, quod tamē ſupraſcriptus *Will. Snellius* per 36 prop. lib. 3. *Eucl.* expeditius faciliuſque abſolvit, hic modo.

Expoſitum eſto triangulum ABC, á quo ex O puncto extra ipſum dato recta OM educenda ſit abſumens triangulū PMC, quod ſit totius ABC pars quarta.



Secetur AC baſis ratione data, ſitque EC ipſius $\frac{1}{4}$, erit itaque *BEC* totius trianguli pars quarta: já rectāgulo BC in CE ſtatuatur rectangulū æquale FH GD. hinc contra latus BC ab dato puncto O parallela ſit OD, cui æqualis ſtatuatur HI vel ML ad quā conſtruatur parallelogrammū HMLI ipſi DFHG ęquale, tumque latitudini inventę HM ponatur æqualis CN. jam CN continuetur in M ut rectangulum CMN æquetur rectangulo NC in CD. Id autem per 36 propoſitionem 3 lib. *Eucl.* præſtari poteſt. ſitque inventa NC cui æqualis ponatur NM, recta ab dato puncto O cum M connexa abſumet triangulum MCP æquale dato BCE. Cujus demonſtratio ita habet. Quia rectangulum CM in MN æquatur rectangulo EC in CB, erit per 14 prop. lib. 6 *Eucl.* ut DC ad CM, ſic MN ad NC: & compoſite, ut DM ad MC, ſic MC ad NC: ſed ut DM ad MC, ita ob paralleliſmum OD ad PC: ex æquo igitur, ut MC ad CN, ſic OD ad PC: quare

 rectan-

rectangulum extremarum MC in PC æquatur rectangulo mediarum OD in NC, hoc est per fabricam BC in *CE*: erit itaque, ut *MC* ad *C*B, sic EC ad CP: ideoque cum duo trianguli ad *C* crura reciproca sint triangula erunt æqualia. quamobrem MCP triangulum é dato externo puncto absum ptina est æquale ipse *B*CE, quemadmodum imperabatur.

Datum triangulum é puncto extra aut intra ipsum dato data ratione secare, tanquam consectarium retuli ad problema secundum in suscitata à nobis Apollonij de spatij sectione Geometria, ubi longe istis generaliora praestantur. Veruntamen quia factio, quæ hic exhibetur nonnihil tadij & minus concinnitatis habet, illam quam in istis soleo insistere viam nunc ostenderem, si per diagramma licitum esset, quod iterata editione curabitur. Hæc eadem, & quomodo à dato quolibet spatio datum spatium recta à dato intra vel extra puncto unica recta absumi possit, Clarissimus Simon Stevinus *in Illustrissimi Principis nostri* Hypomnematum Mathematicorum *syntagmate luculentissime commonstravit, quem vide.*

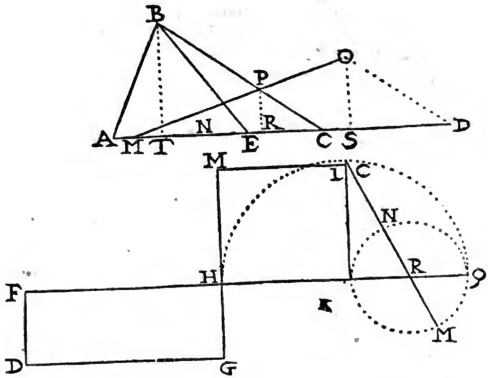
Verum age ut istum linearum ductum numeris quoque exprimamus, sit AB 13, *B*G 15, AC 14, CD 8, OD 9. rectangulum sub BC 15 & EC $3\frac{1}{2}$ (est enim EC quarta pars basis AC) ut DEH erit $52\frac{1}{2}$, cui æquatur HMLI, ponitur autem ML æqualis ipsi OD 9, quare HM cui æquatur NC erit $5\frac{5}{6}$. Huic sit æqualis IQ. Et rectangulo sub NC $5\frac{5}{6}$ CD 8 sit æquale quadratum perpendicularis IC, erit itaque ipsa IC $\sqrt{46\frac{2}{3}}$. hinc quadratum ab IR $2\frac{1}{1}\frac{1}{2}$ dimidio lineæ IQ additum ad quadratum ICdabit potentiam lineæ RC, ideoque etiam magnitudine $\sqrt{55\frac{1}{1}\frac{2}{4}\frac{5}{4}}$ cum qua composita RM $2\frac{1}{1}\frac{1}{2}$ exhibet totam CM $\sqrt{55\frac{1}{1}\frac{2}{4}\frac{5}{4}} + 2\frac{1}{1}\frac{1}{2}$. Huc præterea addatur CD eritque tota MD $10\frac{1}{1}\frac{1}{2} + \sqrt{55\frac{1}{1}\frac{2}{4}\frac{5}{4}}$. Vnde proportio, quemadmodum MD $10\frac{1}{1}\frac{1}{2} + \sqrt{55\frac{1}{1}\frac{2}{4}\frac{5}{4}}$ ad DO 9, sic MC $\sqrt{55\frac{1}{1}\frac{2}{4}\frac{5}{4}} + 2\frac{1}{1}\frac{1}{2}$ ad CP $\sqrt{69\frac{8}{1}\frac{4}{3}\frac{9}{2}\frac{}{4}} - 3\frac{9}{1}\frac{}{3}\frac{}{2}$. dehinc perpendiculares sunto BT PR: quare ob similitudinem CB 15 ad BT 12, ita CP $\sqrt{69\frac{8}{1}\frac{4}{6}\frac{9}{2}\frac{}{4}} - 3\frac{9}{1}\frac{}{2}$ ad PR $\sqrt{44\frac{2}{3}\frac{2}{2}\frac{1}{6}} - 2\frac{1}{1}$, quæ multiplicata per MC dabit 42, cujus dimidium 21 est area trianguli PCM: atqui tanta est area trianguli BCE, est enim $\frac{1}{4}$ totius ABC.

DC	$3\frac{1}{2}$
NC	$5\frac{5}{6}$
LF	$\sqrt{46\frac{2}{3}}$
LR	$\sqrt{55\frac{1}{1}\frac{2}{2}\frac{5}{4}}$
MC	$\sqrt{55\frac{1}{1}\frac{2}{4}\frac{5}{4}} + 2\frac{1}{1}\frac{1}{2}$
NC	$\sqrt{55\frac{1}{1}\frac{2}{4}\frac{5}{4}} - 2\frac{1}{1}\frac{1}{2}$
PC	$\sqrt{69\frac{8}{1}\frac{4}{6}\frac{9}{2}\frac{}{4}} - 3\frac{9}{1}\frac{}{3}\frac{}{4}$
PR	$\sqrt{44\frac{2}{3}\frac{2}{2}\frac{1}{6}} - 2\frac{1}{1}$

Quin etiam si MC $\sqrt{55\frac{1}{1}\frac{2}{4}\frac{5}{4}} + 2\frac{1}{1}\frac{1}{2}$ multiplicetur per CP $\sqrt{68\frac{8}{1}\frac{4}{6}\frac{9}{2}\frac{}{4}} - 3\frac{9}{1}\frac{}{3}\frac{}{2}$, factus $52\frac{1}{4}$ æquabitur facto à BC 15 in CE $3\frac{1}{2}$: quamobrem calculus legitimé est institutus.

Ejusdem generis exemplum aliud: Dantur latera trianguli AB*C* A*B* 13, *AC* 21 *BC* 20, unde ex O puncto recta educta defectum est triangulum M*C*P $\frac{1}{7}$ totius *ABC*, est autem parallela OD æqualis distantiæ *C*D $12\frac{1}{7}$, quæruntur latera MP, P*C*, CM.

Triangulum



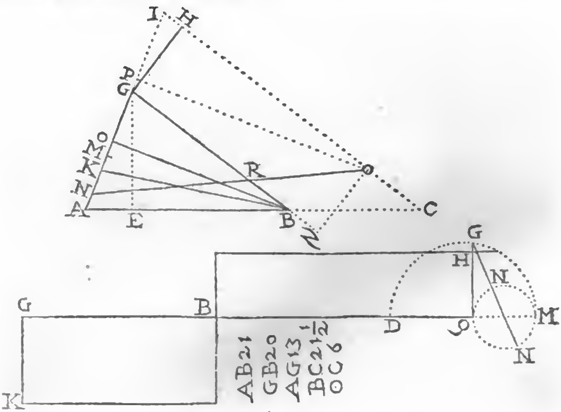
Triangulum BCE sit $\frac{5}{7}$ totius, reliqua fabrica ex præmissis satis obvia est. Inveniatur itaque ut supra MH seu NC $11\frac{1}{7}$, & CR $\sqrt{171\frac{9}{37}}$ tum MC $\sqrt{171\frac{9}{37}} + 5\frac{3}{7}$, quemadmodum hic vides. Hinc quia in exposito exemplo CD DO æquantur, CN & PC quoque æquabuntur, eritque utraque $\sqrt{171\frac{9}{37}} - 5\frac{3}{7}$. denique PM dabitur $\sqrt{.108\frac{8}{1}\frac{1}{2}\frac{1}{7}} + \sqrt{3\frac{2}{3}\frac{1}{6}\frac{6}{6}\frac{1}{6}\frac{4}{2}\frac{1}{7}}$. Non est necesse omnia latera trianguli MPC indagare, sola enim basis MC & ad fabricæ constructionem & ejusdē δοκιμασίαν satis esse potest. sunt enim triangula MCP MDO similia, ideoque inter se ut homologorum laterum quadrata. quare ut quadratum ab MD, ad rectangulum ab MD in dimidiam perpēdicularem OS, (tanta enim est area trianguli MOD) ita quadratum ab MC ad aream trianguli MPC.

KL $11\frac{5}{7}$
CN $11\frac{1}{7}$
CR $\sqrt{171\frac{9}{37}}$
CM $\sqrt{171\frac{9}{37}} + 5\frac{3}{7}$
CN vel MN $\sqrt{171\frac{9}{37}} -$ $5\frac{3}{7}$.
PC $\sqrt{171\frac{9}{37}} - 5\frac{3}{7}$
DM $18\frac{1}{2}\frac{1}{6} + \sqrt{171\frac{9}{37}}$
OD $12\frac{1}{2}$
OS $7\frac{1}{7}$
BC 12
TC 16
MS $\sqrt{171\frac{9}{37}} + 8\frac{1}{1}\frac{1}{6}$
SD 10
SC $2\frac{1}{2}$
PR $\sqrt{61\frac{4}{6}\frac{3}{2}\frac{1}{7}} - 3\frac{1}{2}\frac{9}{7}$
RC $\sqrt{109\frac{4}{6}\frac{1}{2}\frac{9}{7}} - 4\frac{1}{2}\frac{2}{7}$
RM $10\frac{2}{2}\frac{1}{7} + \sqrt{6\frac{5}{6}\frac{3}{2}\frac{4}{7}}$
PM $\sqrt{.180\frac{8}{1}\frac{1}{2}\frac{1}{6}} + 2\frac{2}{3}\frac{1}{5}\frac{6}{6}\frac{1}{6}\frac{4}{2}\frac{1}{7}$

MD	dimid. OS	quadrat. MC
$18\frac{1}{16}+\sqrt{171\frac{9}{25}}$	$3\frac{3}{4}$	$202\frac{1}{2}\frac{8}{5}+\sqrt{21495}$ [illegible]
$18\frac{1}{16}-\sqrt{171\frac{9}{25}}$		$18\frac{1}{16}-\sqrt{171\frac{9}{25}}$
$+327$ [illegible]		$+$ [illegible]
$-171\frac{9}{25}$		$-$ [illegible]
$156\frac{1}{4}$ divisor.	$3\frac{3}{4}$	[illegible] ad 42 aream

trianguli MCP. Eſt enim area totius ABC 126, cujus pars tertia ſunt 42.

Potuit idem probari multiplicata perpendiculari PR $\sqrt{61}$ [illegible] $-3\frac{9}{25}$ per baſin MC $\sqrt{171\frac{9}{25}}+5\frac{1}{5}$ facti enim dimidium 42 eandem aream exhibet. quemadmodum oportuit. Multiplicationes iſtæ non ſunt tam operoſæ atque prima fronte videntur: nam numerorum originem á quibus ſunt facti penitius intuenti eorum ſymmetria eſt in promptu.



Ad hunc numerum accedat ejuſdem generis exemplum tertium. Ab eodem triangulo ſint abſumendæ $\frac{1}{3}$, ſitque OC parallela tantum partium 6, BC $12\frac{1}{2}$. hic, quia punctum O nimium vicinum eſt baſi AB, continuetur latus AG & parallela CO donec cōcurrant in I: ſitque GK $\frac{1}{3}$ lateris AG hinc applicetur rectangulum KG in BG ad parallelam OI, latitudo inde exiſtens ſit GM. tum media proportionalis inter IG GM ſit QG, inde inveniatur ut ſupra

IC	$31\frac{1}{2}\frac{9}{1}$
OI	$25\frac{1}{2}\frac{9}{1}$
BO	$19\frac{1}{1}\frac{9}{5}$
OP	$25\frac{7}{5}\frac{7}{5}$
AI	$20\frac{3}{4}\frac{1}{5}$
GI	$7\frac{3}{4}\frac{1}{2}$
AB	21
GB	20

Conti-

continuata GN. cui ponatur æqualis in basi AG ipsa GN: denique ducta NO recta absumet triangulum GRN magnitudinis optatæ. Id numeris dumtaxat comprobabo, quia Geometrica demonstratio ab antecedente non dissidet.

Rectæ CI, GI ex similitudine triangulorum AGB AIC haud operose invenientur, dantur enim latera AB 21, BG 20, GA 13. tũ BC $12\frac{1}{2}$, OC 6. fiat igitur ut AB 21 ad BG 20, sic AC $33\frac{1}{2}$ ad CI $31\frac{1}{2}\frac{9}{1}$, unde subducta OC 6 reliquam facit OI $25\frac{1}{2}\frac{9}{7}$. hinc ut AB 21 AG 13, sic BC $12\frac{1}{2}$ ad GI $7\frac{1}{4}\frac{1}{2}$. tũ rectangulũ sub KG in GB $173\frac{1}{3}$, applicatum ad OI $21\frac{1}{3}\frac{9}{1}$, dabit latitudinem GM vel HQ $6\frac{4}{6}\frac{7}{8}$. inter GI & GM proportione media est GQ $\sqrt{51\frac{3}{4}\frac{1}{6}\frac{7}{8}}$. inde jam facile invenietur GL $\sqrt{62\frac{5}{7}\frac{3}{7}\frac{8}{4}\frac{1}{8}\frac{9}{8}}$, huc addita QL $3\frac{4}{1}\frac{7}{3}\frac{}{6}$ exhibet magnitudine totam GN $\sqrt{62\frac{5}{7}\frac{3}{7}\frac{8}{4}\frac{1}{8}\frac{9}{8}} + 3\frac{4}{1}\frac{7}{3}\frac{}{6}$. Dehinc jam ut experiaris an triangulum GNR imperatum spatium comprehendat, quadrato basin GN & fiet $20\frac{5}{2}\frac{7}{7}\frac{5}{7}\frac{7}{4}\frac{5}{4} + \sqrt{7\,2\,3\,\frac{3}{2}\frac{6}{7}\frac{0}{6}\frac{8}{7}\frac{7}{7}\frac{6}{6}\frac{8}{6}\frac{7}{1}\frac{5}{2}}$. Vnde proportio.

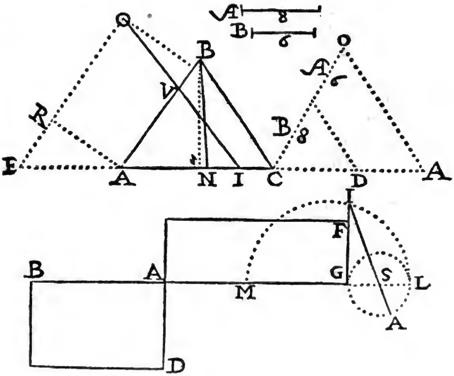
AG	13
BC	$12\frac{1}{2}$
OC	6.
IC	$31\frac{1}{2}\frac{9}{1}$
OI	$25\frac{1}{2}\frac{9}{7}$
BO	$19\frac{}{1}\frac{5}{3}$
OP	$25\frac{}{6}\frac{7}{7}$
AI	$20\frac{1}{4}\frac{1}{8}$
GI	$7\frac{1}{4}\frac{1}{2}$
HQ	$6\frac{4}{6}\frac{7}{8}$
GQ	$\sqrt{51\frac{3}{4}\frac{1}{6}\frac{7}{8}}$
GN	$\sqrt{62\frac{5}{7}\frac{3}{7}\frac{8}{4}\frac{1}{8}\frac{9}{8}} - 3\frac{}{1}\frac{4}{3}\frac{7}{6}$
MN. GL	$\sqrt{62\frac{5}{7}\frac{3}{7}\frac{8}{4}\frac{1}{8}\frac{9}{8}} - 3\frac{}{1}\frac{4}{3}\frac{7}{6}$
GL	$\sqrt{62\frac{5}{7}\frac{3}{7}\frac{8}{4}\frac{1}{8}\frac{9}{8}}$
NI	$11\frac{2}{2}\frac{3}{8}\frac{9}{7}\frac{}{6} + \sqrt{62\frac{5}{7}\frac{3}{7}\frac{8}{4}\frac{1}{8}\frac{9}{8}}$.

Vt NI	ad dimidiam OP	ita quadratum á GN
$11\frac{2}{2}\frac{3}{8}\frac{9}{7}\frac{}{6} + \sqrt{62\frac{5}{7}\frac{3}{7}\frac{8}{4}\frac{1}{8}\frac{9}{8}}$	$12\frac{3}{6}\frac{6}{7}$	$20\frac{5}{2}\frac{7}{7}\frac{5}{7}\frac{7}{4}\frac{5}{4} + \sqrt{7\,2\,3\,\frac{3}{2}\frac{6}{7}\frac{0}{6}\frac{8}{7}\frac{7}{7}\frac{6}{6}\frac{8}{6}\frac{7}{1}\frac{5}{2}}$
$11\frac{2}{2}\frac{1}{8}\frac{9}{3}\frac{}{6} - \sqrt{62\frac{5}{7}\frac{3}{7}\frac{8}{4}\frac{1}{8}\frac{9}{8}}$		$12\frac{2}{2}\frac{3}{8}\frac{9}{7}\frac{}{6} - \sqrt{62\frac{5}{7}\frac{3}{7}\frac{8}{4}\frac{1}{8}\frac{9}{8}}$.
$4\,8\,\frac{8}{8}\frac{4}{7}\frac{1}{7}\frac{0}{6}\frac{0}{7}\frac{0}{7}\frac{0}{6}$	divisor	$3\,1\,7\,\frac{4}{7}\frac{6}{9}\frac{6}{2}\frac{5}{7}\frac{0}{8}\frac{0}{8}\frac{0}{6}\frac{0}{7}$

Multiplicato jam tertium $3\,1\,7\,\frac{4}{7}\frac{6}{9}\frac{6}{2}\frac{5}{7}\frac{0}{8}\frac{0}{8}\frac{0}{6}\frac{0}{7}$ per secundum $12\frac{3}{6}\frac{6}{7}$ & factum $4\,8\,8\,4\,\frac{1}{7}\frac{0}{7}\frac{0}{7}\frac{0}{6}\frac{0}{4}$ dividito per primum $4\,8\,\frac{8}{8}\frac{4}{7}\frac{1}{7}\frac{0}{6}\frac{0}{7}\frac{0}{7}\frac{0}{6}$. hic, quia numeri utrobique æquantur, divisoris nomen per nomen dividendi divisum dabit quotum quæsitũ 84 pro area trianguli RGN: est autem area totius ABG 126, cujus $\frac{2}{3}$ sunt 84: quare RGN continet $\frac{2}{3}$ totius, & ABRN reliquum trientem, quemadmodum imperabatur.

Esto & quartum exemplum ubi O punctum statuatur á parte lateris AB. sitque triangulum ABC secandum ratione data, quam habet A ad B.

Continuetur basis CA donec occurrat parallelæ OE in E, tumque ut supra edoctus es invenito punctum I, recta OI connexa absumet spatium optatũ VAI, cujus ratio ad BVIC eadem sit datæ A, ad B.



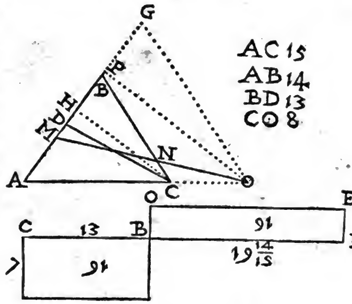
Vt eandem factionem arithmetico epilogiſmo comprobemus ſunto latera trianguli ABC, AB 14 AC 15, BC 13, parallela OE 18, perpēdicularis RA 8. hic propter æqualitatem angulorum BAC REA facile invenietur EA 10: Inde etiam reliquarum magnitudo quantā hic adſcripſimus, denique etiam perpendicularis ab O in EC demiſſa erit $14\frac{2}{7}$ porro autē quadratum ab AI $\sqrt{77\frac{7}{9}}+3\frac{1}{3}$ eſt $88\frac{8}{9}+\sqrt{3456\frac{64}{81}}$. Vnde proportio quemadmodum EI $13\frac{1}{3}+\sqrt{77\frac{7}{9}}$ ad dimidiam perpendicularem ab O demiſſam $7\frac{1}{7}$, ſic quadratum $88\frac{8}{9}+\sqrt{3456\frac{64}{81}}$ ad aream trianguli AIV 48, quod de 84 totius ABC area ſubductum relinquet BCIV 36.

AN	$1\frac{4}{7}$
NC	$6\frac{3}{7}$
AK	$8\frac{2}{7}$.
KC	$6\frac{1}{7}$
AE	10.
FG	$6\frac{2}{7}$
TG	$\sqrt{66\frac{2}{3}}$
TS	$\sqrt{77\frac{7}{9}}$
TA	$\sqrt{77\frac{7}{9}}+3\frac{1}{3}$

Hunc numerum denique claudat exemplum ubi O punctum ſtatuatur in cōtinuata AC, ſitque ABC triangulum ex eo puncto biſectum.

Recta OG parallela contra BC occurrat continuato laterai AB in G, & biſecetur AB in D, atque 91 rectangulum ſub DB 7 & BC 13 comprehenſum applicetur ad OG $19\frac{14}{15}$, latitudo ortiva ſit BH $4\frac{13}{23}$. quemedmodum ſupra es eductus cōtinuetur BH in M ut rectangulū BM in MH æquetur rectangulo ſub HB $4\frac{13}{23}$ in BG $7\frac{7}{15}$ (namque ut AC 15 ad CO 8, ſic AB 14 ad BG $7\frac{7}{15}$) dabiturque tota MB $\sqrt{79\ldots}+2\frac{13}{46}$. inde acta OM biſecabit triangulum veritatem ejus per numeros

numeros ita explorabis, inventa perpendiculari OP $18\frac{2}{7}$, quemadmodū GM 9[illegible] $+\sqrt{}$ 39[illegible] ad dimidiam OP $9\frac{1}{7}$, sic quadratum ab *MB* 44[illegible] $+\sqrt{}$ 819[illegible] ad areā trianguli BMN 42. *C*umque totius triāguli *ABC* area sit 84 reliquum trapezium *AC*NM illi erit æquale: quare tot exemplis hujus sectionis veritas illustrata sit.



Problema 8

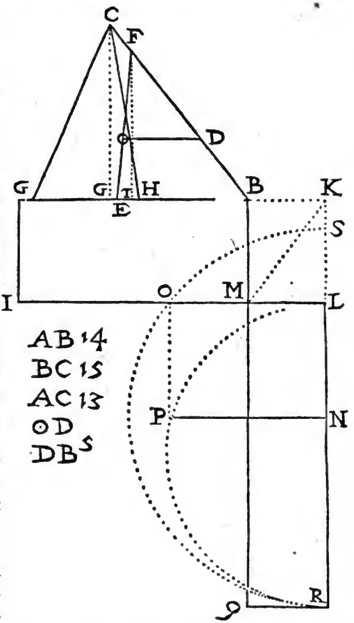
Datum triangulum à puncto intra ipsum dato secare ratione data.

Exponatur triangulum *ABC* & intra ipsam punctum per quod agenda sit recta datum triangulum bisecans. bisecetur basis A*B*, & abs dato puncto O contra eam sit parallela OD, tumque ad OD, sive ei æqualem *B*K, applicetur rectangulū sub *C*B & BH comprehensum, quale est *B*GIM, sitque latitudo inde existēns LR, quæ continuetur in S, ut LS & BD æquales sint, quas inter media proportionalis inveniatur OL, denique eadem OL sive ei æqualis PN statuatur proportione media inter segmenta inventæ RL, tumque alterutri segmento statuatur æqualis BF, quæ hic ipsi RN æqualis posita est, recta ab F per datum punctum O educta absumet triangulum BEE æquale dato H*B*C.

Fabricæ autem demonstratio, quam autor hic nullam affert, talis est. Cum sit per fabricam ut tota RL ad LN, sic RN hoc est BF ad LS seu DB, erit quoq; dividendo RL ad RN, ut RN seu BF ad FD: sed ut BF ad FD, sic, ob parallelismū, B E ad OD. quamobrem ex æquo ut RL ad RN seu BF, sic BE ad OD: quamobrem parallelogrammum BF in BE æquatur æquiangulo parallelogrammo sub RL in OD seu LS, hoc est ipsi CB in BH: quare etiam triangula eorundem dimidia CBH FBE inter se æquabuntur. factum igitur est, quod oportuit.

Si diagramma ex arbitrio nostro deformatum fuisset non tantum fabrica longe foret expeditior, sed & ipsa demonstratio non paulo concinnior: nunc aliena lineamenta nobis fuerunt consectanda.

Hujus

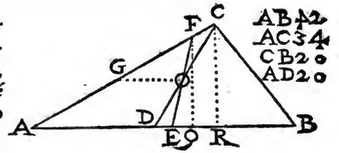


Hujus veritatem numeris exploremus ſecundum laterum quantitatem hic expoſitã, & præterea hoc caſu OD, D*B* æquentur.

Rectangulum BGIM comprehenſum ſub CB & H*B* ſemiſſe ipſius AB 105 diviſum per D*B* 5 exhibet lineam LR 21, tum rectãgulum ab LR 21 & OD 5 comprehenſum erit 105. cujus latus $\surd 105$ eſt media proportionalis LO, huic æqualis PN eſt quoque conſtituta medio loco proportionalis inter ſegmenta RN & NL, erit itaque RN $10\frac{1}{2} + \surd 5\frac{1}{4}$, NL $10 - \surd 5\frac{1}{4}$. æquatur autem ipſi RN recta BF, quamobrem hoc caſu BE quoque ipſi NL æqualis erit, cum OD D*B* ex theſi æquales ſint. Vt verò tertium quoque latus FE invenias demittito perpẽdiculares CG, *F*T, erit itaque ut *B*C 15 ad CG 12, ita B*F* $10\frac{1}{2} + \surd 5\frac{1}{4}$ ad FT $8\frac{2}{5} + \surd 3\frac{9}{25}$. Et queẽadmodũ CB 15, ad BG 9, ſic *F*B $10\frac{1}{2} + 5\frac{1}{4}$ ad BT $6\frac{3}{10} + \surd 1\frac{89}{100}$. quare reliqua TE erit $4\frac{1}{5} - \surd 13\frac{11}{25}$. jam latus ſimul utriuſque quadrati ab ET $\frac{777}{25} - \surd \frac{592704}{625}$ & TF $\frac{1848}{25} + \surd \frac{592704}{625}$ eſt $\surd 105$ quæſita EF.

Quin etiam ejus ſegmenta IO OE hinc per proportionem haud difficulter expedies. ut *F*B ad FE, ſic DF ad FO $\surd 26\frac{1}{4} + 1\frac{1}{4}$: et quemadmodum FB ad *F*E, ſic D*B* ad OE $\surd 26\frac{1}{4} - 1\frac{1}{4}$, ſegmenta addita reſtituent totam OE $\surd 105$, ut ſupra. Porrò autem utrum recta FE hoc caſu per O punctum tranſeat ita conſtabit. ſi per proportionem OD inveniatur partium 5, hoc modo, ut FE $\surd 105$ ad OF $\surd 26\frac{1}{4} + 1\frac{1}{4}$, ſic EB $10\frac{1}{2} - \surd 5\frac{1}{4}$ ad OD 5. area trianguli FED dabitur multiplicata perpendiculari per baſin E*B*, facti dimidium 42 erit area dati trianguli ABC, quemadmodum oportebat. Denique etiam quia crura FB, EB, HB, C*B* eundem angulum comprehendunt erunt quoque reciproce proportionalia. Vt FB ad *B*H, ſic *B*C ad BE: ideoque rectangulum FB in BE æquale rectangulo HB in *B*C.

Esto etiam alterum exemplum. Dantur latera trianguli ABC, AB decempedarum 42 AC 34, CB 20, tum AG vel GO 20, recta per O punctum educenda esto quæ è dato triangulo absumat triangulum 160 decempedarum quadratarum.



Respondeo AF esse $26\frac{2}{3} + \sqrt{4\frac{4}{9}}$, AE $26\frac{2}{3} - \sqrt{4\frac{4}{9}}$, FE $\sqrt{176\frac{16}{9}}$ quæstio ista antecedenti affinis est, investigato itaque ut illic edoctus es AE AF segmenta, quarum magnitudinem hic à latere expressam vides. Iam ob EQ & QF crura anguli recti dabitur EF basis. Verum ut id quoque liquidò constet utrum FE per O punctum educta sit, proportio illud coarguet, hoc modo, ut FA $26\frac{2}{3} + \sqrt{4\frac{4}{9}}$, ad AE $26\frac{2}{3} - \sqrt{4\frac{4}{9}}$, sic FG $13\frac{2}{3} + \sqrt{4\frac{4}{9}}$ ad GO 13, tanta autem eadem quoque est ex thesi, quare O punctũ est in recta FE. Denique si multiplices perpendicularem FQ in dimidiam basin AE factus erit 160 area trianguli AFE. quam ex lateribus solis vulgata via itidem investigare liciut.

AG 13
GO 13
AR 30
RB 12
CR 16
perpend. FQ $12\frac{4}{9} + \sqrt{\frac{1}{3}\frac{5}{7}\frac{6}{9}}$
EQ $\sqrt{14\frac{1}{3}\frac{7}{7}\frac{8}{9}} - 3\frac{}{}$
FE $\sqrt{176\frac{16}{9}}$
GF $13\frac{2}{3} + \sqrt{4\frac{4}{9}}$.

Summa omnium laterum

$52\frac{4}{9} + \sqrt{176\frac{16}{9}}$

semissis $26\frac{2}{3} + \sqrt{44\frac{4}{9}}$

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differentiæ $\sqrt{44\frac{4}{9}} - \sqrt{4\frac{4}{9}}$, $26\frac{2}{3} - \sqrt{44\frac{4}{9}}$, $\sqrt{44\frac{4}{9}} + \sqrt{4\frac{4}{9}}$

$26\frac{2}{3} + \sqrt{44\frac{4}{9}}$ | $\sqrt{44\frac{4}{9}} + \sqrt{4\frac{4}{9}}$
$26\frac{2}{3} - \sqrt{44\frac{4}{9}}$ | $\sqrt{44\frac{4}{9}} - \sqrt{4\frac{4}{9}}$

$+ 684\frac{4}{9}$ | $44\frac{4}{9}$
$- 44\frac{4}{9}$ | $4\frac{4}{9}$

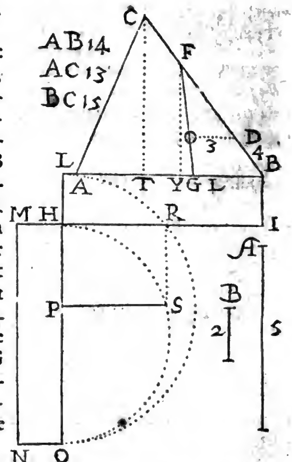
640 | 640 | 40
| 40 |
| 256400 |

1 | 6 | 0 area trianguli FAE, ut supra.

In hoc abaco minus verbosum me fecit operis facilitas, qnod attentus lector haud difficulter animadvertet, modo hujus numerationis nõ sit rudis & expers.

Denique hoc ultimum exemplum sit. Triangulum ABC secandum esto recta per O punctum intra ipsam educta, ut 2 ad 5, laterum quantitatem & linearum ad determinationem necessariarum in ipso diagrammate vides expressam.

Respondeo itaque FB esse $10 + \sqrt{20}$, BG $7\frac{1}{2} - \sqrt{11\frac{1}{4}}$, FG $\sqrt{115\frac{1}{5}} + \sqrt{1531\frac{1}{4}}$. perpendicularē FY $8 + \sqrt{12\frac{4}{5}}$. quæstio hæc ab antecedentibus illud habet diversum, quod OD 3 & DB 4 hic non æquentur: ideoque FB lineam invenies hoc modo, secetur basis AB 14 secundum rationem expositam in L: tum parallelogrammū sub LB in BC videlicet LHIB applicetur ad BD, latitudo inde existens esto HQ. Inde linea HR proportione media inter OD & inventam HQ statuatur media proportionalis inter segmenta ipsius HQ, ea erunt HP & PQ, segmento alterutri ut PQ æqualis sit FB, recta ab F per O educta absumet triangulum FBG æquale triangulo CLB. atque ideo FGB ad reliquum segmentum ACFG habet rationem datam quam 2 ad 5. Si hujus fabricæ vestigia in numerorum epilogismo sequaris, deprehendes PQ vel FB esse $10 + \sqrt{20}$.



Cum enim dentur AB 14, BC 15, AC 13, BD 4 OD 3, & propter datam rationem AL 10, ad LB 4. rectangulū sub CB 15 in BL 4 erit 60, ea divisa per OD 3 dabunt quotum HQ 20, media proportionalis in HQ 20 & BD 4 est $\sqrt{80}$, quæ si statuatur proportione media inter segmenta rectæ HQ, dabit PQ $10 + \sqrt{20}$, PH $10 - \sqrt{20}$. tumque alterutri segmento (modo majus sit quam BD) absumatur æqualis linea BF: ea nunc æquetur ipsi PQ $10 + \sqrt{20}$. Cum itaque triangula CBL FBG sint æqualia, rectangulum CB in BL divisū per FB $10 + \sqrt{20}$ dabit in quoto GB $7\frac{1}{2} - \sqrt{11\frac{1}{4}}$. porró perpendicularis BI habetur per proportionē, ut BC 15 ad CT 12, ita BF $10 + \sqrt{20}$ ad FY $8 + \sqrt{12\frac{4}{5}}$: & quemadmodum BC 15 ad BT 9, ita BF $10 + \sqrt{20}$ ad BY $6 + \sqrt{7\frac{1}{5}}$ inde differentia inter BG $7\frac{1}{2} - \sqrt{11\frac{1}{4}}$ & BY $6 + \sqrt{7\frac{1}{5}}$ erit YG $\sqrt{36\frac{9}{20}} - 1\frac{1}{2}$. deinde quadrata ab YG & YF videlicet $38\frac{7}{10} - \sqrt{328\frac{1}{25}}$ & $76\frac{4}{5} + \sqrt{3276\frac{4}{5}}$ addita conflant quadratum lineæ FG $115\frac{1}{2} + \sqrt{1531\frac{1}{4}}$. datur itaque FG magnitudine $\sqrt{.115\frac{1}{2}} + \sqrt{1531\frac{1}{4}}$. Dehinc utrum FG per O punctum transeat proportio docebit. Namque ut FB $10 + \sqrt{20}$ ad BG $7\frac{1}{2} - \sqrt{11\frac{1}{4}}$, sic FD $6 + 20$ ad DO 3.

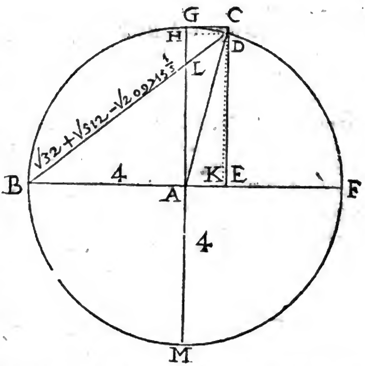
Deniq; basis BG $7\frac{1}{2} - \sqrt{11\frac{1}{4}}$ multiplicata per perpendicularē $8 + \sqrt{12\frac{4}{5}}$ dabit 48, cujus dimidium 24 est optata area trianguli FBG, sed & $\frac{2}{7}$ totius ABC 84 sunt 24: quare factum est quod oportuit.

In isto paradigmate utrumque segmentum pro BF assumi potuit. namque priore casu BF cū ipsi PQ ponetur æqualis erit $10 + \sqrt{20}$ & BG $7\frac{1}{2} - \sqrt{11\frac{1}{4}}$, FG $\sqrt{.114\frac{1}{2}} + \sqrt{1531\frac{1}{4}}$. & perpendicularis FY $8 + \sqrt{12\frac{4}{5}}$. At si FB assumatur $10 - \sqrt{20}$ quanta est HP, tum BG erit

erit $7\frac{1}{2} + \sqrt{11\frac{1}{5}}$, *& FY* $8 - \sqrt{12\frac{4}{5}}$ *BY* $6 - \sqrt{7\frac{1}{5}}$, *YG* $\sqrt{36\frac{9}{10}} + 1\frac{1}{2}$. *& FG* $\sqrt{.115\frac{7}{8}} + \sqrt{1531\frac{1}{4}}$. *fac periculum & numeros accurate quadrare deprehendes.*

PROBLEMA 9.

Si vertex perpendicularis radio æqualis à puncto sectionis diametri proportionaliter secta cum centro connectatur, & recta ab hujus connexa & peripheria communi sectione cum diametri termino connexa à diametro ad primam normali intersecetur, hujus & inscripta & diametri mutua segmenta quæruntur.



Diameter BF secetur proportionaliter, vel ut vulgò loquimur, secundum mediam & extremam rationem in puncto E, unde excitetur perpēdicularis EC radio æqualis, & connectatur AC secās peripheriam in D, tū connectatur DB, quam alia diameter GM priori perpēdicularis intersecet in puncto L, posita diametro BF partium 8, quæruntur segmenta inscriptæ BL, LD: & diametri ML LG. Principio, quia BF proportionaliter secta est in E ejus segmenta BE $\sqrt{80} - 4$ EF $12 - \sqrt{80}$ dabuntur per 36 propositionē hujus libri secundi hinc subducto radio BA de BE reliqua AE erit $\sqrt{80} - 8$. itaque quadratum ab AE $144 - \sqrt{20480}$ additum ad quadratum EC 64 conflabit quadratum AC $160 - \sqrt{20480}$: atque ideo ipsa AC erit $\sqrt{.160 - \sqrt{20480}}$. hinc á termino peripheriæ D demittatur perpendicularis DK, erunt itaque triangula ADK ACE similia: unde proportio, ut AC $\sqrt{.160 - \sqrt{20480}}$ ad AE $\sqrt{80} - 8$, ita radius AD 4 ad AK $\sqrt{.8 - \sqrt{51\frac{1}{5}}}$. tum quadratum AK $8 - \sqrt{51\frac{1}{5}}$ subductum de quadrato AD 16 relinquit quadratum DK $8 + \sqrt{51\frac{1}{5}}$, eritque ipsa DK $\sqrt{.8 + \sqrt{51\frac{1}{5}}}$. jam BA 4 addita ad AK dabit totam BK $4 + \sqrt{.8 - \sqrt{51\frac{1}{5}}}$. Hinc quadratum á BK videlicet $24 - \sqrt{51\frac{1}{5}} + \sqrt{.512 - \sqrt{209715\frac{1}{5}}}$ additum ad quadratum á DK $8 + \sqrt{51\frac{1}{5}}$ dabit quadratum lineæ BD $32 + \sqrt{.512 - \sqrt{209715\frac{1}{5}}}$, atque ideo ipsam BD $\sqrt{.32 + \sqrt{.512 - \sqrt{209715\frac{1}{5}}}}$.

Investigatio segmentorum BL, LD paulo est operosior; primum quia ABL KCD triangula similia sunt, erit

ut BK	ad KD	sic BA	ad AL
$4 + \sqrt{.8 - \sqrt{51\frac{1}{5}}}$	$\sqrt{.8 + \sqrt{51\frac{1}{5}}}$,	4	$\sqrt{.160 - \sqrt{20480}} + 8 - \sqrt{80}$

ad AL addita AM 4 dabit totam ML $\sqrt{.160 - \sqrt{20480}} + \sqrt{12} - \sqrt{80}$ item

 qua-

quadratum lineę AL $304 - \sqrt{81920} - \sqrt{.174080 - \sqrt{30282874880}}$ additum ad quadratum radij AB 16 dabit quadratum lineæ BL $320 - \sqrt{81920} - \sqrt{.}$ $174080 - \sqrt{30282874880}$ cujus latus præfixa nota erit $\sqrt{.320 - \sqrt{8190} - \sqrt{.174080 - \sqrt{30282874880}}}$, ut nota $\sqrt{.}$ primo loco posita omnes numeros afficere intelligatur. Hinc rursum.

ut BK	ad DK,	sic AK vel HD	ad HL.
$4 + \sqrt{.8 - \sqrt{51\frac{1}{7}}}$	$\sqrt{.8 + \sqrt{51\frac{1}{7}}}$,	$\sqrt{.8 - \sqrt{51\frac{1}{7}}}$	$\sqrt{80} - 8 - \sqrt{.136 -}$

$\sqrt{18483\frac{1}{7}}$: quadratũ lineæ HL $280 - \sqrt{77875\frac{1}{7}} - \sqrt{.156160 - \sqrt{24385893171\frac{1}{7}}}$ additum ad quadratum ab HD $8 - \sqrt{51\frac{1}{7}}$ dabit quadratum LD $288 - \sqrt{81920} - \sqrt{.156160 - \sqrt{24385893171\frac{1}{7}}}$. atque ideo eandem dabit magnitudine $\sqrt{.}$ $288 - \sqrt{81920} - .156160 - \sqrt{24385893171\frac{1}{7}}$. Vniversalium numerorum numerationis leges repetes é tractatu surdorũ numerorum, quem huic libro præfiximus.

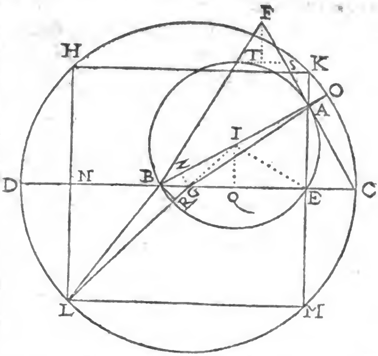
Operosissimam solvendi hujus Zetematis viam autor est secutus: potuit enim alia constructionis formula magna hujus laboris pars declinari Sed illud silentio hic transmittere neque. GL & LM segmenta diametri facillime absque ulla proportione dari, quia GL æquatur differentiæ inter BE $\sqrt{80} - 8$, & AC $\sqrt{.160 - \sqrt{20480}}$. Itemque ML æquatur summæ ipsius EF $12 - \sqrt{80}$ & ejusdem AC $\sqrt{.160 - \sqrt{20480}}$. demonstrationem in proximam editionem differre cogor.

PROBLEMA 10.

Triangulum æquilaterum BCF super segmento diametri DC descriptum ita secat latus inscripti quadrati eidem diametro perpendicularis in A, ut circulus per verticem trianguli B descriptus in A puncto latus trianguli contingens relinquat æqualia reliquorum laterum segmenta PF EC: posita PF $\sqrt{32} + 4$ quanta erit area trianguli BGL, cum G centrum sit majoris circuli.

Primum

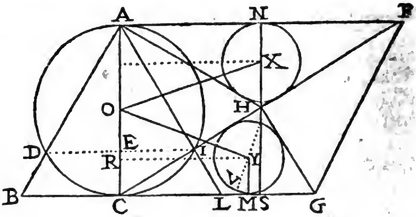
Primum id notandum minoris circuli contactum A incidere in mediũ lateris FC, quia enim PF & E*C* ponuntur æquales, etiam quadrata F*A* & AC, atque ideo ipsas lineas æquari est necesse. Cumq; E*C* detur $\sqrt{32}+4$, AC erit ejus duplum, & duplum ipsius *A*C erit B*C*. quare AC $\sqrt{128}+8$ & BC $\sqrt{512}+16$, & *E*B tripla ipsius E*C* erit $\sqrt{288}+12$. hinc GE multiplicata in BE dabit quadratum diametri circuli minoris BA $576+\sqrt{294912}$, cujus latus est pro ipsa *B*A $\sqrt{384}+\sqrt{192}$. & radius *B*I $\sqrt{96}+\sqrt{48}$.



Ad investigationem majoris diametri similitudine est opus. concipias itaque diametrum DC partium 2, tum EC erit $1-\sqrt{\frac{1}{3}}$, unde proportio. Cum F*C* erit $1-\sqrt{\frac{1}{3}}$ diameter DC erit 2, itaque posita E*C* $\sqrt{32}+4$ diameter D*C* erit $\sqrt{1152}+32$: quare radius GC $\sqrt{288}+16$: subductaque G*C* de *C*B $\sqrt{512}+16$ reliqua erit G*B* $\sqrt{32}$, hinc subducta G*B* de *B*Q $\sqrt{72}+6$ (est enim dimidia ipsius *B*E) relinquet GQ $6+\sqrt{8}$. atque inde é GQ & QI dabitur distãtia centrorum GI $\sqrt{}.\,80+\sqrt{4608}$ Rursum é datis NL & N B dabitur BL $\sqrt{}.\,448+\sqrt{165888}$. perpendicularis BR æqualis ipsi RG est 4. quare radius GL $\sqrt{288}+16$ multiplicatus per 2 dabit aream trianguli BGL $\sqrt{1152}+32$. Porró etiam ut BI $\sqrt{96}+\sqrt{48}$ ad IQ, sic *B*G $\sqrt{32}$ ad GZ $\sqrt{8}$. & BZ $\sqrt{24}$. hinc é datis GZ & GO datur ZO $\sqrt{}.\,536+\sqrt{294912}$; cum qua addita BZ exhibet totã BO $\sqrt{}.\,536+\sqrt{294912}.+\sqrt{24}$. Denique etiam BA subducta de BO reliquam faciet AO $\sqrt{}.\,536+\sqrt{294912}.-\sqrt{216}-\sqrt{182}$.

PROBLEMA II

Inter parallelas AF BG circulus eas contingens descriptus, & à contactu A *inscripta AD continuata est in* B, *hinc DI est o parallela contra BG: Inde à contactu C recta CI educta occurrat parallelæ in* F, *sitque AF æqualis ipsi BG, & GH perpendicularis in* CF, *& connectatur AH, deinque in triangula AH*F *CHG inscribuntur circuli eorum latera contingentes, datis AD 12 DB 4, quæruntur circulorum diametri & centrorum distantia.*

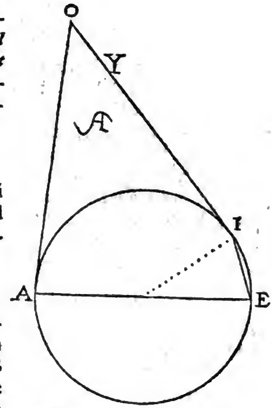


Principio, quia AD & DB dantur, datur tota AB 16, & rectangulum AB in BD 64 æquale quadrato tangẽtis BC, datur itaque BC 8, atque ideo perpendicularis seu diameter AC $\sqrt{192}$, sed BC CL æquantur ob parallelissimum DI: quare ABL triangulum est æquilaterum. itemque ADI: atque ea propter CI radio æqualis erit $\sqrt{48}$, & EI 6, & CE $\sqrt{12}$. Inde proportio, quemadmodum CE $\sqrt{12}$ ad EI 6, ita CA $\sqrt{192}$ ad AF 24: & sic CI $\sqrt{48}$ ad CF $\sqrt{768}$; cumque AF BG per thesin æquentur, dabitur quoque reliqua GC 16, & FG æqualis AB 16: quare CF $\sqrt{768}$ bifecatur à perpendiculari MH estq; CH vel HF aut AH $\sqrt{192}$, & ideo HG 8, & GS 4, namq; est ut CG 16 ad GH 8, sic GH 8 ad GS 4, & reliqua CS vel AN aut NF 12. datis itaque lateribus triangulorum AHF CHG dantur inscriptorum circulorum diametri: verum quia illud triangulũ æquicrurũ, hoc autẽ rectangulũ est, facilius ita rete expidies, ut invenias radium, quia FX angulum F bisecat, cum circulus utrumque latus tangat & HN detur $\sqrt{48}$, secato HN ratione crurum HF ad FN, minus segmentum $24 - \sqrt{432}$ erit NX; hinc NX vel AT subducta de AO $\sqrt{48}$ dabit reliquam TO $\sqrt{768} - 24$, datur autem quoque TX vel AN 12; quamobrem OX distantia quoque dabitur $\sqrt{.488 - \sqrt{1769472}}$. Rursum ad inveniendam diametrum circuli subducas basin CG 16 de summa crurum CH & HG $\sqrt{192} + 8$, reliqui $\sqrt{192} - 8$ dimidium $\sqrt{48} - 4$ erit radius YM, cui æquatur CR: quare reliqua erit OR 4. Et subducto radio, qui ipsi HP æquatur, de latere HC relinquetur PC $\sqrt{48} + 4$, cui æqualis item est ipsa MC vel RY: quare datis RY & RO dabitur basis anguli recti OY $\sqrt{.80 + \sqrt{3072}}$ distantia centrorum. Denique cum detur CS 12 CM $\sqrt{48} + 4$ dabitur reliqua MS $8 - \sqrt{48}$: et addita RO 4 ad OT $\sqrt{768} - 24$ dabitur tota RT $\sqrt{768} - 20$, tũ quadrata TR & MS addita dabunt quadratum YX atque ipsam YX $\sqrt{.1280 - \sqrt{1486848}}$. Quamobrem omnium centrorum distantia cognita est, quemadmodum imperabatur.

BD 4. AD 12
AB 16. BC 8
AC $\sqrt{292}$. EC $\sqrt{48}$
DE vel EI 6
AF 24 CF $\sqrt{768}$
CH vel HF $\sqrt{192}$
GH 8 GS 4
HS $\sqrt{48}$ HP 4
YR vel CM $\sqrt{48} + 4$
YM vel CR $\sqrt{48} - 4$
OY $\sqrt{.80 + \sqrt{3072}}$
MS $8 - \sqrt{48}$
RO vel OT $\sqrt{778} - 24$
RT $\sqrt{768} - 20$
YX $\sqrt{.1280 - \sqrt{1486848}}$

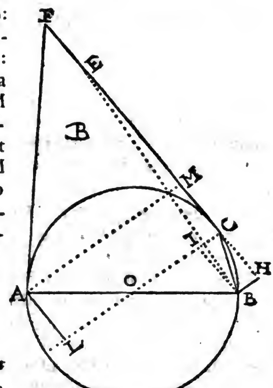
PROBLEMA 12

In circulum diametri AE partium 22 inscripta est EI à quo descriptum undecangulum subundecuplum fit undeculguli à diametro, hinc ab I tangens IY diametro æqualis continuata est in O, ut tota IO æquetur subtensæ EY, inde ad reliquum diametri terminum connexa est ◡AO, cujus magnitudo quæritur.



Zetema hoc anno 1596 postridie Nonarũ mensis Iunij propalam affixum est. quod postquam mihi oblatum est ita solvi quemadmodum é construxione hic videre est.

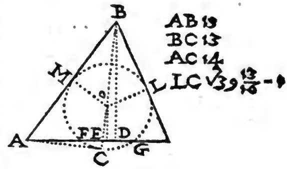
Principio cum figuræ similes habeant rationem homologorum laterum duplicatam erit ut 11 ad 1, sic quadratum diametri AB 484 ad quadratum inscriptæ BC 44, ideoque ipsa BC datur longitudine √44. hinc IC erit 2, cui æquatur BH, atq; ideo IB vel HC √40: quare HE 22 + √40. atque ea propter dabitur basis anguli recti BE √.528 + √77440: datur itaque ei æqualis FC. de qua subducta MC ipsi HC æqualis faciet reliquam FM √.528 + √77440. — √40. præterea LK æqualis IC de diametro subducta relinquet LC vel AM 20. Denique datis cruribus FM MA datur basis recti FA √.968 + √77440 — — √.84480 + √1982464000, ut √.ad omnes hujus numeri partes pertinere intelligatur.



PROBLEMA 13

E basi dati trianguli ABC circulus reliqua latera contingens absumit segmentum FG, dato lateris unius segmẽto LC a contactu L ad basin, quæritur absumpti segmenti magnitudo.

Sunto latera *AB* 15, *B*C 13, *A*C 14, & LC $\sqrt{39\frac{13}{16}}-1$. Respondeo ſegmentum quæſitum FG eſſe partium 6, & circuli diametrum $16-\sqrt{52}$. BE biſecet angulum verticis, & *B*D perpendicularis ſit. Iam per 3 prop. lib. 6, & 12 prop. lib. 2 invenientur *A*E $7\frac{1}{2}$, E*C* $6\frac{1}{2}$, AD 9, D*C* 5, DB 12, DE $1\frac{1}{2}$, BE $\sqrt{146\frac{1}{4}}$. Hinc tollatur L*C* $\sqrt{39\frac{13}{16}}-1$ de B*C* 13, reliqua erit *B*L, cui æqualis eſt BM $14-\sqrt{39\frac{13}{16}}$ Porrò, cum angulus AB*C* biſectus ſit ex radio inſcripti circuli dabitur quoque perpendicularis *A*R $\sqrt{55\frac{5}{13}}$, & *B*R $\sqrt{169\frac{8}{13}}$. Vnde proportio, ut BR $\sqrt{169\frac{8}{13}}$ ad AR $\sqrt{55\frac{5}{13}}$, ſic *B*M $14-\sqrt{39\frac{13}{16}}$ ad MO $8-\sqrt{13}$, cujus duplum dabit integram diametrum $16-\sqrt{52}$. datis igitur OM & MB dabitur quoque OB $\sqrt{260}-\sqrt{52\frac{13}{16}}$, ea ſubducta de E*B* reliquam facit OE $\sqrt{52\frac{13}{16}}-\sqrt{16\frac{1}{4}}$. Iam ut B*E* ad *B*D, ita OE ad OT $\sqrt{52}-4$, cujus quadratum $68-\sqrt{3328}$ de radij quadrato $77-\sqrt{3328}$ ſubductum relinquit 9 quadratum à TF, & ipſam FT 3, cujus duplum ſit inſcripta FG 6, quemadmodum quærebatur.



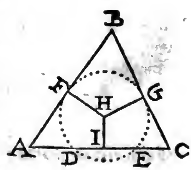
AE $7\frac{1}{2}$ EC $6\frac{1}{2}$
AD 9 DC 5
D*B* 12 DE $1\frac{1}{2}$
BE $\sqrt{146\frac{1}{4}}$
R $\sqrt{169\frac{8}{13}}$ AR $\sqrt{55\frac{5}{13}}$.

Iſtud zetema propoſui júveni in hiſce non infeliciter verſato, qui poſtquam id recte & legitimé, (quamvis alia conſtruxionis formula) ſolviſſet zetema illud quod ſequitur mihi repoſuit, hoc modo.

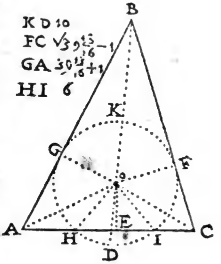
PROBLEMA 14.

Datis ſegmentis laterum trianguli GC $\sqrt{39\frac{13}{16}}-1$, FA $\sqrt{36\frac{13}{16}}+1$, baſi AC 14, interſegmento DE 6, quærantur FB, BG.

In iſto zetemate aliquid & deſideratur & redundat: & primum quidem id redundat quod baſis A C quantitatem datam exponat, cum ea non dari ſed ex ſegmentis AF & G*C* et interſegmento DE ipſa AD & EC atq; ideo tota A C debuerit inveniri, modo diameter circuli quoque detur. Neque enim ſequitur quia in hoc paradigmate FA, F*C* & DE æquales ſunt lineis ijſdem in antecedente queſtione, hunc circulum FDEG illi æquari. quod exemplo illuſtrius erit.



Diameter



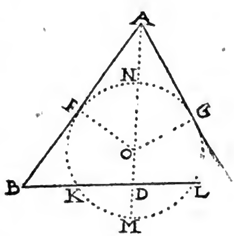
Diameter circuli hujus duo latera contingentis & basin secantis sit partiũ 10, tangens FC $\sqrt{39\frac{13}{16}}-1$, AG $\sqrt{39\frac{13}{16}}+1$, intersegmentum HI 6, quæruntur latera AB, BC & AC.

Respondeo, AB $\sqrt{173\frac{81}{729}}+6\frac{8}{27}$, BC $\sqrt{173\frac{81}{729}}+5\frac{8}{27}$ AC 14. Invenietur autem ipsa AC hoc modo. quadratum IE (quæ est dimidia inscriptæ HI) 9 subductum de quadrato radij OI 25 reliquum facit quadratum perpendicularis OE 16. Et rursum quadratũ radij OF 25 additum ad quadratum ab FC dabit quadratum OC $65\frac{13}{16}-\sqrt{159\frac{1}{4}}$, inde subductum quadratum OE 16 reliquum facit quadratum EC $49\frac{13}{16}-\sqrt{159\frac{1}{4}}$, itaque ipsa EC longitudine datur $7-\sqrt{\frac{13}{16}}$. eandem viam secutus deprehendes AE $7+\sqrt{\frac{13}{16}}$ quare tota AC erit 14. jam veró, quemadmodum antecedente problemate commonstravi, non est difficile invenire latera AB $\sqrt{173\frac{81}{729}}+7\frac{1}{27}$, & BC $\sqrt{173\frac{81}{729}}+5\frac{2}{27}$, quemadmodum initio respondimus.

At si diameter circuli statuatur $16-\sqrt{52}$, tum demum etiam latera erunt eadem, quę in problemate 13.

PROBLEMA 14.

Si circulus expositi trianguli duo crura contingat & basin secet, radio circuli, basis intersegmento, & crurum segmentis à contactu ad basin datis, ipsa latera invenire.



Vt in diagrammate radius OG sit 14, intersegmentum KL 24, BF 20, GC 16, quæruntur latera BC AB AC.

Respondeo, BC esse $\sqrt{544}+20$.

AB $17\frac{2019}{3319}+\sqrt{240\frac{1158960}{11015761}}+\sqrt{26\frac{2139014}{11015761}}+\sqrt{19\frac{7280541}{11015761}}-\sqrt{\frac{7067192}{11015761}}-\sqrt{\frac{5559700}{11015761}}$ AC $13\frac{2015}{3319}+\sqrt{240\frac{1158960}{11015761}}+\sqrt{26\frac{2139014}{11015761}}+\sqrt{19\frac{7280541}{11015761}}-\sqrt{\frac{6017190}{11015761}}-\sqrt{\frac{5559700}{11015761}}$.

Eritq; area triang. ABC in numeris absolutis major quam $705\frac{372}{1000}$ minor autem quam $705\frac{373}{1000}$. Et latera reducta ad numeros explicabiles, BC $43\frac{32380}{100000}$ minus $43\frac{32381}{100000}$ majus vero, AB $41\frac{22614}{100000}$ minus vero at si pro 4 in fine 5 reponas majus vero, AC $37\frac{22614}{100000}$ minus & $37\frac{32615}{100000}$ majus vero.

Cum olim hoc Zetemation autori nostro proposuissem, memini hos numeros mihi ad quæsiti solutionem ab ipso exhibitos, cumque istos a meis quos ex abaco meo de prompseram diversos viderem, é vestigiò quidem ab ipso commissum putabam, quia bis, idque diversa via, eosdem numeros inveneram: tandem sensi novissimos ejus numeros symmetros esse, quos ubi addidissem plane cum meis consentire deprehendi, sunt autem isti.

$$AB\ 17\tfrac{2019}{3319} + \sqrt{240\tfrac{1152960}{116377761}} + \sqrt{20\tfrac{1424480}{116377761}} + \sqrt{13\tfrac{2421499}{116377761}}$$

$$AC\ 13\tfrac{2019}{3319} + \sqrt{240\tfrac{1152960}{116377761}} + \sqrt{20\tfrac{1424480}{116377761}} + \sqrt{13\tfrac{2421499}{116377761}}$$

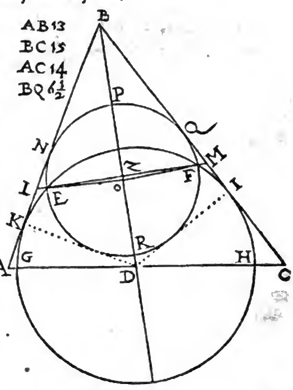
Quamobrem numeri illi non é sex, sed é quatuor tantum nominibus sunt compositi: atqui solœcismus est logisticus symmetros numeros divisos proponere, aut non protinus reducere: monui itaque statim autorem de numerorum suorum symmetria; verum tædio antecedẽtis abaci, aut memoriæ aliquo lapsu id omissum existimo.

Est autem AF vel AG segmentum à contactu ad verticem A. $\sqrt{240\tfrac{1152960}{116377761}} + \sqrt{20\tfrac{1424480}{116377761}} + \sqrt{13\tfrac{2421499}{116377761}} - 2\tfrac{1300}{3319}$ *.hic numerus ad absolutos reductus valet* $21\tfrac{2261462}{10000000}$*: quare AB* $41\tfrac{2261462}{10000000}$*, & AC* $37\tfrac{2261462}{10000000}$*. Horum vero numerorum veritatem per triangulorum canonem quoque comprobavi. Intelligatur OD perpendicularis é centro in subjectam BC, erit itaque BO* $\sqrt{596}$*, OC* $\sqrt{452}$*, unde per tabulas sinuum invenientur anguli, quemadmodum hic adscriptos vides FBD 52 gr. 10 min.* $20\tfrac{6}{100}$ *sec. GCD 61 gr. 0 min.* $46\tfrac{61}{100}$ *sec. quorum summa 113, 11,* $6\tfrac{11}{100}$ *æquatur angulo in centro FOG, nam iste cum angulo ad A duobis rectis quoque æquatur, hujus dimidium AOG 56, 35,* $33\tfrac{34}{100}$*. Hinc proportio, Vt sinus totus 10000000 ad tangentem anguli AOG 56, 45,* $33\tfrac{36}{100}$ *15161533, ita OG 14 ad AG* $21\tfrac{2261462}{10000000}$ *ut supra. Cum qua addita BF 20 exhibet totam AB* $41\tfrac{2261462}{10000000}$*, & AC quoque* $37\tfrac{2261462}{10000000}$*. Vides itaque calculum nostrum plane legitimum cum triangulorum canone ad amussim consentire.*

	grad.	min.	secu.
OBF	34	50	$31\tfrac{27}{100}$
OBD	17	10	$48\tfrac{78}{100}$
FBD	52	10	$20\tfrac{6}{100}$
OCG	41	11	$9\tfrac{32}{100}$
OCD	19	49	$37\tfrac{30}{100}$
GCD	61	0	$46\tfrac{61}{100}$
FOC	113	11	$6\tfrac{72}{100}$
AOG	36	65	$33\tfrac{37}{100}$

PROBLEMA 16.

Si semicirculus super base dati trianguli descriptus reliqua latera contingentem intersecet, data contactus circuli à vertice trianguli distantia horum communem inscriptam invenire.



Exponantur latera AB 13, DB 15, AC 14 & distanta BQ $6\frac{1}{2}$ quæriur EF. Constat omnium circulorum centra esse in linea BD angulum ABC bisecante, inveniatur itaque DI radius semicirculi inscripti, qui perpenpicularis est lateri BC, erit autem is partiũ 6, & cum segmentũ basis DC sit $7\frac{1}{2}$, dabitur IC $4\frac{1}{2}$, cumq; BQ detur $6\frac{1}{2}$ reliqua QC erit $8\frac{1}{2}$, quare IQ 4: prætera BQ BN, BI BK ab eodẽ puncto eosdẽ circulos contingentes æquantur, atque ideo KN QI quoque æquales erunt: sed & QI bisecatur in M, quia quadrata ab IM MQ æquantur eidẽ rectangulo ab EM in MF.

Sunt

Sunt autem triangula BID BQO similia, quamobrem ut *BI* $10\frac{1}{2}$ ad ID 6, sic *B*Q $6\frac{1}{2}$ ad radium QO $3\frac{5}{7}$ jam autem datis cruribus BQ QO dabitur basis *B*O $\sqrt{56\frac{9}{196}}$. cumque LB BM æquentur, BZ angulum B bisecans basin quoque LM bisecat eidemque perpendicularis est. atque ideo ut BD $\sqrt{146\frac{1}{4}}$ ad BI $10\frac{1}{2}$, sic BM $8\frac{1}{2}$ ad BZ $\sqrt{54\frac{1}{2}\frac{8}{66}}$, ea de BO subducta reliquam facit OZ $\sqrt{\frac{15}{3717}}$, hujus quadratum de quadrato radii subductum dabit quadratum FZ $\sqrt{13\frac{5}{67}}$, cujus duplum EF $\sqrt{55\frac{9}{67}}$. quemadmodum petebatur.

Radius semicirculi super base trianguli descripti reliqua latera contingens, cujus inventionem autor hic non explicat, potuit inveniri ex proportione invento primum circuli inscripti radio. Vt si concipiamus NQR circulum inscriptum omnia latera contingere, ejus radius per 34 problema libri tertij dabitur 4, & indidem BQ 7, quare OB $\sqrt{65}$. Vnde proportio, ut OB $\sqrt{65}$ ad OQ 4, ita BD $\sqrt{146\frac{1}{4}}$ ad DI 6 verum ista via operosior est.

Quamobrem etiam hoc modo facilius idem radius invenietur, secetur basis AC in D ratione crurum AB ad BC, erit itaque DC $7\frac{1}{2}$, & concipiatur a vertice B perpendiculares demissa in AC ea erit 12: Vnde proportio, ut CB *15 ad perpendicularem 12, ita BC $7\frac{1}{2}$ ad DI 6.*

Denique tertia, quam in istis usurpo via hæc est. Area trianguli per semissem summæ crurum quæ semicirculus contingit divisa dabit radium in quoto.

Area hujus trianguli 84, summa crurum AB & BC 28, dimidum 14, *per quem* 84 *divisa dabit in quoto radium DI 6. Namque duo triangula* DBC DBA *si bases intelligas A*B *& BC sunt æquealta, habentia pro altitudine radios perpendiculares DK vel DI.*

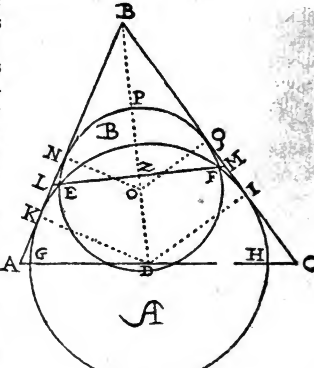
Cæterum ad inventionem BI, *quæ distantia est verticis B ad contactum semicirculi I, therrema tale habeo.*

*Vt summa crurum quæ semicirculus tangit ad omnium laterum summam, Ita crurum eorundem & basis differentia ad duplam contactus à vertice distantiam. A*B *& BC sunt 28, laterum summa 42,* AC *14 differentia ab* 28 *est itidem* 14, *inde proportio, quemadmodum 28 ad 42, ita* 14 *ad* 21, *duplā BI, quare ipsa* BI *$10\frac{1}{2}$. Hinc jam datur* CI $4\frac{1}{2}$ *ut supra, demonstratio attentius consideranti satis erit obvia. Atque hinc si libeat datur quartus modus inveniendi radium DI: cum enim detur* DC $7\frac{1}{2}$ *(nam* B*D bisecans angulum D, secat quoque basin ratione crurum) & isto epichiremate detur* CI $4\frac{1}{2}$, *dabitur quoque crus reliquum anguli recti DI 6. Atque hinc rursum datis DI 6, IB $10\frac{1}{2}$ dabitur quoque bisecans* B D $\sqrt{146\frac{1}{4}}$.

Problema 17

Si semicirculum super base dati trianguli descriptum & duo reliqua latera contingentem circulus alter saltem eadem duo latera contingens intersecet, ut communis subtensa sit latus quadrati semicirculo super basin completo inscripti, quæritur diameter secundi circuli.

Latera eadẽ dentur quæ prius, ſitque EF latus quadrati in circulum, cujus G E F H ſemicirculus eſt, inſcripti. Cũ itaq; radius ſemicirculi DI ſit inventus partiũ 6, latus quadrati circulo huic inſcripti $\sqrt{72}$ erit inſcripta EF, quæritur radius OQ? Reſpondeo $19\frac{4}{4}\frac{1}{5}-\sqrt{124\frac{1}{2}\frac{7}{4}\frac{9}{0}\frac{6}{1}}$. cujus veritas hoc modo demonſtratur,



Vt BI ad BD, ita BQ ad BO
$10\frac{1}{2}$ $\sqrt{146\frac{1}{4}}$, $17\frac{1}{1}\frac{5}{4}-\sqrt{45\frac{2}{4}\frac{5}{5}}$ $\sqrt{1\,8\,3\,\frac{8}{9}\frac{1}{6}\frac{8}{0}\frac{5}{4}}$
$-\sqrt{1\,1\,\frac{8}{2}\frac{0}{4}\frac{9}{0}\frac{8}{1}}$

Vt DB ad BI, ſic BM ad BZ
$\sqrt{146\frac{1}{4}}$ $10\frac{1}{2}$, $13\frac{1}{1}\frac{3}{4}-\sqrt{23\frac{4}{4}\frac{3}{5}}$ $\sqrt{146\frac{1}{4}}$
$-\sqrt{18}$.

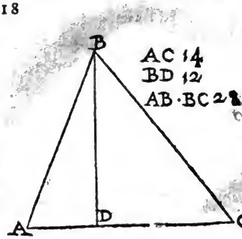
Tum BZ ſubducta de BD reliqua erit OZ $\sqrt{1\,4\,\frac{9}{2}\frac{7}{4}\frac{6}{0}\frac{0}{1}}-\sqrt{1\,1\,\frac{8}{2}\frac{0}{4}\frac{9}{0}\frac{8}{1}}$. hujus quadratum $2\,6\,\frac{7}{2}\frac{8}{4}\frac{5}{0}\frac{8}{1}-\sqrt{7\,0\,7\,4\,\frac{1}{3}\frac{4}{7}\frac{2}{6}\frac{5}{4}\frac{9}{8}\frac{2}{0}\frac{0}{1}}$, de quadrato radij OF $3\,1\,\frac{1}{2}\frac{0}{4}\frac{7}{0}\frac{6}{1}-\sqrt{7\,0\,7\,4\,\frac{1}{3}\frac{4}{7}\frac{2}{6}\frac{5}{4}\frac{9}{8}\frac{2}{0}\frac{0}{1}}$ ſubductum, relinquet quadratum FZ 18, atque ideo ipſam longitudine $\sqrt{18}$, cujus duplum $\sqrt{72}$ eſt inſcripta EF, quod argumẽto eſt magnitudinem radij OQ á nobis legittime inventam eſſe.

BI	$10\frac{1}{2}$
IC	$4\frac{1}{2}$
DI	6
ZM	$\sqrt{47\frac{3}{4}\frac{7}{5}}-\sqrt{5\frac{4}{4}\frac{9}{5}}$
FM	$\sqrt{47\frac{3}{4}\frac{7}{5}}-\sqrt{44\frac{2}{4}\frac{3}{5}}$
EM	$\sqrt{47\frac{3}{4}\frac{7}{5}}+0\frac{1}{4}\frac{5}{5}$
QM	$\sqrt{23\frac{4}{4}\frac{3}{5}}-3\frac{3}{5}$
QI	$\sqrt{95\frac{2}{4}\frac{3}{5}}-6\frac{4}{5}$
BQ	$17\frac{1}{1}\frac{3}{4}-\sqrt{35\frac{2}{4}\frac{9}{5}}$
MB	$13\frac{1}{1}\frac{3}{4}-\sqrt{23\frac{4}{4}\frac{3}{5}}$

Vnum hoc notabo in hac zeteſi non opus fuiſſe lineam BZ operoſa proportione inquirere. Cum enim EF latus inſcripti quadrati ſit, ſegmentũ radij DZ æquatur ipſi ZM ſemiſſi lateris quadrati, data autem eſt BD $\sqrt{146\frac{1}{4}}$ & *EM* $\sqrt{72}$, *ZM* $\sqrt{18}$: *quare BZ erit* $\sqrt{147\frac{1}{4}}-\sqrt{18}$.

PROBLEMA 18

Data trianguli baſe altitudine crurum ſumma invenire triangulum.



AC 14
BD 12
AB·BC 28

Datur baſis trianguli AC 14, altitudo BD 12, ſumma crurũ AB BC 28, quæruntur ſingula AB 13, BC 15. idque non adhibito algebricarum poſitionum ſubſidio. Problema iſtud ad me miſit Leovardia clariſsimus medicus *Ioannes Wilhelmus Velſius*, ad cujus ſolutionem hanc viam ingreſſus ſum.

Primũ multiplicata perpendiculari in dimi-

diam

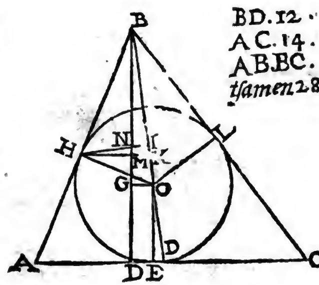
diam basin inveni aream quæsiti trianguli ABC 84, eamque quadravi unde existunt 7059. Idē iste numerus existeret continua multiplicatio é dimidio collectorum laterum & singulorum ab eo differentis. Datur autem omnium semissis 21, & basis datæ ab eo differentia 7, quorum factus 147, per quem 7056 divisus exhibet in quoto 48, hunc igitur à reliquis duabus differentijs factum consequens est. atqui simul utriusque reliquorum duorum laterum differentiæ á semisse æquantur basi AC 14; Id enim facile patet supra é demonstratione areæ triangularis problemate 35 lib. 3. jam dividatur AB 14 in duo segmenta per 5 propos. lib. 2. *Eucl.* ut factus ab ejus segmentis sit 48. segmenta itaque ea erunt 8 & 6 quare si subducas 8 de 21 semisse relinquinur A B crus vnum 13, si 6 dabitur *B*C crus alterum 15. Operis totius formulam hic subjeci.

semissis laterum

perpend.	12	84		21	21	~~111~~	
dimid. basis	7	84	basis	14	7	~~1010~~(48	factus á differentijs
	—	—		—	—	147	laterum quæsitorum
area	84	336	differ.	7	147		
		672					
		—					
areæ quadr.		7056					

Dehinc secetur 14 in duo segmenta, hoc modo: de 49 quadrato á semisse datæ basis subducatur 48 reliqui latus est 1, hic ad semissem additus & ab eodem subductus dabit segmenta optata 8 & 6.

Hoc idem problema cum proposuissem jam supra nobis nominato *Willebrordo Snellio* RF. quæsiti solutionem attulit istac magis concinnam, idque tribus aut quatuor formulis, quarum præcipuam hic adscribam. Triangulum de quo quæritur sit A*BC* cujus in agulu ab ignotis cruribus comprehensū bisecet recta *B*O, & in basin datam perpendicularis sit BD, itemque à centro inscripti circuli OE, tum HI à contactu H perpendicul. in bisecantē, et HM in *B*D, sitque GD ęqualis radio OE. Principio itaque multiplicata

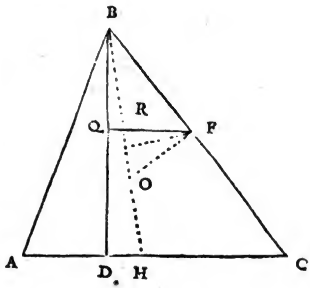


perpendiculari *B*D 12 in dimidiam basin AC 7 fit area trianguli *A*BC 84, ea per dimidiam laterū summā divisa exhibet in quoto radium OE 4 jam dimidia differentia basis à laterū omniū sūma est H*B* vel BL 7. datis itaq; cruribus anguli recti BH HO datur basis BO $\sqrt{65}$, & perpendicularis HI $\sqrt{12\frac{4}{65}}$, OI $\sqrt{3\frac{61}{65}}$, BI $\sqrt{36\frac{61}{65}}$. Porró cum detur perpendicularis *B*D 12 & EO vel GD 4 reliqua BD

erit

erit 8, atque data BO $\sqrt{65}$ BG 8 datur quoque GO 1 : ſunt vero triangula OBG NBI ſimilia, quamobrem ut BG 8 ad GO 1, ita BI $\sqrt{44\frac{0}{5}\frac{1}{5}}$ ad NI $\sqrt{\frac{2}{5}\frac{4}{5}\frac{0}{5}\frac{1}{5}}$, ea ſubducta de HI relinquit HN $\sqrt{3\frac{0}{5}\frac{6}{5}\frac{2}{5}\frac{3}{5}}$ baſin anguli recti ad M. jam ut OB $\sqrt{65}$ ad BG 8, ſic NH $\sqrt{3\frac{0}{5}\frac{6}{5}\frac{2}{5}\frac{3}{5}}$ ad HM $2\frac{4}{5}\frac{1}{5}$: atque inde ob datam HB datur quoque MB $6\frac{1}{5}\frac{6}{5}$, ſunt autem HBM ABD triangula ſimilia, & datur perpendicularis BD 12, hinc proportio, ut BM $6\frac{1}{5}\frac{6}{5}$ B ad BH 7, ita DB 12 ad BA 13 . quæ ſubducta de 28 relinquunt 15 pro latere BC. quare factum eſt quod oportuit. Poſtquam hæc via detecta eſt complures modi ad quæſiti ſolutionem extra algebram jam patent.

Sed eandem formulam ab altero latere quoque experiri libet.



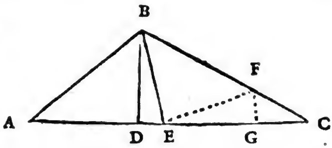
Exponatur iterum triangulum ABC deturque baſis AC 14, perpendicularis BD 12, ſumma crurum AB & DC 28, quæruntur ſingula. hic in rectam biſecantem angulum B ſit perpendicularis FM, & in BD perpendicularis FQ. Cum hic, ut ſupra BF differentia, & FO radius, atque ideo BO nota ſint. unde per 8 prop. 16 Euc. MF MO MB innoteſcent, & triangula BDH, BQR, RFM ſimilia ſint. erit ut DB ad GH ſic, FM ad RM $\sqrt{\frac{1}{5}\frac{4}{5}\frac{9}{5}}$, que de BM ſubducta relinquit BR $\sqrt{31\frac{1}{5}\frac{7}{5}}$. cumque triangula BRQ BHD ſimilia ſint, fiat ut BH ad BD, ſic BR ad BQ $5\frac{3}{5}$. denique etiam, quia BFQ BCD triangula ſimilia ſint, ut BQ $5\frac{3}{5}$ ad BF 7, ſic BD 12 ad BC 15: quare reliqua AB erit partium 13 : factum igitur eſt quod petebatur.

HB $\sqrt{146\frac{1}{4}}$
EO 4
DH $1\frac{1}{2}$
FO 4
BF 7
FM $\sqrt{12\frac{4}{5}\frac{4}{5}}$
BM $\sqrt{36\frac{0}{5}\frac{1}{5}}$
MR $\sqrt{\frac{1}{5}\frac{4}{5}\frac{9}{5}}$
RB $\sqrt{31\frac{1}{5}\frac{7}{5}}$
BQ $5\frac{3}{5}$

PROBLEMA 18.

Data trianguli baſe aera & crurum ratione ipſa crura invenire.

In

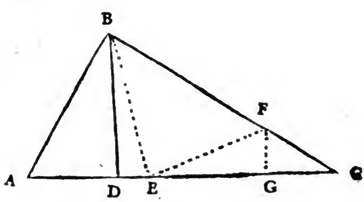


In expoſito triangulo ABC datur baſis AC 15, area 45, ratio AB ad BC quæ 2 ad 3, quæruntur crura AB BC. Reſpondeo AB √.468—√165888, BC √.1053—√839808.

Dividito aream datam 45 per dimidium baſis AC $7\frac{1}{2}$ quotus 6 erit altitudo BD. hinc BE biſecans angulum ABC ſecat baſin AC in E ratione crurum: quare AE erit 6, EC 9: atque ita quoque ſunt inter ſe triangula ABE EBC: quare ABE erit 18, & EBC 27. Statuatur lateri BA æqualis recta BF, & connectatur EF, erunt itaque ABF EBF triangula æquilatera & æqualia, quare datur EF 6, & area trianguli EFC 9, quæ per dimidiam baſin EC diviſa babit perpendicularem FG 2. Itaque data baſe anguli recti EF 6 & crure FG 2, dabitur crus reliquum EG √32, & GC 9—√32. Atqui triangula FGC BDC ſunt ſimilia itaque ut FG 2 ad GC 9—√32 ſic BD 6 ad DC 27—√288, tumque reliqua AD √288—12. dantur itaque crura rectiãguli BDC, itemque ipſius BDA, dabitur ideo quoque BC & BA, quemadmodum imperabatur.

Arguta & concinna ſolutio, verum id mihi hic notandum, hujus problematis ſolutionem fere geminam ſemper eſſe. ut hic manente eadem area & baſi erit BC √.1053+√839808 & AB √.468+√165888. Cujus cum me monuiſſet Geometrica noſtra hujus problematis ſolutio, quam alibi retulimus, video tamen etiam hinc præſtari poſſe, quod prout potero exprimam. Concipias F punctum verſus B adſcendere ut E ſit inter G, & C, itemque BD perpendicularem extra cadere, hoc eſt angulum ad A eſſe obtuſum: jam EG non eſſet de EC ſubducenda, ſed ad eam addenda. reliqua ſi hoc animo percipis non erunt difficilia. ſi diagramma eſſet ad manum res planiſſima eſſet.

Eſto exemplum ſecundũ. ſit AC baſis 13, ratio crurum AB ad BC ut 2 ad 3, perpendicularis BD 6, quæruntur crura.



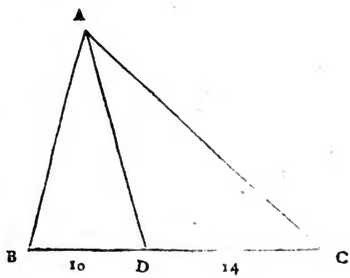
Respondeo AB $\sqrt{52}$, BC $\sqrt{117}$. Solutionis via eadem quæ supra. Vnde area EFC datur $7\frac{4}{7}$, & perpendicularis FG 2, atque ideo EG $4\frac{4}{7}$, GC 3. quare DC per proportionem datur 9. tumque BC $\sqrt{117}$. Porro AD 4, hinc jam AB $\sqrt{52}$. ut petebatur.	AE $5\frac{1}{7}$. vel EF EC $7\frac{4}{7}$. area AEB $15\frac{3}{7}$ EBC $23\frac{1}{7}$ EFC $7\frac{4}{7}$ GF 2 EG $4\frac{4}{7}$

Potuit item responderi BC $\sqrt{1464\frac{8}{17}}$ BA $\sqrt{621\frac{1}{2}}$; fac periculum & plane quadrare deprehendes.

PROBLEMA 19.

Datis basis segmentis in quæ à recta angulum verticis bisecante dividitur, dataque laterum summa aream invenire.

AD bisecet angulum BAC, sitque BD 10, DC 14, summa crurum BA & AC 48, quæritur area trianguli ABC. Respondeo eam esse $\sqrt{55296}$. Cum enim AD angulum A bisecet erit per 3 propos. lib. 6 *Euclid.* BA ad AC ut BD ad DC, dividatur itaque 48 ut segmenta habeant eam rationem quam 10 ad 14. dabitur AB 20, AC 28. datis itaque lateribus dabitur area, ut supra.



PROBLEMA 20.

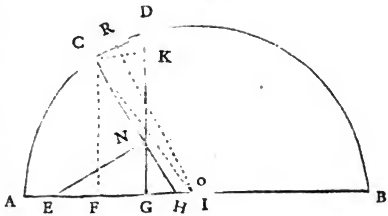
Super data diametro AB partum 25 constructum est triangulum rectangulum ENH, cujus basis EH 9 secatur proportionaliter a perpendiculari NG, ea cum latere minore NH continuata occurrunt peripheriæ in D & C, posita distantia AE 3 quæritur inscripta CD.

Respondeo $\sqrt{.}\,330\frac{1}{4} + \sqrt{.}\,912 - \sqrt{772245}. - \sqrt{427\frac{11}{16}} - \sqrt{.}\,548\frac{1}{4} - \sqrt{276125} - \sqrt{.}\,241464 - \sqrt{35025939045} - \sqrt{.}\,508964\frac{7}{8} - \sqrt{25585849642 5\frac{41}{64}}$.

Intelligas notam radicis universalis $\sqrt{.}$ primo loco positam omnes numeros complecti, reliquas autem $\sqrt{.}$ tantum bina nomina attingere, & $\sqrt{427\frac{11}{16}}$ esse numerum solitarium, ut $\sqrt{.}$ sit radix universalis numeri è sex nominibus compositi. Id clarius infra videbis cum amor hunc numerum facta resolutione ad explicabiles revocabit.

Istud

Istud Zetema istud supra jam nobis nominatus *Willebrordus Snellius* proposuit & soluit. quam viam in hujus solutione. secutus sim paucis indicabo. Primum secui basin EH per 5 propos. lib. 2 *Euclidis* secundum mediam et extremam rationem: Erit itaque EG $\surd 101\frac{1}{4}-4\frac{1}{2}$ & GH $13\frac{1}{2}-101\frac{1}{4}$. Iam NH crus anguli recti quod inter basin EH & segmentū GH proportione est medium erit $\surd 101\frac{1}{4}-4\frac{1}{2}$, nam EG & NH hic æquantur: præterea cum radius AI sit $12\frac{1}{2}$, AH 12, relinquetur IH $\frac{1}{2}$. sitq; in continuatam CH perpēdicularis IO, erunt itaque IHO, EHN triangula similia. ideoque ut EH 9 ad HN $\surd 101\frac{1}{4}-4\frac{1}{2}$, sic IH $\frac{1}{2}$ ad HO $\surd\frac{5}{16}-\frac{1}{4}$. atque ideo perpendicularis OI $\surd.\surd\frac{5}{64}-\frac{1}{8}$, cujus quadratum de quadrato radij relinquit quadratum OC. quare ipsa OC datur $\surd.156\frac{1}{4}-\surd\frac{5}{64}$. unde subducta HO facit HC $\surd.156\frac{1}{4}-\surd\frac{5}{64}.-\surd\frac{5}{16}-\frac{1}{4}$. (*Notabis in hoc numero* $-\frac{1}{4}$ *referri ad* $\surd\frac{5}{16}$ *atque ideo ita fuisse vere scribendum* $\surd.156\frac{1}{4}-\surd\frac{5}{64}.+\frac{1}{4}-\surd\frac{5}{16}$. *illum tamen modū notandi autor retinuit, ut numeratio ita expeditor sit, in quo tamen summa attentione & cura opus est ne quid committatur. quod lectorem monuisse opere precium duxi.*) Hinc jam quia triangula ENH FCH similia sunt, ut EH 9 ad EN $\surd.\surd 8201\frac{1}{4}-40\frac{1}{2}$, ita HC $\surd.156\frac{1}{4}-\surd\frac{5}{64}.-\surd\frac{5}{16}-\frac{1}{4}$ ad FC $\surd.\surd 30615\frac{5}{16}-78\frac{1}{2}-\surd.\surd\frac{5}{16}-\frac{1}{2}$, tumque DG media proportionalis inter AG & GB est $\surd.\surd 79380-141$. de qua subducta CF vel GK reliquam facit DK $\surd.\surd 79380-141-\surd.\surd 30615\frac{5}{16}-78\frac{1}{2}-\surd.\surd\frac{5}{16}-\frac{1}{2}$. porrò, quemadmodum EH 9 ad NH $\surd 101\frac{1}{4}-4\frac{1}{2}$, ita HC $\surd.156\frac{1}{4}-\surd\frac{5}{64}.-\surd\frac{5}{16}-\frac{1}{4}$ ad HF $\surd.234\frac{7}{8}-30615\frac{5}{16}+\surd\frac{5}{16}-\frac{3}{4}$, unde subducta HG $13\frac{1}{2}-\surd 101\frac{1}{4}$ reliqua est FG vel CK $\surd.234\frac{7}{8}-\surd 30615\frac{5}{16}.+\surd 112\frac{5}{16}-14\frac{1}{4}$. vel quia hic numerus quadrandus est commodioris numerationis gratia notetur hoc modo $\surd.234\frac{7}{8}-\surd 30615\frac{5}{16}.-14\frac{1}{4}-112\frac{5}{16}$, cujus vis hæc sit, videlicet de $\surd.234\frac{7}{8}-\surd 30615\frac{5}{16}$ subducendum esse $14\frac{1}{4}-\surd 112\frac{5}{16}$. quadratum itaque á DK $550\frac{1}{4}-\surd 228445\frac{5}{16}-\surd.508964\frac{7}{8}-\surd 255858496425\frac{45}{64}$. additum ad quadratum KC $\surd 209101\frac{1}{4}-220+\surd.912-\surd 772245-\surd.241464-\surd 3502593904 5-\surd.548\frac{1}{4}-\surd 276125$, dabit quadratum lineæ CD. atque ideo ipsam CD quantam supra expressimus. Hic numeris ad absolutos revocabitur initio à novissimis facto $\surd.508964\frac{7}{8}-\surd 255858496415\frac{45}{64}$. & primum quidem novissimum hoc membrum ad millesimas reductum dabit



2 5 5 8 5 8 4 9 6 4 2 5 7 9 3 1 2 5 0 0 0 0 0 0 0 0 0 0 0 0

1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Cujus latus est 505824 [illegible] (tot autem milesmias usurpo ut nobis detur subtensa CD in partibus radij secundum tabulas Rhetici, quas *Otto Valentinus* publicavit) is numerus subductus de 508964 [illegible] ad easdem millesimas reductis relinquet 34 [illegible]. Hinc postpositis 12 circulis latus erutum dabit 56 [illegible]. Hoc latus est ultimi membri nota — affectum. Hinc eodem modo ex antepenultimo membro $\surd.241464 - \surd 3502593904 5$ latus erutum dabit 4 [illegible], itidem subducendum; tum antepenultimi membri $\surd.548\frac{1}{4} - \surd 276125$ latus 23 [illegible], itidem subducendum: deinde latus é $\surd.427$ [illegible] est 20 [illegible] quod itidem subducendum sũma horũ omniũ 134 [illegible], porrò numeri $\surd.912 - \surd 772245$ latus est 5 [illegible] quod ad $330\frac{1}{4}$ additum conflat summam 336 [illegible] unde illa summa est subducta relinquit [illegible], cujus ad extremũ latus 4 [illegible] exhibet in numeris absolutis rectam CD. Iam si ad magnitudinem radij canonũ Rhetici hanc inscriptam reducere cupias, proportione id concludes, ut hujus circuli diameter 25 ad suam inscriptam CD [illegible] ita 20000000000 ad 3749851531, cujus dimidium 1874925765 erit sinus peripheriæ dimidiæ.

Hic numerus in tabulis sinuum accuratè non datur; sed proxime minor 1874765590, cui debetur peripheria 10 grad. 48 min. 20 secund. & 1875241805 proxime major, quorum differentia 476215. Hinc dati numeri differentia á numero minore est 160175. Vnde proportio, cum differentiæ 476215 debeantur 10 scrupula secunda, ergo 160175 debebuntur secunda 3 [illegible], eritq; tota CRD 21 gr. 36 min. 46 [illegible] secund.

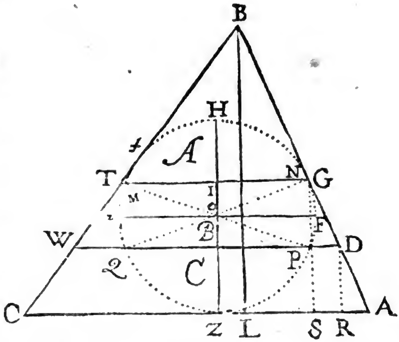
Eandem peripheriam etiam ita invenies, si GD sinum peripheriæ AD, quæ est $\surd.\surd 79380 - 141$ revoces ad numeros absolutos 1 [illegible], posita autem diametro 2000000000 erunt ijdem 9490865171 sinus 71 gr. 38 min. 17 [illegible] sec. Præterea cum perpendicularis FC sit $\surd\ \surd 30615\frac{1}{4} - 78\frac{1}{2}. - \surd.\surd\frac{1}{4} - \frac{1}{2}$, hic ad explicabiles reductus erit 9 [illegible]. Cujus resolvendi modus hic est latus é $\frac{1}{4}$ erutum dabit [illegible], hinc $\frac{1}{2}$ subducta relinquit [illegible] cujus latus [illegible]. dehinc latus é $30615\frac{1}{4}$ erutum dabit 174 [illegible] inde subducantur $78\frac{1}{2}$, reliqui 96 [illegible] latus erit 9 [illegible], hinc jam subductum latus ultimi membri supra inventum dabit CF 9 [illegible], qualium diameter AB datur partium 25, itaque qualium 20000000000 talium erit CF 7663278561 [illegible] sinus peripheriæ AC, atque ideo dabitur AC 50 grad. 1 min. 30 [illegible] secund. subducta AC de AD reliqua erit CD 21 gr. 36 min. 46 [illegible] secund. plane ut supra. totius operis specimen exhibui ut exemplorum sequentium ratio tanto planius constaret.

PROBLEMA 21.

Triangulo cui circulus sit inscriptus lineis rectis contra latus parallelis in tres æquas partes diviso, quærantur segmenta parallelarum circulo inscriptarum, & circularium segmentorum area.

exponatur

Exponatur triangulum A*B*C, cujus latera AB 13, AC 14, BC 15, idque á parallelis TG, wD trifariam dividatur, quæruntur inscriptæ MN, QP, & areæ MHN, QMNP, PZQ? Hic primum inveniantur per 19 propoſ. 6 lib. *Eucl.* segmenta laterum BG, GD, DA; DT, TW, *WC*, & parallelæ GT, D*W*. cum enim area dati trianguli A*B*C sit 84, erit, quemadmodum area 84, ad quadratū lateris AB 169, ita 56 ($\frac{2}{3}$ areæ dictæ) ad quadratū BD $112\frac{2}{3}$, unde ipsa BD $\sqrt{112\frac{2}{3}}$: & sic 28 ($\frac{1}{3}$ totius) ad quadratum BG $56\frac{1}{3}$. quare ipsa *B*G $\sqrt{56\frac{1}{3}}$. Hanc eandem analogiam secutus invenies segmentorum quantitatem eam, quam hic expressimus. Præterea diameter circuli inscripti est partium 8. Et triangula A*B*L, AGS, ADR sunt similia, atque inde innotescunt DR seu KZ, & GS seu IZ. Iam quia IN est media proportionalis inter HI & IZ diametri segmenta, dabitur ipsa $\sqrt{.\sqrt{12288}-96}$, & ejusdem dupla inscripta MN $\sqrt{.\sqrt{196608}-384}$. Similiter media proportionalis inter HK & KZ est KP, cujus dupla est PQ. Iam OI, OK, IM, KQ reductæ valent OI $1\frac{717968}{10000000}$ OK $1\frac{79795897}{100000000}$ QK $3\frac{5731419}{10000000}$, IM $3\frac{85373817}{100000000}$. quæ in partibus radij 10000000 erunt IM 9634330, KQ 8932855, OI 2679491 vel 2679492, OK 4494897 hic est sinus peripheriæ Q3. atque inde datur ipsa peripheria Q3 26 gr. 42 min. $39\frac{1}{2}$ secu. Hinc reliqua pars inferior QZ illius complementum 63 gr. 17 min. $20\frac{1}{2}$ sec. & HM, 74 gr. 27 min. 28 sec. ejusque complementum M3 15 gr. 32 min. 32 sec. Si istæ peripheriæ ad expositæ diametri HZ partes revocentur, dabitur QZ vel PF $1\frac{86477794}{100000000}$. inventionis modus hic est inquisito inscripti circuli peripheriam multiplicata diametro per $3\frac{14159265}{100000000}$ et habebis $25\frac{1327412}{10000000}$. Hinc proportio,



*B*G $\sqrt{56\frac{1}{3}}$
*B*D $\sqrt{112\frac{2}{3}}$
GA $13-\sqrt{56\frac{1}{3}}$
AD $13-\sqrt{112\frac{2}{3}}$
T*C* $15-\sqrt{75}$ TB $\sqrt{75}$
T*W* $15-\sqrt{150}$ B*W* $\sqrt{150}$
GS vel IZ $12-\sqrt{48}$
DR vel KZ $12-\sqrt{96}$
IH $\sqrt{48}-4$. KH $\sqrt{96}-4$
KI $\sqrt{96}-\sqrt{48}$.
IO $8-\sqrt{48}$. KO $\sqrt{96}-8$
IM $\sqrt{.\sqrt{12288}-96}$
NM $\sqrt{.\sqrt{196608}-384}$
KQ $\sqrt{.\sqrt{24576}-144}$.
PQ $\sqrt{.\sqrt{393216}-576}$
Hoc est in absolutis.
OI $1\frac{717957}{10000000}$
OK $1\frac{79795897}{100000000}$
QK $3\frac{5731419}{10000000}$
IM $3\frac{85373817}{100000000}$

ut

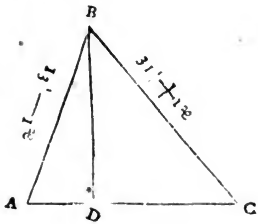
ut	ad	ita	ad
360 gr.	25.13274122,	26 gr. 42 min. 39½ sec.	1.86477764.

tanta igitur est peripheria Q3. Eodem modo dabitur HM 5.1981135, & QZ 4.4184976, & M3 1.836518. jam peripheria M3 per radium OM multiplicata dabit aream sectoris MO3 & NOF 4.3452372. & OI per IM multiplicata dabit aream trianguli MON 4.1324177, utraque summa addita conflat aream segmenti F3MN 8.4806249. haud aliter dabitur segmentum F3PQ 13.8834711 hæc area antecedenti addita conflat aream totius segmenti intermedij MNPQ 22.3640960 ut proxime. Porrò multiplicata peripheria HM per radium HO dabitur area sectoris HMON 20.7924540 inde subducta area trianguli MON, reliqua erit area sectionis MNH 16.6600360. Haud aliter invenietur area sectionis PQZ 11.2492680. Horum trium segmentorum areæ in unam summam collectæ dabunt 50.2654825, quam areæ circuli æqualem esse convenit. Vt istius periculum facias invenito circuli aream per proportionem, sunt enim circuli inter se quemadmodum quadrata à diametris

ut quad. diam.	ad aream circuli	sic quad. diam.	ad aream circ.
4	3.141592653	64	50.26548245 | 40

PROBLEMA 22.

Data trianguli area base & crurum summa ipsa crura invenire.



Libet hoc loco istud problema Algebrice tantum expedire. Detur area trianguli ABC 84, basis AC 15, summa crurum AB & BC 27. Statuamus minus latus AB esse $13\frac{1}{2} - 1$℞, erit itaque BC majus $13\frac{1}{2} + 1$℞. summa collectorum laterum est 42, dimidium 21, inde singula subducta dabunt tres differentias 6, $7\frac{1}{2} + 1$℞, $7\frac{1}{2} - 1$℞, numerus à tribus differentijs & dimidio continua multiplicatione factus æquabitur quadrato areæ datæ 84. operis formula hæc est.

$7\frac{1}{2} + 1$℞   21 $56\frac{1}{4} - 1$ʒ
$7\frac{1}{2} - 1$℞   6   126
—— quadratum
$56\frac{1}{4} - 1$ʒ   126 $7087\frac{1}{2} - 126$ʒ areæ trianguli

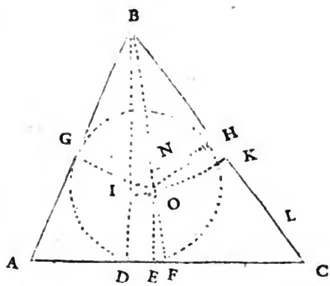
$7087\frac{1}{2} - 126$ʒ Æqualia 7056
7056
——
$31\frac{1}{2}$ Æqual. 126ʒ
$\frac{1}{4}$ Æqual. 1ʒ
$\frac{1}{2}$ Æqual. 1℞

Quamobrem una positio, seu unum latus, vel 1℞ æquatur $\frac{1}{2}$, hæc ad $13\frac{1}{2}$ addita & ab eodem subducta dabit AB 13, & BC 14. quemadmodum imperabatur.

Problema 23.

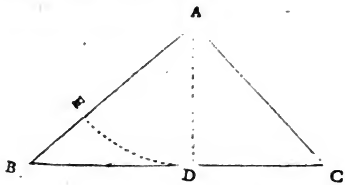
Idem per canones sinuum.

Constructionem in exposito diagrammate vides. dehinc jam area 84 per dimidiā basin $7\frac{1}{2}$ divisa datur perpendicularis BD $11\frac{1}{5}$, eadem area 84 per dimidiam perimetrum divisa exbet radiū inscripti circuli OE 4. Porrò cum GA & AE ab A ad contactum, itemque si CE CH æquales sint, & AC sit 15, etiam AG & HC erunt 15, quæ de 27 subducta relinquūt 12 pro HB & BG, quare singulæ erunt partiū 6. dantur itaque crura anguli recti BHO, atque ideo etiam basis BO $\sqrt{52}$. Hinc DI statuatur æqualis radio OE, & connectatur OI, ea erit parallela contra basin AC. eritque reliqua IB $7\frac{1}{5}$. inde proportio ut IB $7\frac{1}{5}$ ad BO $\sqrt{52}$, sic $11\frac{1}{5}$ ad BF $\sqrt{125}$ [illegible] unde DF datur $\frac{2}{5}\frac{2}{9}$, & EF $\frac{2}{9}$, & DE vel OI $\frac{2}{5}$. Vt verò deinceps ad canonum epilogismum accingamur, quantitatem lineæ BO $\sqrt{52}$ ad numeros explicabiles revocemus, videlicet 7 [illegible]. Si itaque B centrum circuli statuamus, cujus radius BO, fiet OI linea sinus anguli OBI. ut BO 7 [illegible] ad OI $\frac{2}{5}$, ita sinus totus 10000000 ad sinum 554700, cui respondet angulus 3 gr. 10 min. $47\frac{3}{5}$ sec. Itemque, ut BO 7 [illegible] ad OH 4, ita sinus totus 10000000 ad 5547000 sinum anguli OBH, cui respondet peripheria 33 gr. 41 min $24\frac{1}{5}$ sec. pro angulo OBH vel OBG. ad hunc additus angulus OBI dabit totum HBI vel CBD 36 gr, 52 min. $11\frac{3}{5}$ sec. idem de OBG subductus reliquum faciet angulum ABD. cum itaque in triangulo rectangulo DBC detur angulus acutus DBC datur quoque reliquus BCD 53 gr. 7 min. $48\frac{2}{5}$ sec. Hujus sinus datur é tabulis 8000009 vel ut calculus accurate, congruat 8000000, unde proportio ut sinus 8000000 ad radium 10000000, ita BD $11\frac{1}{5}$ ad BC 14: quare reliqua AC erit 13. Potuit idem nonpaulo facilius expediri per tabulas tangentiū & secantiū hoc modo, sit BH radius, & OH tangens anguli HBO. erit itaque ut BH 6 ad HO 4, ita radius 10000000 ad tangentem anguli OBH 6666667, cui respondet peripheria 33 gr. 41 min. $24\frac{1}{5}$ sec. haud aliter invenietur angulus IBO 3 grad. 10 min. $47\frac{2}{5}$ sec. Horum summa 36 grad. 52 min. $11\frac{3}{5}$ sec. exhibet angulum IBH ut supra. Hinc si BD sit radius erit BC secans anguli DBC. itaque é tabulis dabitur 12499906: hoc est ob calculi rotundationem 12500000, hinc proportio, ut radius 10000000 ad tangentem 12500000 ita BD $11\frac{1}{5}$ ad BC 14, ut supra.



PROBLEMA 24.

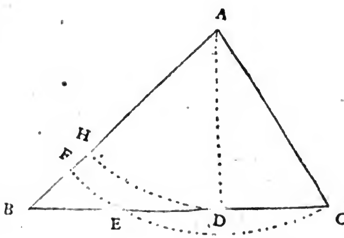
Data trianguli base, area, & angulo ad basin reliqua latera invenire.



Detur basis *BC* 24, area 144. angulus ad basin A*B*C 40 grad, 36 min. $4\frac{2}{3}$ sec. *A*ngulus datus *AB*C 40 gr. 36 min. $4\frac{2}{3}$ sec. de 90 gr. subductus relinquet angulũ BAD 49 gr. 23 min. $55\frac{1}{2}$ secund. assimpto itaque *A*D pro radio 10000000, *B*D erit tãgens anguli *BA*D 11666666. Vndè proportio, ut 10000000 ad 11666666 ita AD 12 ad DB 14, quare reliqua DC erit 10. atque inde per 47 propos. 1 lib. *Euclid*- dabuntur AB $\sqrt{340}$, *A*C $\sqrt{244}$. quemadmodum petebatur.

PROBLEMA 25.

Data trianguli area, angulo ad basin, crurum summa & ratione differentiæ segmentorum basis ab differentiam crurum quæruntur latera.



In exposito diagrammate datur area trianguli *A*B*C* 192, angulus à majore crure & base comprehensus AB*C* 36 grad. 52 min. $11\frac{1}{3}$ sec. itemque summa crurum BA & AC $\sqrt{1568}$, ratio *B*E ad crurum differentiam *B*F quæ 7 ad 5, omnium laterum magnitudo quæritur. Respondeo *B*C esse $\sqrt{800}$, AB $\sqrt{512}$, *A*C $\sqrt{288}$. Centro *A* intervallo lateris minoris decircinetur peripheria CEF, ea secet basin in E & crus reliquum in F. cum igitur per 36 propos. 3 lib *Eucl.* rectangulum *C*B in *B*E æquetur rectangulo ex C*A* & AB in segmentnm B*F*, ideó *C*A plus AB, BC, BE, BF proportionales erunt, quare ut 7 ad 5, sic CA & AB $\sqrt{1568}$ ad BC $\sqrt{800}$. dehinc area 192 per ejus semissem divisa dabit perpendicularem AD $\sqrt{184\frac{8}{25}}$, quamobrem data trianguli base, altitudine, crurum summa ipsa sigiliatim dabuntur per problema 18 etiamsi angulus ad B non detur, quia verò hic angulus datur operis ratio

ratio hæc erit. angulus *B* de recto subductus reliquum facit angulum *B*AD 53 gr. 7 min. $48\frac{1}{7}$ sec. sit *A*D radius & BD tangents anguli *B*AD 13333333 & A*B* secans 16666666. Inde proportio ut 10000000 ad 13333333 sic AD $\surd 184\frac{4}{7}$ hoc est 13 [illegible] ad BD. & quemadmodum 10000000 ad 16666666, sic AD 13 [illegible] ad AB. In istoc exẽplo ex reductione constat AD ad D*B* habere rationem quam 3 ad 4, ad *AB* quam 3 ad 5 ut proxime. quare in numeris inexplicabilibus, quoque dabitur *B*A $\surd 512$, ea subducta de $\surd 1568$ relinquit AC $\surd 288$. quemadmodum petebatur.

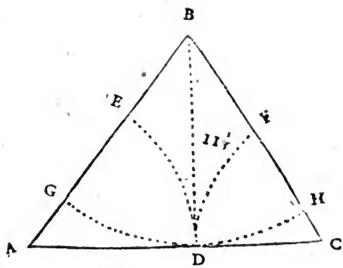
Problema 26.

Datis lateribus tiranguli invenire angulos.

Sit AB 14, *B*C 13, C*A* 15, quæruntur anguli A, B, C. Respondeo angulum

A esse	53 grad.	7 min.	$48\frac{1}{7}$	secund.
B	67 grad.	22 min.	$48\frac{2}{7}$	secund.
C	59 grad.	29 min.	23	secund.
Sum.	180 grad.	o	o.	

Investigato primum perpendicularem BD $11\frac{1}{7}$, & segmenta basis AD $8\frac{4}{7}$, DC $6\frac{3}{7}$. Itaque sit BD $11\frac{1}{7}$ ad AB 14, ita radius 10000000 ad 12500000 secantem : quare angulus ABD erit 36 gr. 52 min. $11\frac{2}{7}$ secund. qui de 90 gr. deducti relinquunt 53 gr. 7 min. $48\frac{1}{7}$ sec. pro angulo *B*AD. Et rursum, ut BD $11\frac{1}{7}$ ad BC 13, ita radius 10000000 ad secantem 11607143, cui debentur 30 gr. 30 min. 37 sec. qui de 90 deducti relinquent 59 gr. 29 min. 23 sec. pro angulo *B*CD. denique *C*BD ABD additi conflant totum angulum A*B*C 67 gr. 22 min. $48\frac{2}{7}$ sec. Factionis veritas per 32 propos. 1 lib. *Euclidis* constare potest, si tres isti anguli additi duo rectos æquent, ut supra vidisti.

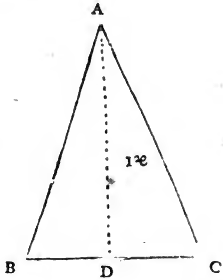


Problema 27.

Data trianguli æquicruri base, & ratione quam habet cruris quadratum ad aream trianguli quæruntur crura.

Esto

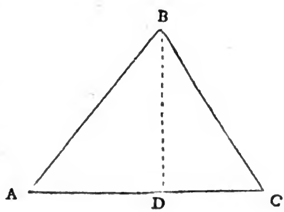
Esto *BC* partium 2, ratio quadrati cruris AB ad triangulum A*B*C quæ 3 ad 1, quæruntur crura AB, vel AC, & anguli A, B, C. Respondeo crus AB vel AC esse $\sqrt{3\frac{3}{4}} + \sqrt{\frac{3}{4}}$, vel etiam $\sqrt{3\frac{3}{4}} - \sqrt{\frac{3}{4}}$. angulum A 41 gr. 48 min. $37\frac{7}{5}$ sec. angulũB 69 gr. 5 min. $41\frac{43}{133}$ sec. Statuamus enim AD esse 1ꝝ, inde per 47 prop. 1 lib. *Euclid.* dabitur AC $\sqrt{.}$ 1ʒ + 1. dehinc multiplicata perpendiculari AD in dimidiam basin DC datur area trianguli 1ꝝ, cujus triplum æquatur quadrato lateris AC 1ʒ + 1, atque ideo 1ꝝ æquatur $1\frac{1}{2} + \sqrt{1\frac{1}{4}}$: & latus A*B* vel AC $\sqrt{3\frac{3}{4}} + \sqrt{\frac{3}{4}}$. angulorum magnitudo ex antecedente theoremate facile innotescet. Sed ob æquationis leges latus AB quoq; assumi potuit $\sqrt{3\frac{3}{4}} - \frac{3}{4}$, atque de casu angulus A obtusus, erit 138 grad. 11 min. 37 secund. & angulus B vel C 20 grad. 54 min. $18\frac{1}{2}$ sec. paulo amplius.



PROBLEMA 28.

Data trianguli base, & ratione quam habet quadratum lateris unius ad aream trianguli, & perpendicularis ad basis segmentum, quæruntur latera.

Exponatur basis *AC* 15, ratio quadrati a latere A*B* ad aream trianguli quæ $2\frac{1}{3}$ ad 1, itemque ratio perpendicularis BD ad segmentum AD 4 ad 3 quæruntur, ejus latera & anguli. Sit BD 4ꝝ & AD 3ꝝ, hinc A*B* datur 5ꝝ. Hinc multiplicata basi 15 in dimidiã perpendicularem BD 2ꝝ datur area 30ꝝ, cujus $2\frac{1}{3}$ sunt 70ꝝ æqualia quadrato ab AB 25ʒ. Atq; ideo 1ꝝ æquatur $2\frac{4}{5}$, & 5ꝝ pro AB 14: 4ꝝ BD $11\frac{1}{5}$: 3ꝝ AD $8\frac{2}{5}$. quare reliqua DC $6\frac{3}{5}$. atque ideo BC 13. angulorum magnitudo per propositionem 26 haud difficulter invenietur.

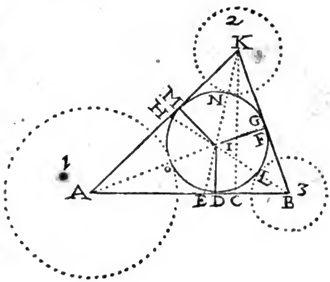


Non esset admodum difficile utrumque hoc Zetema geometrica fabrica expedire, nisi diagrammatum sculptor, nos moraretur.

PRO-

Problema 26.

Datâ trianguli base, altitudine, crurum summa quæruntur diametri circulorum ex apicibus trianguli descriptorum & circulum inscriptum contingentium.



Basis expositi trianguli AB sit 20, summa crurum AK & KB 36, perpendicularis KC 14, quæruntur diametri circulorũ cujus centra sint A, K, B & circulum triangulo dato inscriptum contingentium. Data base altitudine crurum summa ipsa sigillatim dari supra jam ostendimus problemate 18, unde AK deprehendetur $18 + \sqrt{12\frac{1}{2}}$, BK $18 - \sqrt{12\frac{1}{2}}$. radij inscripti circuli IM, IG, ID, sunt partium 5. Hinc, quia MA FB æquantur basi AB relinquitur pro MK & KF simul 16, cujus dimidium MK 8. quamobrem datis cruribus recti KM & MI dabitur basis KI $\sqrt{89}$, unde subductus radius IN relinquit radium KN $\sqrt{89} - 5$. reliquorum AO & LB eadem est ratio, datis itaque radijs dabuntur eorum diametri quarum magnitudinem in hac tabella exhibemus.

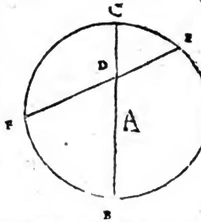
AC	2
AK	$18 + \sqrt{12\frac{1}{2}}$
BK	$18 - \sqrt{12\frac{1}{2}}$
diameter inscripti circuli 10.	
radius ID, IF, IM	5
KM, KF	8
MA, AD	$10 + \sqrt{12\frac{1}{2}}$
DB, BF	$10 - \sqrt{12\frac{1}{2}}$
IK	$\sqrt{89}$
AI	$\sqrt{.137\frac{1}{2} + \sqrt{5000}}$
IB	$\sqrt{.137\frac{1}{2} - \sqrt{5000}}$
radius NK	$\sqrt{.89} - 5$, &
diameter	$\sqrt{356} - 10$
radius AO	$\sqrt{.137\frac{1}{2} + \sqrt{5000}}. - 5$
diameter	$\sqrt{.550 + \sqrt{80000}} - 10$
radius BL	$\sqrt{.137\frac{1}{2} - \sqrt{5000}}. - 5$
diameter	$\sqrt{.550 - 80000} - 10.$

Problema 30.

Si eidem circulo inscriptæ diameter & adiameter se mutuo intersecent, data adiametro data ratione segmenti adiametri ad majus segmentum diametri, itemque ratione quam habet potentia reliqui segmenti ad potentiam intersegmenti diametri inter ipsam & centrum intersecti, quæritur diameter & mutua utriusque segmenta.

 Exponatur

Exponatur diameter BC, inſcripta alia non per centrum EF eſto data partium 18, ſitque ratio ſegmenti DF ad ſegmentum diametri DB ſubdupla: & ratio quadrati à DE ad quadratum ab DA quæ 11 ad 2, quæritur diameter BC, & utriuſque inſcriptæ mutua ſegmenta BD, DC, ED, DF. Reſpondeo BC eſſe $11\frac{5}{9} + \sqrt{69\frac{4}{8}\frac{1}{1}}$. & radium AC $5\frac{7}{9} + \sqrt{17\frac{3}{8}\frac{1}{4}}$. BD. $\sqrt{123\frac{4}{7}\frac{4}{2}\frac{1}{9}} + \sqrt{3\frac{1}{2}\frac{1}{7}}$. DC $8_{\frac{}{2}\frac{4}{7}} - \sqrt{7\frac{1}{7}\frac{2}{2}\frac{9}{9}}$. FD $16_{\frac{}{2}\frac{8}{7}} - \sqrt{30\frac{6}{7}\frac{5}{2}\frac{8}{9}}$. DE $\sqrt{30\frac{6}{7}\frac{5}{2}\frac{8}{9}} + 1\frac{1}{2}\frac{9}{7}$.



Stamamus enim FD $\sqrt{11}$ȝ, AD $\sqrt{2}$ȝ. Cumque rectangulum BDC rectangulo FAE per 35 prop. 3 lib. *Eucl.* æquetur, ſequitur BD DE, FD DC proportinales eſſe: atqui BD eſt dupla ipſius DE, quare & FD ipſius DC dupla quoque erit: quare DC erit $\sqrt{2\frac{3}{4}}$ȝ, & reliqua DE $18 - \sqrt{11}$ȝ, & BD $36 - \sqrt{44}$ȝ huc addita DC $\sqrt{2\frac{3}{4}}$ȝ dabit totam diametrum BC $36 - \sqrt{24\frac{3}{4}}$ȝ, & radium AC $18 - \sqrt{6\frac{3}{16}}$. ſed AD $\sqrt{2}$ȝ addita ad DC $\sqrt{2\frac{3}{4}}$ȝ eundem radium exhibet $\sqrt{2\frac{3}{4}}$ȝ $+ \sqrt{2}$ȝ. Quamobrem $\sqrt{2\frac{3}{4}}$ȝ $+ \sqrt{2}$ æquantur $18 - \sqrt{6\frac{3}{16}}$. addito utrimque $\sqrt{6\frac{3}{16}}$ȝ, fient $\sqrt{17\frac{3}{16}}$ȝ $+ \sqrt{2}$ȝ æqualia 18.

Iam ſi utrumq; æquationis membrum dividas per $\sqrt{17\frac{3}{16}} + \sqrt{2}$, fiet 1ꝛ æquale $\frac{18}{\sqrt{17\frac{3}{16}} + \sqrt{2}}$ id eſt, peracta diviſione 1ꝛ æquale $\sqrt{24\frac{1}{7}\frac{0}{2}\frac{4}{9}} - \sqrt{2\frac{1}{7}\frac{9}{2}\frac{2}{9}}$, cujus quadrati $19\frac{6}{7}\frac{4}{2}\frac{8}{9} - \sqrt{144\frac{1}{3}\frac{7}{3}\frac{9}{7}\frac{2}{4}\frac{2}{7}\frac{0}{7}}$ per 11 multiplicati latus exhibet magnitudinem ipſam DF $\sqrt{\,}$ 216 $\frac{1}{7}\frac{2}{2}\frac{8}{9} - \sqrt{\,}$ 17445 $\frac{6}{3}\frac{8}{3}\frac{3}{7}\frac{8}{4}\frac{3}{4}\frac{0}{7}$. ejuſdem quadrati duplicati latus dabit rectam AD $\sqrt{\,}$. 39 $\frac{2}{7}\frac{9}{2}\frac{6}{9} - \sqrt{\,}$ 5 7 7 6 $\frac{7}{3}\frac{1}{3}\frac{6}{7}\frac{8}{4}\frac{0}{4}\frac{0}{7}$. horum latera erunt quæ ſupra à nobis ſunt notata. reliqua jam ſunt faciliora.

δοκιμασία.

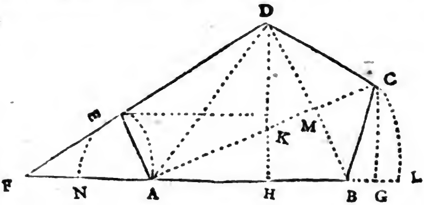
Addito interſegmentum AD ad radium BA, dabitur totum ſegmentum BD $\sqrt{123\frac{4}{7}\frac{4}{2}\frac{1}{9}} + 3\frac{1}{2}\frac{1}{4}$, datur3 DC $8_{\frac{}{2}\frac{4}{7}} - \sqrt{7\frac{1}{7}\frac{2}{2}\frac{9}{9}}$. horũ factus $\sqrt{6580\frac{2}{3}\frac{7}{3}\frac{4}{7}\frac{8}{4}\frac{2}{4}\frac{8}{7}} - 3\frac{1}{7}\frac{0}{2}\frac{3}{9}$ æquatur facto ab DF $16_{\frac{}{2}\frac{8}{7}} - \sqrt{30\frac{6}{7}\frac{5}{2}\frac{8}{9}}$ in DE $\sqrt{30\frac{6}{7}\frac{5}{2}\frac{8}{9}} + 1\frac{1}{2}\frac{9}{7}$. Quare operis ratio tota eſt legittima.

PROBLEMA 35.

Datis quinquanguli ABCDE lateribus AB 14, BC 7, CD 10, DE 12, AE 5 diagonio AC 17, anguloque ad E recto, quæritur quanta ſint continuationes EF, AF laterum BA BE, ad mutuum occurſum productorum.

Hoc

Hoc zetema solvit *Willebrordus Snellius RF.* & postea quoque *Nathanael Classonius* numeri verò adeó sunt vasti & enormes ut neque typographum cudendo neque lectorem eos relegendo fatigandos existimem,



ideoque duntaxat eorum valorem in numeris absolutis exhibebo, quorum veritatem per triangulorum canones comprobabo. FE $6\frac{64843}{100000}$, FA $8\frac{31874}{100000}$. in quibus ne quidem $\frac{1}{10000}$ unius unitatis à vero desit aut redundat. Vt eosdem per dictos Canones assequaris perpendicularem CG $\sqrt{46\frac{53}{100}}$ ad numeros absolutos revocato $6\frac{8213351}{10000000}$ is est sinus anguli CAB. fiat itaq;, ut 17 ad $6\frac{821335155}{1000000000}$, ita sinus totus 10000000 ad sinuum 4012550, cui debentur 23 gr. 39 min. $23\frac{96}{100}$ sec. pro angulo BAC, cui æqualis quoque est KDM. Porro perpendicularis ab D in AC est $\sqrt{58\frac{13}{100}}$ vel in numeris absolutis $7\frac{6244008}{10000000}$ hic sinus est anguli DAM, unde proportio ut AD 13 ad $7\frac{62440008}{100000000}$, sic 10000000 ad sinuũ 5864923 cui in canone respondent 35 gr. 54 min. $30\frac{53}{100}$ sec. quare totus angulus DAH datur 59 gr. 33 min. $54\frac{52}{100}$ sec ejus complementum 30 gr. 26 min. $5\frac{48}{100}$ sec. pro ADH. Præterea quia crura anguli recti AE ED dantur, dabitur quoque angulus ADE 22 gr. 37 min. $11\frac{56}{100}$ sec. quare totus FDB quoq; dabitur 53 gr. 3 min. $16\frac{74}{100}$ sec. jã triãgulũ FOA triãgulo FDH simile est ob angulũ ad *F* cõmunẽ, ad E & H rectos. quare datur angulus *F*AO idem qui *F*DH 53 gr. 3 min. $16\frac{84}{100}$ sec. Itaque assumpta EA pro radio, fiet *F*E tangens FA secans, unde proportio, quemadmodum sinus totus 10000000 ad tangentem anguli FAE 13296861, ita AO 5 ad E*F* $6\frac{64843}{100000}$. Et quemudmodum radius 10000000 ad secantem ejusdem anguli 16637480, sic *EA* 5 ad *AF* $8\frac{31874}{100000}$. ut supra. Vel divisionis opere per solas sinuũ tabulas: statuatur FA radius, tumq; AE erit sinus anguli AFE 6010525. Hinc proportio ut 6010525 ad radium 1000000, sic AE 5, ad AF $8\frac{31874}{100000}$ ut supra. Hujus periculum pluribus modis facere tilicebit.

Quia nos autor huc quasi invitat, quamvis neque istius diagrammatis apparatus nostræ constructioni respondeat, neque per temporis angustia novum exhibere possim, linearum quæsitarum magnitudinem numeris suis hic duntaxat adscribam, reliquam operis exegesin alias exhibituri.

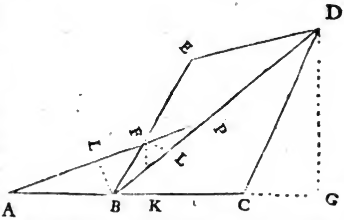
EF $\dfrac{\sqrt{9305600211574169505 28} + \sqrt{201659102524410531 84000 - \sqrt{20227718684988909608 00 - 100160540340}}}{14037215639}$

AF $\dfrac{\sqrt{19507250405341639584000} + \sqrt{22930211911556832215680 - \sqrt{3961309428281275180000 - 116704363615}}}{14037215639}$

AL $\dfrac{\sqrt{199600800} + \sqrt{73053480}}{2203}$

PROBLEMA 32.

In exposito diagrammate dantur latera quadranguli BC decempedarum 10, CD 14, DE 9, EB 12. item trianguli FB 4, BA 7, AF 10, quæritur si AF latus continuatum occurrat diagonio BD, quanta sit area trianguli BFP.



Hoc zetema mihi ab accurato Geodæta *Adriano Ockersonio* Amsterodamense transmissum; qui primum quæsiverat tantum diagonium BD, cujus solutionem repetas é zetemate 50 libri antecedentis. Postea coram mihi narravit aream figura quadrangula comprehensam ad mensurandum sibi propositam, qualem hic vides, cujus omnium laterum quidem mensura libera daretur, sed lignorũ asserumque struices in ipsa area diagonij dimensionem non concederent: ideoque ut nihilominus hujus figuræ certum aliquem modum constitueret, id assecutum dimensis lateribus trianguli externi ABF: Postulavit itaque ut quantitatem rectarum FP PB in numeris exhiberem. quas per canones triangulorum investigavi. easdem quoque discipulo meo *Petro Cornelij* quales nam veré essent investigandos proposui, qui adeò mirabiles vasti et enormes ei acciderunt, ut non solum ad describendum molesti, sed ad experimẽdum intricati & fastidio sint. Ego solam absolutorum investigationem sequar, qui ne quidem $\frac{1}{1000000}$ unius decempedæ à vero absint, quemadmodum in præmisso problemate quoque usurpavimus. In constructione vides perpendiculares BL in AF, & FL in BF, FK in BC. Inveni itaque angulum FBK 51 gra. 19 min. $4\frac{12}{100}$ sec. hinc quia FK datur $\sqrt{9\frac{3}{4}}$, dabitur sinus anguli A 3122499 qualium AF radius 20000000, atque ideo angulus KAF 18 gr. 11 min. 41 $\frac{12}{100}$ sec. qui de externo FBK subductus relinquit AFB 33 gr. 7 min. $22\frac{66}{100}$ sec. itaque BPF reliquus ad duos rectos 146 gr. 52 min. $37\frac{30}{100}$ sec. Porrò cum in 50 zetemate libri 4 inventa sit diagonius BD $\sqrt{.240\frac{5}{8}}$ — $\sqrt{29807\frac{6971}{8876}}$, hoc est in numeris absolutis $20\frac{1230813}{10000000}$. & perpẽdicularis DG $\sqrt{108\frac{93423}{14377}}$ + $\sqrt{5\frac{41495}{14137}}$ hoc est $12\frac{717760111}{1000000000}$. fiat itaque ut BD $20\frac{1230813}{10000000}$ ad DG $12\frac{717760111}{1000000000}$, ita sinus totus 10000000 ad sinum 6257485 cui competunt 38 gr. 44 min. $13\frac{72}{100}$ sec. pro angulo DBG, qui de FBK subductus relinquit angulum FBP 12 grad. 44 min. $13\frac{72}{100}$, quare tertius BPF erit 20 gr. 32 min. $32\frac{30}{100}$. Dantur itaque anguli trianguli BPF & latus BF, quare & reliqua latera dabuntur BP $6\frac{229000}{1000000}$, BF $2\frac{4829326}{10000000}$ & perpendicularis FL $\frac{8712156}{10000000}$, & segmenta PL $2\frac{3250415}{10000000}$, LB $3\frac{9639561}{10000000}$. si hujus periculum facere libeat nihil commissum deprehendes.

BK	$2\frac{1}{2}$
FK	$\sqrt{9\frac{3}{4}}$
AK	$9\frac{1}{2}$
AF	10
FP	$2\frac{48293}{100000}$
BP	$6\frac{129000}{1000000}$

ἀμάρ-

ἁμάρτημα μνημονικὸν est, cum enim Geodæta diligentißimus Adrianus Ockersonius *solam diagonium BD quæsivisset, longo post tempore eam quæstiunculam istoc auctariolo conlocupletavi, ut quærerem aream trianguli BFP, & latera FP PB. Quorum magnitudo in numeris irrationalibus neque insolens, neque stupenda est, solis enim binomijs definitur: quod verò adeo mirabiles & intricati acciderint, id vel ab abaci errore, vel ex minus idonea constructione evenisse arbitror. quamvis enim omnis geometricorum lineamentorum constructionis formula uera sit & necessaria, non tamen secundum omnes æqué est expedita ac facilis numerationis via. Constructionis typum, & numerationis modum quia à typographis urgemur nunc quidem exhibere non licet, itaque solam linearum quæsitarum & areæ magnitudinem ex adversarijs nostris huc transcribam*

FP $\dfrac{\sqrt{44755691662 5} - 66425}{242686}$

BP $\dfrac{\sqrt{.128630631 3201} + \sqrt{99782168941645286293606 5}}{\sqrt{58896494596}}$

area trianguli B P F. $\dfrac{\sqrt{34211250706815} - \sqrt{8431868274375}}{\sqrt{47117195 6588}}$

Hoc est in numeris absolutis *FP* $2_{\circ}\,4\,8\,2\,9\,2\,8\,5$ PB $6_{\circ}\,8\,2\,9\,0\,0\,9\,0$.

PROBLEMA 33.

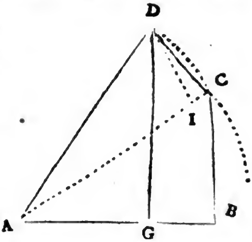
Datis lateribus quadranguli	*AB*	$9 + \sqrt{768} + \sqrt{20}$.
	AD	$\sqrt{.1239 + \sqrt{190512}} - \sqrt{500}$
	DC	$\sqrt{.132 + \sqrt{960}}$
	BC	$\sqrt{768}$

& angulo ad B recto, quæritur perpendicularis DG.

Istud zetema à viro in hisce artibus versatissimo *Nicolao Petersonio Daventriense* Amsterodami programmate typis excuso evulgatum anno salutis 1584, ad cujus solutionem *Guilselmus Goudanus* ab ipso invitur. quod postmodum anno 1599 *Samuel Kroppius* Doeveranus horum studiosissimus legittimé & apodictice explicavit. & deprehendit perpendicularem DG $7 + \sqrt{588} - 5$. hoc est in numeris absolutis $29_{\circ}$. $1\,2\,6\,4\,3$, ut proximé.

Sit

Si per triangulorum canonem hujus solutionẽ instituere cupias, numeros propositos primum ad absolutos & explicabiles revocato, quemadmodum hic vides. jam si BC assumatur pro radio



AD	40 [illegible]		
DC	12 [illegible]		
AB	41 [illegible]		
BC	27 [illegible]		
DI	8 [illegible]		
	gr.	min.	sec.
CA*B*	33.	56.	9 [illegible]
DAI	11.	35.	23 [illegible]
D*A*G	45.	31.	34 [illegible]

fiet *A*B tangens & AC secans anguli ACB. itaque qualium B*C* 10000000 talium erit *AB* tangens 14861338 cui respondent 56 gr. 3 min. 50 [illegible] sec. pro angulo ACB. Hujus complementum CA*B*, erit 33 gr. 56 min. 9 [illegible] sec. Porrò AC vel é quadratis crurum AB *B*C vel é secantium tabulis dabitur 49 [illegible] decempedarum. dantur itaque latera trianguli ADC, atque ideo perprndicularis DI 8 [illegible]. quamobrem qualium AD 10000000 talium DI 2000053 quippe sinus anguli DAI, cui respondent 11 gr. 35 min. 23 [illegible] sec. atque hinc jam totus angulus DA*B* datur 45 gr. 31 min. 34 [illegible] sec Hujus sinus est perpendicularis DG, fiat itaq; ut radius 10000000 ad sinũ 7135684, sic AD 40 [illegible] ad DG 29 [illegible]. ut supra. Hujus generis questiones & geometrica & Algebrica analysi docui solvere in eo, quiem adversus *Guilielmum Coudanum* edidi libello.

Hujus Zetematis numeros & lineamenta quemadmodum potero hic exprimam DI perpendicularis intelligatur in AC, erit itaque

$$\text{AI } \frac{1372+\sqrt{218700}+\sqrt{845}+\sqrt{11760}}{\sqrt{.1637+\sqrt{248832}+\sqrt{61440}+\sqrt{6480}}}$$

$$\text{CI } \frac{265+\sqrt{19440}+2645+\sqrt{972}}{\sqrt{.1637+\sqrt{248832}+\sqrt{61440}+\sqrt{6480}}}$$

$$\text{quadratum DI } \frac{130482+\sqrt{1404349488}+\sqrt{40147440}-\sqrt{97416980}}{1637+\sqrt{248832}+\sqrt{61440}+\sqrt{6480}}$$

$$\text{ipsa DI } \frac{\sqrt{127308}+53-\sqrt{240}+\sqrt{125}}{\sqrt{.1637+\sqrt{248832}+\sqrt{61440}+\sqrt{6480}}}$$

*Dehinc fingamus DI perpeddicularem continuatam occurrere basi A*B *in R (hoc autem mente concipiendum, quia lineamenta desunt) erunt itaque* A*BC* AI*R* *triangula similia, atque hinc proportio, ut* AB *ad* BC, *sic AI ad*

IR $\sqrt{144}$.

$$IR\ \frac{\sqrt{1445670912}+12960+\sqrt{9031680}+\sqrt{648960}}{\dfrac{\sqrt{.1637}+\sqrt{248832}+\sqrt{61440}+\sqrt{6480}}{9+\sqrt{768}+\sqrt{20}}}$$

Iam DI addita ad IR dabit totam DR

$$DR\ \frac{23375+\sqrt{1817546988}+\sqrt{8489045}+\sqrt{6613440}}{\dfrac{\sqrt{.1637}+\sqrt{248832}+\sqrt{61440}+\sqrt{6480}}{9+\sqrt{768}+\sqrt{20}}}$$

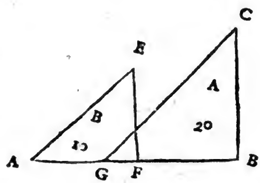
Vnde ad extremum proportio, ut CA ad AB, sic RD ad quæsitam

$$DG\ \frac{23375+\sqrt{1817546988}+\sqrt{8489045}+\sqrt{6613440}}{1637+\sqrt{248832}+\sqrt{61440}+\sqrt{6480}}$$

Qua divisione peracta dabitur ipsa DG $7+\sqrt{388}-5$ *ut supra.*

PROBLEMA 30.

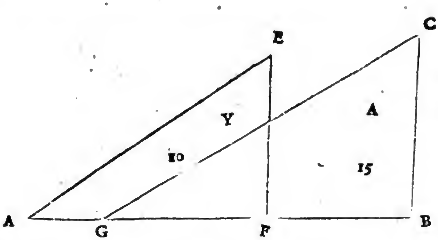
Super data basi AB partium 9 consistunt duo triangula rectangula ad F & B, AFE cujus area sit 10, GBC cujus area 20, laterum autem hæc est affectio, ut AG & BF simul duplicia sint communis segmenti GF, & GE cum BC duplicia ipsius FB, denique AG & BC æqualia ipsis GF & FB.



Respondeo AG esse $6\frac{1}{4}-\sqrt{25\frac{1}{16}}$. GF 3. FB $\sqrt{25\frac{1}{16}}-\frac{1}{4}$. BC $\sqrt{100\frac{1}{4}}-4\frac{1}{2}$. FE $2\frac{1}{16}+\sqrt{2\frac{9}{16}}$. Hoc zetema propositum est amico cuidam meo, qui me ejus solutionem rogavit. Algebricarum positionum hoc est formula. sit AG 1℞, GF 1℞ + 1 A, FB 1℞ + 2 A, summa omnium AB 3℞ + 3 A, itaq; 3℞ + 3 A, æquantur AB 9, & 1A æquale 3 — 1℞: tanta est differentia inter AG & GF, vel GF & FB. atqui cum AG sit 1℞ & differentia inter AG & GF 3 — 1℞ additis 1℞ & 3 — 1℞ dabitur GF 3, & pro FB 6 — 1℞, atq; CB 9 — 2℞. Iam multiplicata CB in dimidiam BG datur area trianguli CBG 81 — 27℞ + 2q, ea igitur æquatur datæ areæ 20. atque ideo 1℞ est $6\frac{1}{4}-\sqrt{25\frac{1}{16}}$. quare datur AG. reliqua hinc sunt in promptu.

PROBLEMA 35.

Si è quatuor lineis pari differentia crescentibus detur triangulum comprehensum è prima & secunda summa in tertiam, item a summa secunda & tertia in quartam, ipsa lineæ quæruntur.



Sunto quatuor lineæ AG, GF, FB, BC, pari intervallo auctæ, deturque triangulum rectangulum AFE 10. sub AF prima secundaque in tertiam FE comprehensum. & detur triangulum rectangulnm GBC 15, à secunda tertiaq; GB in quartam BC comprehensum, quæruntur singulæ AG, GF, *FB*, *BC*. Respondeo AG esse $\sqrt{.}\sqrt{4784\frac{1}{16}} - 63\frac{3}{4}$, GF $\sqrt{.}\sqrt{321\frac{9}{16}} - 8\frac{1}{4}$, *FB* vel FE $\sqrt{.}\sqrt{59\frac{1}{16}} + 6\frac{1}{4}$, BC $\sqrt{.}\sqrt{1476\frac{9}{16}} - 18\frac{3}{4}$, denique progressionis differentiam $\sqrt{.}\sqrt{420} - 20$.

Hoc zetema antecedente difficilius & operosius ei reposui, quod in analysi facile deprchendes: eum quem in his solvendis modum usurpare consuevi tibi impartiar. Sit AG 1ꝛ, GF 1ꝛ + 1A, FB 1ꝛ + 2 A, BC 1ꝛ + 3 *A*. Ideoque ◡*A*F erit 2ꝛ + 1 A & GB 2ꝛ + 3 *A*.

multiplicato AF 1ꝛ + 2 *A*	multiplicato G*B* 2ꝛ + 3 *A*
in F B seu *FE* 2ꝛ + 1 *A*	in BC 1ꝛ + 3 *A*
2ʒ + 5ꝛ*A* + 2*AA*	1ʒ + 9ꝛ*A* + 9*AA*

Itaque 2ʒ + 5ꝛ◡*A* + 2◡*A*◡*A* æquantur 20
2ʒ + 9ꝛ◡*A* + 9◡*A*◡*A* æquantur 30

ut ꝛ◡*A* se mutuo elidant sumito prioris noncuplum, posterioris quintuplum hoc modo.

18ʒ + 45ꝛ◡*A* + 18 ◡*AA* æqualia 180
10ʒ + 45ꝛ *A* + 45 *A A* æqualia 150

itaque differentiæ 8ʒ — 27*AA* æquantur 30
& 1A æquatur $\sqrt{.}\,\frac{8ʒ - 30}{27}$

Quam.

quamobrem differentia progressionis arithmeticæ erit $\sqrt{.\frac{8\mathfrak{z}-30}{27}}$. atqui cum primus terminus statuatur $1\mathfrak{r}$, secundus erit

$$1\mathfrak{r}+\sqrt{.\frac{8\mathfrak{z}-30}{27}},\ \text{tertius}\ 1\mathfrak{r}+\sqrt{.\frac{32\mathfrak{z}-120}{27}},$$

quartus $1\mathfrak{r}+\sqrt{.2\frac{2}{3}\mathfrak{z}-10}$. jam secundus tertiusque additi conflabunt lineam GB $2\mathfrak{r}+\sqrt{.2\frac{2}{3}\mathfrak{z}-10}$. quę summa per quartum terminū BC $1\mathfrak{r}+\sqrt{2\frac{2}{3}\mathfrak{z}-10}$ multiplicata dabit $4\frac{2}{3}\mathfrak{z}-10+\sqrt{.24\mathfrak{z}-90\mathfrak{z}}$ æqualia duplo areæ trianguli GBC 30. æqualitatis leges secutus deprehendes $1\mathfrak{z}\mathfrak{z}$ æquari $720-127\frac{1}{7}\mathfrak{z}$. atque ideo $1\mathfrak{z}$ æquari $\sqrt{4784\frac{1}{16}}-63\frac{3}{4}$, & $1\mathfrak{r}$ tandem æquari $\sqrt{.\sqrt{4784\frac{1}{16}}-63\frac{3}{4}}$. tanta igitur erit prima AG, ab hujus quadrati sextuplo $\sqrt{306180}-510$ subductis 30, reliquoque $\sqrt{306180}-540$ per 27 diviso, quoti latus $\sqrt{.\sqrt{420}-20}$ erit quæsitę progressionis differentia. ista ad primum terminum AG addita dabit secundum GF. simili modo invenientur cæteri EG & BC, quemadmodum supra expressimus.

Veritatis periculum facies additia AG ad GF & summa $\sqrt{.\sqrt{24625}-125}$ per tertium terminum EF $\sqrt{.\sqrt{59\frac{3}{16}}+6\frac{1}{4}}$ multiplicata, horum enim factus est $\sqrt{400}$ seu 20, cujus dimidium 10, æquatur areæ triangali AFE, quemadmodum oportuit. Eodem plane modo factus à composita é secundo tertioque in quartam erit 30 duplus trianguli GBC.

Verum enim veró quia longe & iucundius est & ut ilius est ista é fontibus haurire, quam in rivulis consectari, hujus & antecedentis quæstionis affectiones & symmtomata paulo altius repetam initio ab antecedente problemate facto.

Cum ita proponitur, AG & FB simul æquari duplo mediæ GF, sequitur AG, GF, FB arithmetica progressione proportionales esse: & rursum cum GF & BC ponuntur duplæ ipsius FB, hæ tres lineæ, GF FB BC, itidem erunt continuè arithmeticé proportionales, atque ideo quatuor lineæ in continua progressione arithmetica, hoc est æqualibus differentijs crescentes AG, GF, FB, BC: quare tertium illud quod adjicitur plane redundat, videlicet AG & BC ipsi GB æquari, id enim necessario positam thesin sequitur. Porrò cum detur area rectanguli trianguli GBC 20, ejusq; duplum 40, Zetematis hæc vis erit.

Si é quatuor terminis in Arithmetica progressione continuis primi secundi & tertij summa detur, cum facto ultimæ in secundum & tertium, quæruntur singuli.

Summa primi secundi & tertij sit 9, factus ultimi in secundum & tertium 40.

Ponamus itaque ultimam BC esse $1\mathfrak{r}$, *per quam 40 divisa dabunt in quoto* $\frac{40}{1\mathfrak{r}}$ *summā GF & FB utriusque mediæ quibus utraque extrema AG & BC per Arithmeticæ progressionis leges quoque æquantur, cum autem CB sit* $1\mathfrak{r}$ *AG erit* $\frac{40}{1\mathfrak{r}}-1\mathfrak{r}$, *quæ subducta de AB, 9, relinquit GB* $9-\frac{40}{1\mathfrak{r}}+1\mathfrak{z}$. *atqui eadem GB ante inventa est* $\frac{40}{1\mathfrak{r}}$ *quamobrem* $9-\frac{40}{1\mathfrak{r}}+1\mathfrak{r}$ *æquabuntur* $\frac{40}{1\mathfrak{r}}$ *hoc est, additis utrumque* $\frac{40}{1\mathfrak{r}}$, $\frac{80}{1\mathfrak{r}}$ *æquabuntur* $9+1\mathfrak{r}$ *vel quod idem est 80 æquantur* $9\mathfrak{r}+1\mathfrak{z}$, *ergo* $1\mathfrak{r}$ *æquatur* $\sqrt{100\frac{1}{4}}-4\frac{1}{2}$ *tanta ergo est ultima CB. ideoque prima AG erit,* $6\frac{1}{4}-\sqrt{25\frac{1}{16}}$.

Quamobrem data prima & summa progressionis dabitur differentia, subducta enim prima $6\frac{1}{4} - \sqrt{26\frac{1}{16}}$ de summa 9, reliquus $\sqrt{25\frac{1}{16}} + 2\frac{3}{4}$ per numerum terminorum (qui hic sunt tres) divisus dabit in quoto progressionis differentiam. Atque hinc haud difficulter GF & FB secundus tertiusque invenientur. Hæc fusius notavi, quia satius existimo ista è fontibus ipsis haurire, & ab origine sua quæsiti solutionem arcessere.

Secundo esto quæsitum quoque hoc novissimum in serie quatuor terminorum arithmeticæ progressionis continuæ facto tertij in primum secundum & quarti in secundum & tertium cognitis ipsos terminos invenire.

Sit factus primi & secundi in tertium 20, item factus secundi tertijque in quartum 30. Statuamus pro novissima BC $1x$, itaq; BG secunda & tertia erit $\frac{30}{1x}$ atque istis quoq; æquatur prima AG cum ultima BC, hinc itaque subducta ultima BC reliqua erit prima AG $\frac{30}{1x} - 1x$. Hæc addita ad GB dabit totam AB $\frac{60}{1x} - 1x$, atqui prima AG & tertia FB æquantur duplo mediæ GF, quare tota erit ejusdem tripla: datur itaque GF $\frac{20}{1x} - \frac{1x}{3}$, quæ subducta de GB $\frac{30}{1x}$ relinquet FB $\frac{10}{1x} + \frac{1x}{3}$, dantur itaque CB $1x$, BF $\frac{10}{1x} + \frac{1x}{3}$, FG $\frac{20}{1x} - \frac{1x}{3}$, GA $\frac{30}{1x} - 1x$. itaque prima & secunda AF $\frac{50}{1x} - 1\frac{1}{3}x$ multiplicata per EF seu BF $\frac{10}{1x} + \frac{1x}{3}$ dabit $\frac{4500 + 30z - 4zz}{9z}$ æqualia 20 duplo trianguli AFE. hoc est 1125 æqualia $1zz + 37\frac{1}{2}z$, id est $1z$ æquale $\sqrt{1476\frac{9}{16}} - 18\frac{3}{4}$, & $1x$ æquale $\sqrt{.\sqrt{1476\frac{9}{16}} - 18\frac{3}{4}}$, atq; inde jam haud difficulter reliqua dabuntur BF $\sqrt{.\sqrt{59\frac{1}{16}} + 6\frac{1}{4}}$. FG $\sqrt{.\sqrt{321\frac{9}{16}} - 8\frac{3}{4}}$, AG $\sqrt{.\sqrt{4784\frac{1}{16}} - 63\frac{1}{4}}$. Vt supra.

Eodem modo ab FB initium fieri potuit. posita enim FB $1x$, datur AF $\frac{20}{1x}$, itaque AB $\frac{20}{1x} + 1x$, per 3 diviso dant GF $\frac{20}{3x} + \frac{1x}{3}$ & AG $\frac{40}{3x} - \frac{1x}{3}$, deniq; BC $1\frac{1}{3}x - \frac{20}{3x}$. Vnde reliqua æquatio in promptu est, & recta FB $1x$ $\sqrt{.\sqrt{59\frac{1}{16}} + 6\frac{1}{4}}$. ut prius. Atque ita Problematum miscellaneorum, & simul quinti libri finis hic esto.

LVDOLPHI

LVDOLPHI à CEVLEN LIB. SEXTVS, de Figuris ordinatis circulo adſcriptis, & alijs quibuſdam huc ſpectantibus, cum uſu Canonis triangulorum in circularium ſegmentorum geodæſia.

Libro ſecundo geometricam quarundam figurarum aſcriptionem quæ circino & regula abſolvi potuit expreſſi, earundem laterũ magnitudinẽ in expoſitis diametri partibus hic inveſtigare docebo. Primum itaque omnium à quadrato nobis initium eſto, à quo per laterum biſectionem gradatim ad octangulum, ſedecangulum, & ſequentia polygona progrediar. & preterea rectarum á centro ad angulos polygoni circumſcripti eductarum, quæ ſecantes ſunt, magnitudinem inveſtigabo. Quæ diagrammate ſubjecto illuſtriora erũt. eſto HG latus quadrati circũſcripti, DC inſcripti, latus octanguli inſcripti PQ circumſcripti WX. IK inſcripti ſedecanguli, MN triginta duorum angulorum latus. *Præterea* infinita BP extremę diametro AB perpendicularis, cui radij per angulos inſcriptorum polygonorum educti occurrant in H, I, P, Q, qui propterea ſecantes vocantur. OH ſecans anguli HOB, & HB ejuſdem tangens. OI ſecans anguli IOB & IB tangens, atque ita porrò. vides itaque pro amplitudine anguli in centro ipſas tangentes & ſecantes quoq; augeri: Haud aliter ſemiſſes laterum circumſcriptorum H6, X6, N6 tangentes erunt ſuarum peripheriarum, & OX, ON earundem ſecantes. His ita prælibatis, eſto nobis.

PROBLEMA I.

Data diametro partium 2, quæruntur latera quadrati inſcripti & circumſcripti, & polygonorum ab his continua biſectione ortorum.

Diametri D4, C5, mutuò ſeſe ad angulos normales interſecent in centro O; poſita itaque diametro partium 2, erit DO radius 1, atque ideo DC latus quadrati recto angulo ſubtenſum $\sqrt{2}$; latus circũſcripti quadrati HG 2, eſt enim diametro æquale. area inſcripti quadrati eſt 2, circũſcripti 4 tangens BH 1, ſecans OH $\sqrt{2}$. Porrò ad inveſtigationem lateris octanguli PQ, ſubducito quadratum lineæ 9D (quæ eſt dimidia totius CD) de quadrato radij OD, reliqui latus dabit lineam 9O $\sqrt{\frac{1}{2}}$, ea de radio 6O ſubducta relinquet lineam 96, $1 - \sqrt{\frac{1}{2}}$, hujus quadratum $1\frac{1}{2} - \sqrt{2}$ additum ad quadratum 9D, latus dabit inſcriptam 6D vel PQ latus octanguli $\sqrt{.2 - \sqrt{2}}$. Hoc autem latus reliquaque deinceps é biſectione nata longe facilius inveniuntur hoc modo.

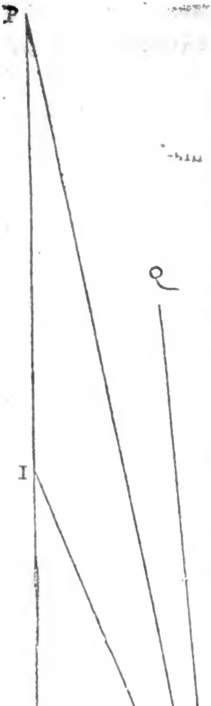
Media proportionalis inter radiũ & differentiam complementi datæ inscriptæ, est inscripta dimidiæ peripheriæ.

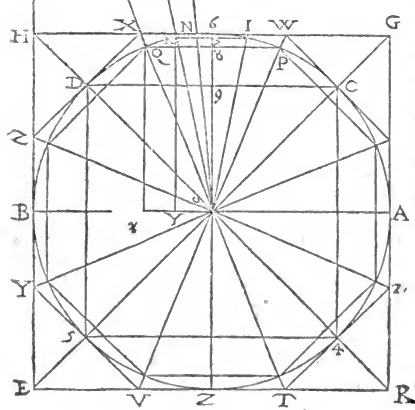
Complementum inscriptæ voco lineam cum qua ipsa æque possit circuli diametro. DC esto latus inscripti quadrati, ejusdem complementum $\surd 2$, id de diametro 2 subductum relinquit $2-\surd 2$, qui numerus per radium 1 multiplicatus idem manebit, unitas enim nihil mutat, hujus itaq; latus $\surd.2-\surd 2$ erit latus octanguli inscripti ut supra. Hujus theorematis veritatem in libro quem de ascriptis olim edidi demonstratam, & pluribus exemplis comprobatam reperies.

Porrò ad inventionem lateris sedecanguli, subducas quadratum lateris octanguli $2-\surd 2$ de quadrato diametri 4 reliqui latus $\surd 2+\surd 2$ est PT complementum lateris octanguli ad semicirculum. hæc de diametro subducta dabit $2-\surd.2+\surd 2$, cujus latus (namque unitas in multiplicatione nihil mutat) $\surd.2-\surd.2+\surd 2$ erit latus sedecanguli.

Et rursum hujus complementi $\surd.2+\surd.2+\surd 2$ differentia a diametro $2-\surd.2+\surd.2+\surd 2$, cujus latus $\surd.2-\surd.2+\surd.2+\surd 2$ est latus polygoni 32 angulorum.

Eadem via latus figure 64 angulorum deprehendes $\surd.2-\surd.2+\surd.2+\surd.2+\surd 2$. atque ita in cæteris consimili in infinitum progressu.





Circumscriptarum autem figurarum latera hoc modo auquires. Investigato primum perpendicularem à centro in latus polygoni inscripti, ea autem æquatur dimidio complementi dati lateris ad semicirculum. Ita perpẽdicularis 8O a centro O in latus octanguli QP erit $\surd.2+2$. Inde

jam.

jam proportio ut O8 $\frac{\sqrt{.2+\sqrt{2}}}{2}$ ad QP $\sqrt{.2-2}$ latus octanguli inscripti, ita radius O6 1 ad WX latus octanguli eidem circulo circumscripti $\sqrt{8}-2$, cujus dimidium $\sqrt{2}-1$ est tangens anguli 6OX. seu peripheriæ 6Q. Hujus quadratum $3-\sqrt{8}$ additum ad quadratum radij O6 1 dabit $4-\sqrt{8}$ pro quadrato secantis OX, ideoque ipsam $\sqrt{.4-\sqrt{8}}$. quare posita diametro 200000000000 tangens peripheriæ 6Q erit 41421356237, & ejusdem secans $108239220002\frac{9}{10}$.

Porrò ad inventionem OI & BI, notabis triangula OQẓ, OIB similia esse. ideoque ut Oẓ vel 8Q ad $\sqrt{.2-\sqrt{2}}$ ad ẓQ, ita radius O 6 ad BI $\sqrt{2}+\sqrt{1}$ (Notabis autem secantem $\frac{}{2}$ KoB, esse quoque secantem anguli reliqui obtusi KoA, qui ambo simul duobus rectus æquales sint. atque ita in cæteris omnibus) jam ut Oẓ $\frac{\sqrt{.2+\sqrt{2}}}{2}$ ad BQ 1, sic OB 1, ad OI $\sqrt{.4+\sqrt{8}}$, quæ secans est peripheriæ BQ 67 gr. 30 min. hoc est posita diametro portium 200000000000 erit 261312592975, indeque dabitur tangens IB 241421356237.

Area octanguli in circulum inscripti invenietur, multiplicata perpendiculari 8O $\frac{\sqrt{.2+\sqrt{2}}}{2}$ in latus $\sqrt{.2-\sqrt{2}}$ sumptoque facti qnadruplo, factus á perpendiculari in latus est $\sqrt{\frac{1}{2}}$, cujus quadruplum $\sqrt{8}$ vel $2\frac{828427124}{1000000000}$ est area octanguli. Haud aliter constabit area octanguli circulo circumscripti, datur enim latus $\sqrt{8}-2$ & perpendicularis à centro ipse radius 1, fit itaque area $\sqrt{128}-8$ sive $3\frac{313708498}{1000000000}$.

Latus sedecanguli inscripti supra inventum est $\sqrt{.2-\sqrt{.2+\sqrt{2}}}$, hujus complementum ad semicirculum $\sqrt{.2+\sqrt{.2+\sqrt{2}}}$, dimidium pro perpendiculari 7O $\frac{\sqrt{.2+\sqrt{.2+\sqrt{2}}}}{2}$. inde proportio, ut 7O $\frac{\sqrt{.2+\sqrt{.2+\sqrt{2}}}}{2}$ ad latus sedecanguli inscripti $\sqrt{.2-\sqrt{.2+\sqrt{2}}}$, ita radius 1 ad latus circumscriptum, leges numerationis surdæ, quam libro primo expressi, secutus invenies $\sqrt{.16+\sqrt{128}}-8+2$, hoc est $\frac{39782473}{100000000}$ cujus octuplum $3\frac{18259787}{100000000}$ est area sedecanguli circumscripti: & $3\frac{06146745}{100000000}$ area sedecanguli inscripti. Idem aliter. quemadmodum $\sqrt{.2+\sqrt{.2+\sqrt{2}}}$ ad $\sqrt{.2-\sqrt{.2+\sqrt{2}}}$, ita radius 1 ad tangentem anguli 6OK vel peripheriæ 6K, hic latus inscripti sedecanguli per suum complementum ad semicirculum dividitur. ista autem divisio lib. 1. cap. 8 à nobis peracta est, libet hic compendiosiorem formulam docere. Intuere subjectum typum.

Dividendus √.—√.2+√2 divisor √.2+√.2+√2
√.—√.2+√2 √.2—√.2+√2 residuum

2—√.2+√2 4
√.2+√2 —2+√2

dividendus √.8+√32.—2+√2 √.2—√2
divisor √2 √.2+√2

quotus √.4+√8.—√2+1, 4
tangens 11 gr. & 15 minutorum. —2

√2 divisor.

Hic quadratū divisoris 2+√ 2+√2 per sui residui quadratum 2—√.2+√2 multiplicavi, facti 2—√2 latus √.2—√2 fuit factus optatus. hic rursum per suum residium multiplicatus exhibuit √.8+√32.—2+√2 qui per √2 divisus produxit quotum √.4+√8—√2+1. Ejusdem anguli secans invenietur si diametrum 2 per complementum dictum √.2+√.2+√2 diviseris quotus enim √.8+√32.—√.80+√6272. erit secans 11 gr. 15. atque hinc jam facili negotio tangens & secans complementi ad quadrātem hoc est 78 gr 45 min. innotescent, videlicet tangens BP √.4+√8.+√2+1 .secans OP √.8+√32.+√.80+√6272, quomodo isti ad explicabiles facta analyse revocentur hic docebo. sit propositus numerus √ 4+√8.—√2+1. Erit primum latus é √2 est $\frac{14142135623}{10000000000}$, huc 1 addita dabit $2\frac{4142135623}{10000000000}$. dehinc latus numeri 8 additum ad primum 4, totiusque latus erutum dabit $2\frac{6131259297}{10000000000}$, unde ille inventus $2\frac{4142135623}{10000000000}$ subductus relinquet $\frac{1989123674}{10000000000}$ itaq; posita diametro 20000000000 tangens 11 gr. 15 min. erit 1989123674. atque is numerus huic loco in tabulis Rhetici ad amussim respondet. Porro si hos eosdē $2\frac{6131259297}{10000000000}$ $2\frac{4142135623}{10000000000}$ addas conflabis tangentem 78 gr. 45 min. 50273394920. secantem autē √.8+√32—√.80+√6272 ad absolutos hoc modo revocabis. eruito latus é postremo membro √.80+√6272, $12\frac{6172881106}{10000000000}$: tumque latus 32 ad numerū primū 8 addes, & ab hac sūma $13\frac{6568542494}{10000000000}$ illū $12\frac{6172880195}{10000000000}$ subduces, reliqui latus 1019591582 erit secans 11 graduum 15 minutorum: at si eosdem addidisses, & summę latus investiges ea erit secans 75 gr. 45 min. videlicet 5125830895 5.

Latus polygoni 32 inscripti est √.2—√.2+√.2+√2 hinc dabitur ejusdem polygoni circumscripti latus √.32+√512.+√.1280+√1605632.—√.16+√128+√8+2.

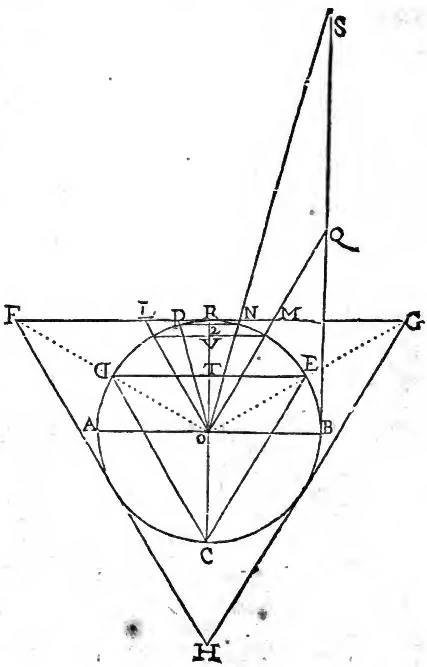
Cujus

Cujus dimidium est tangens 5 gr. 37 min. 30 sec. $\sqrt{.8+\sqrt{3}.+\sqrt{.80+\sqrt{6272.}}}$ $-\sqrt{.4+\sqrt{8}.+\sqrt{2}+1}$. atque tangens complementi 84 gr. 22 min. 30 sec. erit $\sqrt{.8+\sqrt{32}.+\sqrt{.80+\sqrt{6272.}}}+\sqrt{,4+\sqrt{8}.+\sqrt{2}+1}$. qui numeri ad absolutos reducti dabunt tangentem 5 gr. 37 min. 30 sec. 984914030, & tangentem complementi 101531703876 in partibus radij 10000000000.

PROBLEMA 2.

Trianguli æquilateri, & reliquorum polygonorum continua bisectione hinc ortorum circulo adscriptorum latera investigare.

Per ea que supra jam á nobis demonstrata sunt constat latus triãguli æquilateri circulo inscripti posse triplum circularis radij, posito itaque radio 1, latus trianguli æqui-lateri erit $\sqrt{3}$. inde latus circumscriptum ita concluditur, ut OT $\frac{1}{2}$ ad $\sqrt{3}$ ita EK 1 ad FG $\sqrt{12}$. itaque area trianguli inscripti erit $\sqrt{1\frac{11}{16}}$ vel $1\frac{299038105676}{1000000000000}$, & area circumscripti $\sqrt{27}$ vel $5\frac{196152422705}{1000000000000}$. Itaque triangulum circumscriptum est quadruplum inscripti.



Angulus R O G est graduum 60, RG $\sqrt{3}$ tangens, OG secans ejusdem. itaque posito radio partium 20000000000 secãs OG eidem æquabitur quia latus FG duplum est lateris DE. ideoque EO ipsis EG, eritque RG tangens in ijsdem partibus 17320508076.

Esto

Esto *B*Q tangens ipsi RG æqualis, & OQ ipsi OG, erit itaque RM tangens 30 graduum $\sqrt{\frac{1}{3}}$ vel 5773502692, & secans OM $\sqrt{1\frac{1}{3}}$, vel 11547005384.

Hinc $\sqrt{3}$ de diametro 2 subducta relinquunt $2-\sqrt{3}$, hujus latus $\sqrt{.2-\sqrt{3}}$ vel $\sqrt{1\frac{1}{2}}-\sqrt{\frac{1}{2}}$, est latus dodecanguli.

Ita latus polygoni inscripti 24 angulorum invenietur $\sqrt{.2-\sqrt{.2+\sqrt{3}}}$. & 48 angulorum latus $\sqrt{.2-\sqrt{.2+\sqrt{.2+\sqrt{3}}}}$. atque ita in infinitum facta progressione.

Latus dodecanguli circumscripti per similitudinem jam supra expositam invenitur $4-\sqrt{12}$, ejus dimidium $2-\sqrt{3}$ RN tangens 15 graduum, & *B*S tangens anguli complementi NOR $2+\sqrt{3}$. Secans ON 15 graduum est $\sqrt{6}-\sqrt{2}$, secans complementi 75 graduum BS $\sqrt{6}+\sqrt{2}$. est itaque in numeris absolutis tangens 15 gradium 2679491924, tangens 75 grad. 37320528076; secans 15 grad. 10352761804, secans 75 gr. 38637033051 2 posita diametro pariiū 20000000000 est enim $\sqrt{6}+\sqrt{2}$, 3,86370330515.

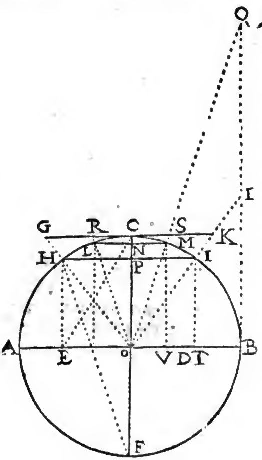
Notabis inventa tangente & secante 75 graduum facillimo negotio quoque inveniri tangentem 7 gr, 30 min. itemque tangentem 82 gr. 30 min. si enim de secante $\sqrt{6}+\sqrt{2}$ subducas tangentem ejusdem peripheriæ $2+\sqrt{3}$, reliquus $\sqrt{6}+\sqrt{3}-2+\sqrt{3}$ erit tangens 7 grad. 30 minut. hoc est semissis lateris polygoni 24 angulorum circulo circumscripti. cujus duplum $\sqrt{24}+\sqrt{8}-4+\sqrt{12}$ erit ipsum latus. potuit verò quoque per proportionem supra commonstratam ita concludi, ut $\sqrt{.2+\sqrt{.2+\sqrt{3}}}$ ad $\sqrt{.2-\sqrt{.2+\sqrt{3}}}$, ita radius 1 ad $\sqrt{.8+\sqrt{48}}.-\sqrt{.7+\sqrt{48}}$, hoc est ut supra $\sqrt{6}+\sqrt{2}-2+\sqrt{3}$. qui in absolutis valent 0,13165249758. At si ad secantem $\sqrt{6}+\sqrt{2}$ addas tangentem $2+\sqrt{3}$, totus erit tangens 82 gr. 30 — $\sqrt{6}+\sqrt{2}+2+\sqrt{3}$, hoc est 7,59575411272. hinc jam secans 7 gr. 30 minutorum datur $\sqrt{.16+\sqrt{192}}--\sqrt{216}+\sqrt{200}$, hoc est 1,00862896058; Et secans 82 gr. 30. $\sqrt{.16+\sqrt{192}}.+\sqrt{216}+\sqrt{200}$, hoc est 7,66129757554: ab hac secante ejusdem tangens subducta dabit tangentem 3 gr. 45 minutorum, quod est dimidium latus polygoni 48 angulorum, $\sqrt{.16+\sqrt{192}+\sqrt{216}+\sqrt{200}}.-\sqrt{6}+\sqrt{2}+2+\sqrt{3}$. ijdem numeri additi conflabunt secantem 86 gr. 15 minutorum $\sqrt{.16+\sqrt{192}}+\sqrt{216}+\sqrt{200}.+\sqrt{6}+\sqrt{2}+2+\sqrt{3}$. Et totum latus polygoni 48 angulorum $\sqrt{.64+\sqrt{3072}+\sqrt{3456}+\sqrt{3200}}-\sqrt{24}+\sqrt{8}+4+\sqrt{12}$. illæ tangentes ad absolutos revocabuntur hoc modo, latera numerorum 6,3,2, addantur ad 2, itemque é 200, 216, 192, ad 16, hujus summæ latus ad priorem summam additæ dabit 15,25705168833 tangentem 86 grad. 30 min. & ab eadem subducta dabit 0,06554346285 tangentem 3 gr. 30 minutorum, cujus duplum 0,13108692563, latus est polygoni 48 angulorum circulo circumscripti.

Secans 86 gr. 15 min. invenietur, si quadres $\sqrt{.16+\sqrt{216}+\sqrt{200}}.+\sqrt{192}+\sqrt{.15+\sqrt{216}+200+\sqrt{196}}$ (hic secundum membrū $\sqrt{.15+\sqrt{216}+\sqrt{200}+\sqrt{196}}$, est quadratum numeri $\sqrt{6}+\sqrt{2}+2+\sqrt{3}$) cumque eo radij quadratum componas, summæ latus $\sqrt{.32+\sqrt{864}+\sqrt{800}+\sqrt{768}+\sqrt{.3392+\sqrt{3321216}+\sqrt{3075200}+\sqrt{2952192}+\sqrt{2764800}+\sqrt{2654208}+\sqrt{2457600}}}$ erit secans 86 gr. 15 min. Si ab hoc tangens ejusdem anguli subducatur, reliquus erit

est tangens 1. grad. 52 $\frac{1}{2}$ minutorum. videlicet semissis lateris polygoni circumscripti 96 angulorũ. √.32. + √864 + √800 + √768 √ + √.3392 + √3321216 + √3075200 + √2952192 + √2764800 + √2654208 + √2457600 − √.16 + √216 + √200 + √192 + √.15 + √216 + √200 + √196. Eorundemque numerorum summa dabit tangentem 88 grad. 7 min. 30 secundorum, √.32 + √864 + √800 + √768 + √.3392 + √3321216 + √3075200 + √2952192 + √2764800 + √2654208 + √2457600. + √.16 + √216 + √200 + √192 + √.15 + √216 + √200 + √192. secans anguli 86 gr. 15 min. posita diametro partium 2000000000000 erit 1528978829867 8, & ejusdem tangens 1525705168826 5, horum summa 30546839986944 est tangens 88 gr. 7 min. 30 sec. eorundemque differentia 3273661041 minor vero, & 3273661042 major vero tangens 1 gr. 52 min. 30 sec. ideoque latus polygoni 96 angulorum circulo circumscripti erit major quam 6547322082, & minor quã 6547322083. Si porrò libeat hunc laborem continuare, invenies secantem 1 gr. 7 min 30 sec. 10005356993437 7 posita diametro 20000000000000 0, & secantem 88 gr. 7 min. 30 sec. 30563203909079 posita diametro 20000000000000, hujus à tangẽte suo differẽtia relinquit tangẽtẽ 3054683998694 4, posita diametro 2000000000000 0 inde dabitur tangens 56 min. 15 sec [illegible] posita diametro partium 2. cujus duplum est latus polygoni 192 angulorum circulo circumscripti. atque ita porrò in infinitum continuando.

PROBLEMA 3.

Quinquanguli ordinati & reliquorum polygonorum continua bisectione hinc ortorum circulo adscriptorum latera investigare.



Quinquanguli ordinati circulo inscripti fabrica hæc est. Sunto diametri normales AB CF, & radius OB, bisecetur in D, tum subtensæ DC statuatur æqualis DE, hic segmentum OE erit latus decanguli circulo inscripti, & subtensa CE latus inscripti quinquanguli, quemadmodum supra libro 2 propositione 58 à nobis demonstratum est. Eorundem autem magnitudinem harum linearum ductum secutus ita investigabis. cum CO sit 1, DO $\frac{1}{2}$ subtensa DC hoc est ipsa DE dabitur √1 $\frac{1}{4}$, inde subducta OD $\frac{1}{2}$ relinquit EO latus inscripti decanguli √1 $\frac{1}{4}$ − $\frac{1}{2}$. Iam datis cruribus anguli recti EOC dabitur basis EC vel ei æqualis HI √.2 $\frac{1}{2}$ − √1 $\frac{1}{4}$ latus inscripti quinquanguli. Et rursum cum HC latus decanguli detur √1 $\frac{1}{4}$ − $\frac{1}{2}$ dabitur quoque HF ejus complementum ad semicirculum: namque quadratum HC 1 $\frac{1}{2}$ − √1 $\frac{1}{4}$ de quadrato diametri CF subductum

relinquet quadratum HF, cujus latus ipsa HE $\surd.2\frac{1}{2}+\surd 1\frac{1}{4}$, hoc numeor de diametro CF 2 subducto, reliqui latus erit latus inscripti vigintanguli $\surd.2-\surd.2\frac{1}{2}+\surd 1\frac{1}{4}$, simili via dabitur latus quadragintanguli $\surd.2-\surd.2+\surd.2\frac{1}{2}+\surd 1\frac{1}{4}$ & octogintãguli $\surd.2-\surd.2+\surd 2+\surd.2\frac{1}{2}+\surd 1\frac{1}{4}$. atq; ita porro. latera figurarum circumscriptarum antecedentium problematum præcepta secutus invenies, & primum quinquanguli circumscripti latus $\surd.20-\surd 320$, cujus semissis tangens CK $\surd.5-\surd 20$. unde secans anguli COK dabitur $\surd.6-\surd 20$. vel $\surd 5-1$. & tangens anguli KOB $\frac{\surd.5+\surd 20}{\surd 5}$, vel $\surd.5+\surd\frac{4}{5}$ & secans OG

$\frac{\surd.10+\surd 20}{\surd 5}$ vel $\surd.2+\surd\frac{4}{5}$.

. Latus decanguli inscripti est $\surd 1\frac{1}{4}-\frac{1}{2}$. perpendicularis à centro in ipsum $\surd.\frac{5}{8}+\surd\frac{5}{64}$ est sinus anguli MOB 72 graduum. jam quemadmodum sinus anguli MOB ad sinum MN, ita OC ad tangentem CB $\surd.1-\surd\frac{4}{5}$ hic est semissis lateris circumscripti decanguli, cujus duplum est ipsum latus $\surd 4-\surd 12\frac{4}{5}$. atq; ideo secans $\surd.2-\surd\frac{4}{5}$. Hinc secans anguli QOB 72 graduum $\surd 5+1$, ejusdemq; tangens $\surd.5+\surd 20$. Idem aliter. subducito $\surd.1+\surd\frac{4}{5}$ de $\surd.2+\surd\frac{4}{5}$ reliquus $\surd.1-\surd\frac{4}{5}$ erit tangens 15 graduum, ijdemque numeri inter se additi dabunt $\surd.5+\surd 20$ tangentem 72 graduum.

Porrò hic secans cum tangente compositus, dabit $\surd 5+1+\surd.5+\surd 20$ tangentem 81 gr. Et rursum $\surd.5+\surd 20$ de secante subductus relinquet $\surd 5+1-\surd.5+\surd 20$ tangentem 9 graduum, qui est semissis lateris circumscripti vigintanguli, atque ideo ipsum latus erit $\surd 20+\surd 2-\surd.20+\surd 320$.

Dehinc quadratum tangentis 81 gr. $11+\surd 80+\surd 200+\surd 38720$ additũ ad quadratum radij 1 dabit $12+\surd 80+\surd 200+\surd 38720$ quadratum secantis 81 gr. atque ideo ipsam secantẽ $\surd.12+\surd 80+\surd 200+\surd 38720$. ideoque secantem complementi seu 9 graduum $\surd.12+\surd 80-\surd.200+\surd 38720$. Iam subducta tangente 81 graduum de ejusdem secante reliqua erit tangens $4\frac{1}{2}$, qui est semissis lateris quadragintãguli $\surd.12+\surd 80+\surd.200+\surd 38720.-\surd 5+1+\surd.5+\surd 20$. vel si é pripre membro $\surd.12+\surd 80+\surd.200+\surd 38720$ latus eruas habebis $\surd 4\frac{1}{2}+\surd 2\frac{1}{2}+\surd.5+\surd 5$. atq; ideo tangens $4\frac{1}{2}$ fuerit $\surd 4\frac{1}{2}+\surd 2\frac{1}{2}+\surd.5+\surd 5.-\surd 5+1+\surd.5+\surd 20$, cujus duplum $\surd 18+\surd 10+\surd 20+\surd 80.-\surd 20+1+\surd.20+\surd 320$, erit latus quadranginranguli. Porrò ex earundem linearum additione conflabitur tangens 85 gr. 30 min. $\surd 4\frac{1}{2}+\surd 2\frac{1}{2}+\surd 5+\surd 5.+\surd 5+1+\surd.5+\surd 320$ numeri isti ad absolutos reducti dabunt secantem 4 gr. 30 min. $1^{1\,0\,9\,2\,1\,9\,8\,4\,8}$, & ejusdem tangentem $^{7\,8\,7\,0\,1\,7\,0\,6\,8}$, cujus duplum $^{1\,5\,7\,4\,0\,3\,4\,1\,3\,6}$ latus circumscripti quadragintãguli. Itemq; secantẽ 85 gr. 30 minutorum dabunt $12^{7\,4\,5\,4\,9\,4\,8\,4\,4\,6}$, & ejusdem tangentem $12^{7\,0\,6\,2\,0\,4\,7\,1\,7\,0}$. Atque ita in cæteris simili in infinitum progressu.

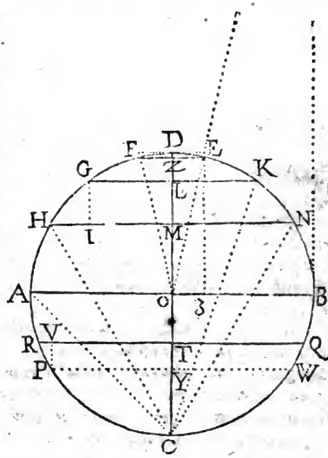
PRO-

PROBLEMA 4.

Quindecanguli ordinati & reliquorum polygonorum continua bisectione hinc ortorum circulo adscriptorum latera invenire.

Inscribatur circulo latus trianguli æquilateri HN, & GK latus qninquanguli eidem parallelum, hic jam subducta $\frac{1}{5}$ de $\frac{1}{3}$ reliquæ erunt $\frac{2}{15}$ cujus dimidium GH $\frac{1}{15}$. atque ideo linea ei subtensa erit latus inscripti quindecanguli. Hujus magnitudem ita inquires, subducito GK latus quinquanguli $\sqrt{}.2\frac{1}{2} - \sqrt{}1\frac{1}{4}$ de latere trianguli $\sqrt{}3$, reliqui dimidium $\frac{\sqrt{}3 - \sqrt{}.2\frac{1}{2} - \sqrt{}1\frac{1}{4}}{2}$ erit HI deinde subducito perpendicularem OM $\frac{1}{2}$ de OL $\sqrt{}\frac{5}{16} + \frac{1}{4}$ reliqua LM vel GI dabitur $\sqrt{}\frac{5}{16} - \frac{1}{4}$. jam quadratum HI $\frac{5\frac{1}{2} - \sqrt{}1\frac{1}{4} - \sqrt{}.30 - \sqrt{}180}{4}$ addatur ad quadratum à GI $\frac{3}{8} + \sqrt{}\frac{5}{64}$, hoc est si ad idem nomen reducatur $\frac{1\frac{1}{2} + \sqrt{}1\frac{1}{4}}{4}$ summæ latus $\sqrt{}.1\frac{3}{4} - \sqrt{}\frac{5}{16} - \sqrt{}.1\frac{7}{8} - \sqrt{}\frac{45}{64}$ dabit inscriptam HG latus quindecanguli.

Latus trigintanguli invenitur ope quinquanguli & sexanguli. sit enim PC $\frac{1}{5}$ totius peripheriæ, & RC $\frac{1}{3}$, harum differentia RP erit totius peripheriæ $\frac{1}{30}$, rectaque RP inscripta latus trigîntanguli: jam RQ statuatur subtensa $\frac{2}{5}$, quę ideo erit $\sqrt{}.2\frac{1}{2} + \sqrt{}1\frac{1}{4}$. & PW $\frac{1}{3}$ subtensa, quę est $\sqrt{}3$. hujus dimidium PY $\sqrt{}\frac{3}{4}$ subductũ de RT $\frac{\sqrt{}.2\frac{1}{2} + \sqrt{}1\frac{1}{4}}{2}$, quæ est dimidia RQ reliquam facit RV $\frac{\sqrt{}.2\frac{1}{2} + \sqrt{}1\frac{1}{4} - \sqrt{}3}{2}$. deinde inveniatur perpendicularis TC $1\frac{1}{4} - \sqrt{}\frac{5}{16}$ é differentia quadratorum RC & RT; tumq; differentia CT & CY $\frac{1}{2}$ dabit TY PV $\frac{3}{4} - \sqrt{}\frac{5}{16}$. deniq; latus summæ quadratorum ab RV $1\frac{3}{8} + \sqrt{}\frac{5}{64} - \sqrt{}1\frac{7}{8} + \frac{45}{64}$ & PV $\frac{7}{8} - \sqrt{}\frac{45}{64}$ dabit RP $\sqrt{}.2\frac{1}{4} - \sqrt{}\frac{5}{16} - \sqrt{}1\frac{7}{8} + \sqrt{}\frac{45}{64}$.



Eodem modo invenies latus sexagintanguli $\sqrt{}.2 - \sqrt{}\frac{15}{16} - \sqrt{}\frac{5}{16} - \sqrt{}.\frac{7}{8} - \sqrt{}\frac{5}{64}$ vel $\sqrt{}.2 - \sqrt{}.\frac{3}{4} + \sqrt{}\frac{5}{8} + \sqrt{}.1\frac{7}{8} + \sqrt{}\frac{45}{64}$, veluti in eo, quem de circulo & adscriptis edidi libro demonstratum est. Et quamuis isti numeri multum inter se dissidere videantur, tamen ad absolutos reducti plane inter se congruere deprehenduntur.

 Ex

Ex iſtis haud difficulter invenientur continua biſectione latera polygonorum 120, 240, 480, 260 & reliquorum deinceps quorumlibet.

Latera polygonorum circumſcriptorum in his invenientur eo quem ſupra uſurpavimus modo. Et primum quidem circumſcripti quindecanguli latus $\sqrt{}.10 - \sqrt{}20. + \sqrt{}3 - \sqrt{}15$. Namque ut perpendicularis à centro in latus inſcripti quindecanguli, ad ipſum quindecanguli latus, ita radius ad latus circumſcriptum. itaque latus inſcriptum $\sqrt{}.\frac{3}{4} - \sqrt{}\frac{5}{16} - \sqrt{}.1\frac{7}{8} - \sqrt{}\frac{45}{64}$ dividendum fuit per iſtam perpendicularem $\sqrt{}.2\frac{1}{4} + \sqrt{}\frac{5}{16} + \sqrt{}.1\frac{7}{8} - \sqrt{}\frac{45}{64}$. verum cum iſtius diviſionis opus difficile & intricatum videatur totius abaci typum hic oculis ſubjiciam.

Dividend. $\sqrt{}.1\frac{3}{4} - \sqrt{}\frac{5}{16} - \sqrt{}.1\frac{7}{8} - \sqrt{}\frac{45}{64}$. diviſor $\sqrt{}.2\frac{1}{4} + \sqrt{}\frac{5}{16} + \sqrt{}1\frac{7}{8} - \sqrt{}.\frac{45}{64}$

$\sqrt{}.2\frac{1}{4} + \sqrt{}\frac{5}{16} - \sqrt{}.1\frac{7}{8} - \sqrt{}\frac{45}{64}$. reſid. $\sqrt{}.2\frac{1}{4} + \sqrt{}\frac{5}{16} - \sqrt{}1\frac{7}{8} - \sqrt{}.\frac{45}{64}$

$3\frac{15}{16}$ $\quad$ $\sqrt{}.1\frac{7}{8} - \sqrt{}\frac{45}{64}$ $\quad$ $5\frac{1}{16}$

$-\frac{5}{16} - \sqrt{}\frac{5}{16}$ $\quad$ 16 $\quad$ $\frac{1}{2} + \sqrt{}6\frac{11}{64}$.

$-1\frac{7}{8} - \sqrt{}\frac{45}{64}$

$2\frac{5}{8} - \sqrt{}\frac{5}{16}$ $\quad$ $\sqrt{}.30 - \sqrt{}180$

$1\frac{7}{8} - \sqrt{}\frac{45}{64}$ $\quad$ $\sqrt{}.3\frac{1}{2} + \sqrt{}11\frac{1}{4}$ factus.

$\sqrt{}.3\frac{1}{2} - \sqrt{}11\frac{1}{4}$

$\sqrt{}.5\frac{1}{2} - \sqrt{}1\frac{1}{4} - \sqrt{}.30 - \sqrt{}180$ factus.

$\sqrt{}.3\frac{1}{2} - \sqrt{}11\frac{1}{4}$ quad. $\sqrt{}.23\frac{1}{4} - \sqrt{}551\frac{1}{4}$ $\quad$ $12\frac{1}{4}$

$-11\frac{1}{4}$

dividend. $\sqrt{}.23 - \sqrt{}500 - \sqrt{}.1020 - \sqrt{}1039610$. $\quad$ 1 diviſor

Tantæ itaque magnitudinis eſt tangens 12 graduum dimidium latus quindecanguli circulo circumſcripti $\sqrt{}.23 - \sqrt{}500 - \sqrt{}.1020 - \sqrt{}1039680$, ut nota $\sqrt{}.$ primo loco poſita totius numeri latus involvat, ejus itaque duplum integrum latus definiet $\sqrt{}.92 - \sqrt{}800 - \sqrt{}.16320 - \sqrt{}266158080$, cujus numeri latus $\sqrt{}27 - \sqrt{}.50 - \sqrt{}2420$ eſt latus quinddcanguli: ejuſdemque dimidium $\sqrt{}6\frac{3}{4} - \sqrt{}3\frac{1}{8} - \sqrt{}.12\frac{1}{2} - \sqrt{}151\frac{1}{4}$ tangens 12 gr. unde ejuſdem ſecans datur $\sqrt{}.24 - \sqrt{}500 - \sqrt{}.1020 - \sqrt{}1039680$.

Eandem operis analogiam ſecutus invenies latus trigintanguli circulo circumſcripti $\sqrt{}.28 - \sqrt{}320 - \sqrt{}.960 - \sqrt{}737280$. hic $\sqrt{}.$ totum numerum propoſitum involvit, cujus latus datur $\sqrt{}.10 - \sqrt{}20. + \sqrt{}3 - \sqrt{}15$: & ejus dimidium tangens 6 graduum $\sqrt{}.2\frac{1}{2} - \sqrt{}1\frac{1}{4}. + \sqrt{}\frac{3}{4} - \sqrt{}3\frac{3}{4}$, unde ejuſdem anguli ſecans dabitur $\sqrt{}.29 - \sqrt{}320 - \sqrt{}.960 - \sqrt{}737280$.

Porrò ex iſtis dabitur tangens 84 gr. $\sqrt{}.23 + \sqrt{}500 + \sqrt{}.1020 + \sqrt{}1039680$, & ſecans $\sqrt{}.24 + \sqrt{}500 + \sqrt{}.1020 + \sqrt{}1039680$.

Itemque tangens 3 gr. (quæ eſt ſemiſſis lateris circumſcripti ſexagintanguli erit $\sqrt{}.24 + \sqrt{}500 + \sqrt{}1020 + \sqrt{}1039680 - \sqrt{}.23 + \sqrt{}500 + \sqrt{}.1020 + \sqrt{}1039680$, ejuſdemq; ſecans $\sqrt{}.25 + \sqrt{}500 + \sqrt{}1020 + \sqrt{}1039680 - \sqrt{}.23 + \sqrt{}500 + \sqrt{}1020 + \sqrt{}1039680$.

Denique

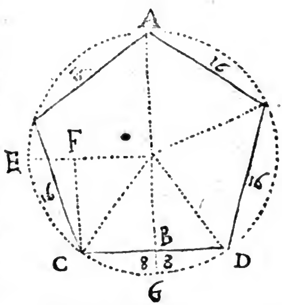
Denique tangens complementi, qui sunt 87 gradus erit $\sqrt{.24 + \sqrt{500} + \sqrt{1020} + \sqrt{1039680}} + \sqrt{.23 + \sqrt{500} + \sqrt{.1020} + \sqrt{1039680}}$, & ejusdem secans $\sqrt{.25 + \sqrt{500} + \sqrt{1020} + \sqrt{1039680}} + \sqrt{.23 + \sqrt{500} + \sqrt{.1020} + \sqrt{1039680}}$.

Numeri isti ad absolutos reducti posita diametro partium 200000000000000 dabunt tangentem 3 gr. 5240777928304, secantem 100317234599792. quare latus circumscripti sexagintanguli erit 10481555956608. & rursum posita diametro 200000000000 exhibebunt tangentem 87 graduum 1908113668773, & secantem 1910732290929.

PROBLEMA 5.

Dato latere quinquanguli ordinati quæritur area, & circumscripti circuli diameter.

Exponatur CD ordinati quinquanguli latus 16 decempedarum, quæritur area, & diameter circuli AG. Respondeo aream quinquanguli esse $\sqrt{.102440} + \sqrt{83886608000}$, hoc est $440\frac{4422}{10000}$ minorem & $440\frac{4423}{10000}$ majorem vero. Itemque diametrum AG $\sqrt{.522 + \sqrt{52428\frac{4}{5}}}$.



Zetematis hujus solutio varia afferri potest. Primum ex similitudine cum enim figuræ simles habeant duplicatam rationem homologorum laterũ, & jam supra inventum sit latus quinquanguli $\sqrt{.2\frac{1}{2} - \sqrt{1\frac{1}{4}}}$ posita diametro partium 2, itemque perpendicularis OB à centro in latus $\sqrt{\frac{1}{16}} + \sqrt{\frac{5}{16}}$. perpendicularis autẽ multiplicata in dimidiã perimetrũ areã ordinati cujusq; polygoni exhibeat, dabitur talis quinquanguli area $\sqrt{.4\frac{29}{32} + \sqrt{3\frac{53}{1024}}}$. Atque ideo quemadmodum quadratum à latere hujus quinquanguli $2\frac{1}{2} - \sqrt{1\frac{1}{4}}$ ad suã aream $\sqrt{.3\frac{29}{32} + \sqrt{3\frac{53}{1024}}}$, ita quadratũ a latere da o 256 ad ejusdem aream $\sqrt{.102400} + \sqrt{838808000}$. Pręterea ut latus illus quinquanguli $\sqrt{.2\frac{1}{2} - \sqrt{1\frac{1}{4}}}$ ad suam diametrum 2, ita datum latus CD 16 ad quesitam diametrum AG $\sqrt{.512 + \sqrt{52418\frac{4}{5}}}$.

Quinetiam ex ista diametro aream dati quinquanguli assequeris, ut enim quadratum diametri 4 ad sui quinquanguli aream $\sqrt{.3\frac{29}{32} + \sqrt{\frac{3053}{1024}}}$, ita hujus diametri quadratum $512 + \sqrt{52418\frac{4}{5}}$ ad sui quinquanguli aream. $\sqrt{102400} + \sqrt{83888608000}$.

Vel hoc modo, ut latus quinquanguli $\sqrt{.2\frac{1}{2} - \sqrt{1\frac{1}{4}}}$ ad perpendicularem $\sqrt{\frac{1}{16}} + \sqrt{\frac{5}{16}}$, ita datum latus 16 ad suam perpendicularem $\sqrt{.64 + \sqrt{3276\frac{4}{5}}}$, ea per 40 dimidiam quinquanguli perimetrum multiplicata dabit aream, ut supra.

Vt etiam ijs serviam qui ab irrationalium numeratione alieniores sunt, easdem has lineas ad explicabiles & absolutos revocabo. Area quinquanguli posita diametro partium 2 est 2.37764129 paulo major & quadratum à latere quinquanguli 1.381966 paulo item majus. inde jam omnium ordinatorum quinquangulorũ areę dabuntur lateribus datis. Namq; ut 1.381966 ad 2.37764129, sic 256 (quadratum à dato latere 16) ad aream suam 440.4422.

Idem aliter per triangulorum canones. Cum angulus quinquanguli æquetur $\frac{6}{5}$ recti (quod é 15 & 32 prop. 1 lib. *Euc.* constare potest) & angulus C bifariam secetur à radio OC erit OCB $\frac{3}{5}$ recti & angulus in centro BOC $\frac{2}{5}$ hoc est 36 graduum, & ACB 54 gr. atque ideo quoque peripheria GC 36 gr. & CE 54 gr. aut si libet dividito 360 per multitudinem laterum (ut hic 5) quotus dabit gradus peripheriæ CD 72, cujus dimidium itidem sit GC 36. hujus sinus 5877852 pro CB, & sinus complementi CE 54 gr. 8090168 ut proxime pro CF vel perpendiculari OB. Vnde proportio, quemadmodum CB 5877852 ad perpendicularem OB 8090168, ita dimidium dati lateris 8 ad perpendicularẽ OB 11.011056. qua in dimidiam perimetrũ 40 multiplicata datur area quinquanguli 440.4422 minor & 440.4423 major vero. Si & hanc & similes alias etiam accuratius subducere postules istis majoribus numeris uti licebit. posita enim diametro 2 erit BD .5877852522925, & OB .8090169943749. Item tangens 54 gr. ad eandem diametrum taxata erit 1.3763819204712. quæ invenitur sinu 54 gr. per sinum 36 gr. diviso. Item si aream quinquanguli per quadratum ab ejus latere dividas quotus erit 1.7204774005889 paulo minus, secundum eandem diametri estimationem. hujus numeri ope omnium quinquangulorum areæ dato latere quam facillime expediri possunt. Namq; si omnium laterum quadruplum multiplices per tangentem 54 gr. factus erit area.

```
 137638192
       320
 ----------
 2752763840
412914576
 ----------
44044221440
```

area 440.44221440 quinquanguli.

Item si numerum ex areæ per lateris quadratum divisione natum per dati lateris quadratum multiplices, factus erit area.

17204774
256

103228644
86023870
14409548

2404422144
$240\frac{4}{1}\frac{4}{0}\frac{2}{0}\frac{2}{0}\frac{1}{0}\frac{4}{0}\frac{4}{0}$ area quinquanguli.

Denique etiam hoc modo perfici poterit. Posita diametro partium 2 inveniẽda sit area quinquanguli. Investigato lineam $\frac{2}{5}$ peripheriæ subtensam, ea jã supra nobis inventa est $\sqrt{.}\,2\frac{1}{2} + \sqrt{1\frac{1}{4}}$ hæc per $1\frac{1}{4}$ (quęsũt $\frac{1}{4}$ numeri 5, qui numerum laterum quinquanguli definit) multiplicata dabit aream quinquanguli $\sqrt{.}\,3\frac{29}{32} + \sqrt{3\frac{53}{1024}}$, ut supra, verum de istis alijsque consimilibus videas 8 caput libri quem de circulo & adscriptis edidi.

Problema 6.

Data diametro circuli quinquanguli ordinato circumscripti ejus latus & aream invenire.

Exponatur diameter circuli 400 decẽpedarũ, quęritur latus inscripti quinquãguli & area? Respondeo latus esse decempedarũ $235\frac{1}{1}\frac{1}{0}\frac{4}{0}\frac{1}{0}\frac{0}{0}\frac{0}{0}\frac{9}{0}\frac{1}{0}\frac{6}{0}\frac{6}{0}\frac{9}{0}\frac{3}{0}$, & aream $95805\frac{6}{1}\frac{5}{0}\frac{1}{0}\frac{6}{0}\frac{2}{0}\frac{9}{0}\frac{5}{0}$. Vt huic zetemati respondeas multiplicato datæ diametri quadratum per $\frac{5}{1}\frac{9}{0}\frac{4}{0}\frac{4}{0}\frac{1}{0}\frac{0}{0}\frac{3}{0}\frac{2}{0}\frac{2}{0}\frac{6}{0}\frac{2}{0}\frac{4}{0}\frac{4}{0}\frac{7}{0}$ numerus ista multiplicatione factus erit area quæsita.

At si latus quæras multiplicato circuli dati radium per istum numerum ad hanc rem factum $1\frac{1}{1}\frac{7}{0}\frac{5}{0}\frac{5}{0}\frac{7}{0}\frac{0}{0}\frac{5}{0}\frac{0}{0}\frac{4}{0}\frac{5}{0}\frac{8}{0}\frac{4}{0}\frac{9}{0}\frac{6}{0}$, numerus hinc existens erit quæsitum quinquanguli latus. ita in exposito exemplo hic numerus per 200 multiplicatus dabit optatum latus $235\frac{1}{1}\frac{1}{0}\frac{4}{0}\frac{1}{0}\frac{0}{0}\frac{0}{0}\frac{9}{0}\frac{1}{0}\frac{6}{0}\frac{6}{0}\frac{9}{0}\frac{3}{0}$.

Areæ investigationem legittimam esse deprehendes, inventa perpendiculari à centro in latus; ea autẽ invenietur radio 200 in istũ numerũ $\frac{8}{1}\frac{0}{0}\frac{9}{0}\frac{0}{0}\frac{1}{0}\frac{6}{0}\frac{9}{0}\frac{9}{0}\frac{4}{0}\frac{3}{0}\frac{7}{0}\frac{4}{0}\frac{9}{0}$ multiplicato, factus enim erit perpendicularis quæsita $161\frac{8}{1}\frac{0}{0}\frac{3}{0}\frac{3}{0}\frac{9}{0}\frac{8}{0}\frac{8}{0}\frac{7}{0}\frac{4}{0}\frac{9}{0}\frac{8}{0}$. denique hæc perpendicularis per omnium laterum semissem multiplicata dabit areã quæsitam, ut supra.

Problema 7.

Data quinquanguli ordinati area ejus latus & circumscripti circuli diametrum invenire.

Esto

Esto area quinquanguli 480000 decempedarum, hoc est, ut nostrorum geodætarum more loquar, jugerum 800, quorũ singula 600 decempedis taxantur, quæritur latus & circumscripti circuli diameter. Respondeo latus quinquanguli esse 528,197244068226, & circuli diametrum 896,6238286741, ipsamque circuli aream 634227,7184127182 minorem, & 634227,7184327183 majorem vero. ad hujus quæsiti solutionem multiplicato datam aream 480000 per ,581234022442887081 facti 278992,136714038579984 latus erit optatum quinquanguli latus, quale supra exhibui & si eandem aream multiplices per 1,4821935878122751 facti latus erit circumscripti circuli diameter quantam supra expressi.

Porrò aream circuli invenies secundum præcepta antecedentia. Vel hoc modo, multiplicata quinquanguli area per 2,8732777981856926, numerus enim inde factus circumscripti circuli areæ æquatur.

PROBLEMA 8.

Data circuli diametro inscripti & circumscripti sexanguli aream invenire.

Exponatur circuli diameter 800 decempedurum, quærentur areæ Respondeo inscripti sexanguli aream esse √172800000000, hoc est 415692,21918 6; minorem, atque si 7 in fine reponas majorem vero. aream autem circumscripti esse √307200000000, hoc est 554256,218422, hujus sexanguli autem latus √213333⅓ hoc est 461,880211335. cujus inveniendi modus iste est. multiplicatis √3 per 1½, datur area inscripti sexanguli √6¾ cujus diameter sit partium 2. Inde proportio ut quadratum diametri 4 ad aream sexanguli √6¾, ita datæ diametri quadratum 640000 ad aream sui trianguli √172800000000.

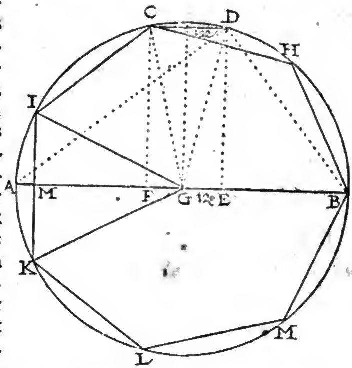
Eodem modo, cum supra deprehenderimus latus circumscripti sexanguli esse √1⅓, hoc per 3 multiplicatum dabit ejus aream √12. Hinc rursum proportio, ut 4 ad √12, ita 640000 ad areã sexanguli circũscripti √307200000000. vel etiam hoc modo, ut 1 quadratum à latere sexanguli ad suam aream. ita 213333⅓ quadratum à latere circumscripti sexanguli ad suam aream √307200000000, ut supra.

PROBLEMA 9.

Data circuli diametro latus & aream inscripti septanguli invenire.

Sit expositi circuli diameter partium 200000000, in quem septangulum ordinatum inscribatur, quæritur ejus latus KI & area. Respondeo latus septanguli esse 86776747,821311624073: & aream 27364101886381043,31 ut proximè.

Hujus



Hujus quantitatem é quatuordecangulo in eundem circulum inscripto indagavi, hoc modo. Assumatur C D latus quatuordecanguli parallelum diametro A B; in qua FE æqualis sit ipsi CD. sitq; FE 1℞, erit igitur perpendicularis DE $\sqrt{}.1 - \frac{1}{4}$ʒ (namque hic ut supra diametrum assumimus partium 2) & EB $1 - \frac{1}{2}$℞, ideoque per 47 prop. lib. 1. BD $\sqrt{}.2 -$ 1℞ & AD $\sqrt{}.2 +$ 1℞ subtensa $\frac{3}{7}$ totius peripheriæ: atqui per antecedentia & primum caput libri quem de circulo & adscriptis edidi invenitur latus septãguli $\sqrt{}.2 - \sqrt{}\,2 -$ 1℞, & latus quatuordecanguli $\sqrt{}.2 - \sqrt{}.2 + \sqrt{}\,2 -$ 1℞ æquale 1℞, atq; ideo $2 - \sqrt{}.2 + \sqrt{}.1 -$ 1℞ æquale 1ʒ, id est 1ʒʒ + 2 æqualia 4ʒ + $\sqrt{}.2 -$ 1℞, vel 1ʒʒ + 1℞ + 2 æqualia 4ʒ + 3℞, atq; inde facta resolutione deprehendes 1℞ æquari 0.445041867912628808577805, modum nostrum hujusmodi æquationes solvendi propediem in lucem proferam, modo Deus coeptis aspiret. hoc latus quatuordecanguli de diametro 2 subducatur, reliqui latus iterum de diametro subductum est quadratum á latere septanguli, cujus radix ipsum inscripti septanguli latus exhibet 0.867767478235116240951536.

Er rursum si latus quatuordecanguli ad diametrum 2 addatur, summæ latus 1.5636629649360596174 est recta AD $\frac{3}{7}$ totius peripheriæ subtensa, quæ per $1\frac{3}{4}$ multiplicata dabit aream septanguli 2.7364101886381043 31 minorem & 2.7364101886381043 31 majorem vero.

Idem aliter, quadratũ à dimidio septãguli latere 0.188255099070633 2 1 3 4 7 3 7 4 de quadrato radij 1 subductum relinquet quadratum perpendicularis à centro in latus septanguli, ejus radix 0.9009688679024191262361 ipsa perpendicularis GM, ea per omnium laterum semissem multiplicata dabit aream septanguli, ut supra. Atq; hinc jam per proportionẽ cujusvis septanguli area data circumscripti circuli diametro facillime invenietur. Namque ut supra quadratum diametri 4 ad areã septanguli jam inventam, ita quadratum datæ diametri 4000000000000000 (datur enim diameter 200000000) ad aream septanguli inscripti, quantam supra respondimus 27364101886381043 $\frac{1}{3}$.

PROBLEMA 10.

Quatuordecanguli inscripti aream invenire.

Multiplicato inventum septanguli latus per $3\frac{1}{2}$ (quæ est $\frac{1}{4}$ numeri laterum quatuordecanguli) factus 3[illegible] minor, et si 9 in fine reponas major vero, erit area quatuordecanguli, quando diameter est 2. Hujus analogia propositum quodlibet quatuordecangulum mensurare haud operosum erit. Si aream vingintioctanguli postules, multiplicato latus quatuordecanguli per 7, factus erit area.

PROBLEMA 11.

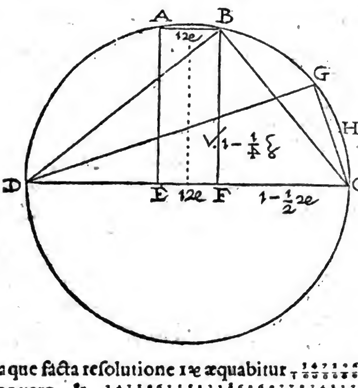
Octanguli inscripti aream invenire.

Multiplicato latus inscripti quadrati $\sqrt{2}$, per 2, quæ sunt $\frac{1}{4}$ numeri laterum octanguli, factus $\sqrt{8}$, vel $2\frac{828427124746190098}{1000000000000000000}$ erit area octanguli atque ita porro.

PROBLEMA 12.

Nonanguli circulo inscripti latus invenire.

Esto diameter, ut supra, partium duarum, quæritur GC latus inscripti nonanguli. Respondeo GC esse [illegible] quam ope octodecanguli assecutus sum.



Sit enim AB latus octodecanguli contra diametrum DC paralleli, proque eo statuamus $1\mathfrak{x}$, ideo BC erit $\sqrt{.2-\mathfrak{x}}$ subtensa $\frac{4}{9}$ totius peripheriæ, & BD $\sqrt{.2+1\mathfrak{x}}$ subtensa $\frac{5}{18}$ totius peripheriæ, ea sudducta de diametro 2, reliqui latus erit GC latus nonanguli $\sqrt{.2-\sqrt{.2+\mathfrak{x}}}$ quare DG erit $\sqrt{.2+\sqrt{.2+\mathfrak{x}}}$ subtensa $\frac{7}{18}$ seu $\frac{4}{9}$ peripheriæ totius, ea subducta de diametro 2, reliqui latus erit HC $\sqrt{.2-\sqrt{.2+\sqrt{.2+1\mathfrak{x}}}}$ latus inscripti octodecanguli, ideoque æquale AB $1\mathfrak{x}$.

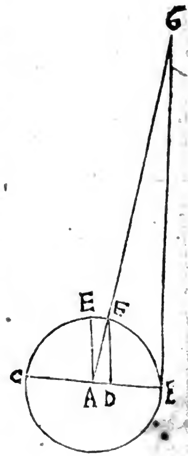
Itaque facta resolutione $1\mathfrak{x}$ æquabitur [illegible] minor vero, & [illegible] major vero, pro latere octodecanguli.

Hoc jam latere cognito quamplurimæ inscriptæ inveniri possunt, ad canonis sinuum tangentium & secantium constructionem quam maxime necessariæ.

Namque

Namque si valorem 1ꝶ seu latus octodecanguli de diametro subducas, reliqui latus [illegible] erit subtensa 80 graduum, cujus dimidium est sinus 40 graduum; quod si idem latus octodecanguli ad diametrum addas, summæ latus 1 [illegible] erit inscripta 100 graduum, ejusque dimidium sinus 50 gr. Itaque si radius assumatur partium 10000000000, dabitur sinus 40 graduum 6427876096 minor, & 6427876097 major vero, sed hic vero vicinior, quam ille, quia [illegible] majores sint quam [illegible]. Et sinus 50 gr. dabitur 7660444431 pauloamplius. Dehinc rursum si supra inventas inscriptas ad diametrum addas & ab eadem subducas numerorum hinc ortorum latera quatuor inscriptas dabunt, é duabus primis una erit inscripta 130 graduum 1 [illegible], hujus dimidium sinus 65 graduum 9063077870; secunda [illegible], subtensa 50 gr. hujus dimidium sinus 20 graduum 4226182617. é duabus novissimis dabitur inscripta 140 graduum 1 [illegible], hujus dimidium sinus 70 graduum 9396926208. & denique inscripta 40 graduum [illegible] seu latus nonanguli, cujus dimidium est sinus 20 graduum 3420201433. Si rursum quatuor inventæ inscriptæ ad diametrum singulæ addantur & subducantur & numerorum hinc existentium latera eruantur dabuntur octo inscriptæ hoc ordine. subtensa 160 gr. 1969615506024416; 20 gr. 347296355333861, 110 gr. 1638304088577983: 70 gr. 1147152872701631: 153 gr. 1952592014239866· 25 gr. 432879227876205· 65 gr 1074599216693647: 115 gr. 1686782891625742 qualium diameter 2000000000000000: atque ideo posita diametro 20000000000 earum semisses eaunt sinus 80 gr. 9848077530: 10 gr. 1736481776: 35 gr. 5735764363: 55 gr. 8191520443: 77 gr. 30 min. 9762960071: 12 gr. 30 min. 2164396139· 57 gr. 30 min. 8433914458: 31 gr. 30 min. 5372996083. Si rursum octo illas jam inventas subtensas ad diametrum 2 addas atque demas & numerorum inde existentium latus eruas orientur aliæ sedecim subtensæ, atque ex his sedecim aliæ novæ triginta duæ, atque hinc denum item aliæ sexagintaquatuor. atque ita in infinitum. cum autem occurrent peripheriæ quæ gradibus & minutis & secundis secundorumque partibus definiuntur, illæ commodę erunt ad investigationem differentiarum, & aliorum sinuum. Sinu autem cujuscunque peripheriæ cognito etiam sinus complementi cognoscitur: est autem complementum differentia dati arcus à quadrante. Namque peripheria quadrante minor cum suo complemento quadrantem æquat. Esto exempli gratia EF peripheria 30 sec. vel quod idem est angulus EAF 37 minutorum 30 secundorum, hujus sinus posita diametro partium 2 erit [illegible], complementum peripheriæ FB æqualis angulo FAB 89 graduum, 22 minutorum, 30 sec. hujus sinus DF valet in iisdem partibus [illegible]. Atque hinc jam & tangens & secans anguli EAF, EAB facile invenientur.

Cæterum



Cæterum in illarum investigatione illud accurate observandum, cum tangens BG propter amplitudinem anguli EAB longissime excurrat priusquam suę secanti AG occurrat in G ad ejus investigationem sinum EF majusculum assumi debere. Verbi gratia statuamus diametrum CB 20000000000 partium, & dicti anguli 89 gr. 22 min. 30 sec. tangentem inveniendam. itaque primum inquirendus est sinus datę peripheriæ EAB & complementi ejusdem FAE qui ex numeris suprascriptis assumantur 9999405050 & 109080915. Erit itaque ob similitudinem triangulorum, ADE, ABG, quemadmodum AD 109080915 ad DE 9999405050, ita AB radius 20000000000 ad BG 916696110405. Itemque ut DA 109080915 ad AE 10000000000, sic AB 10000000000 ad AG 916750652485 cum hęc numeratio legittima videatur nihilominus tamen in isto abaco error arrepsit, & numeri inventi non usquequaque ad amussim veri sunt, nanque utrobique tres novissimæ notæ á veriate sunt alieni, in tangente BG notæ 405, et in secante AG characteres 485. Erroris causa inde est, quod sinus AD primus proportionis terminus tantum novem notis definiatur, tangens verò & secans duodcim; quamobrem ut tangens et secans secundum hanc diametrum accurate habeatur, sinus qui in hac proportione adhibentur ut minimum tot notis constare debent quot in tangente vel secante continentur. ita hic assumendus fuit sinus $109080914\frac{}{1}\frac{9}{0}\frac{4}{0}\frac{1}{0}\frac{8}{0}\frac{2}{0}\frac{3}{0}\frac{7}{0}$ cum ista partium appendicula vel paulo minore, non autem 109080915 tantum, hoc modo.

Vt AD	ad radium AF	ita AF radius	ad secantem
$109080914\frac{}{1}\frac{9}{0}\frac{4}{0}\frac{1}{0}\frac{8}{0}\frac{2}{0}\frac{3}{0}\frac{7}{0}$	10000000000	10000000000	AG

Divisionis hypotyposin quam in istis usurpare consuevi hic subjeci.

10000-

1	1090809149418237
2	2181618298836474
3	3272427448254711
4	4363236597672948
5	5454045747091185
6	6544854896509422
7	7635664045927659
8	8726473195345896
9	9817282344764133

100000000000000000000 | 00
9 9817282344764133
1827176552358670
1 1090809149418237
736367402940433
6 654485489650942
81881913289491
7 76356640459276
5525272830215
5 5454045747091
0
71227083124
6 65448548965
5778534159
5 5454045747
324488412
2 218165829
106326583
9 98172823
8153760
7 7635664
518096
4 436323
81773
7 75356
5417
5

Erit itaque AG secans anguli FAB major quam 916750652974 & minor quam 916750652975 posita diametro partium 20000000000.

paulo minus,

99994050500014976
9 9817282344764133
1821227052373646
1 1090809149418237
730417902955409
6 654485489650942
75932413304467
6 65448548965094
10483864339373
9 9817282344764
666581994609
6 654485489651
12096504958
1 10918091494
1188413464
1 1090809149
0
97604315
8 87264732
10339583
9 9817282
522301
4 436323
85978
7 76356
9622
8

Erit itaque tangens 89 gr. 22 nin. 30 secund. 916696110894 minor vero & 916696110895 major vero.

Quod in utraque hac divisione é dividendo tot circulos rejecerim, & paulatim divisoris characteres conciderim nihil quidquã operis veritati derogat, idque in hoc numerorum abaco versatus facile animadvertet. Praeterea haud obscurum est, quo fondamento hæc tabella, cujus ope divisionis opus peragitur cõstructa sit. Hac itaque formula omnes majores divisiones haud operose absolvuntur. Deniq; etiam illud monendum existimavi quotlibet & inscriptas & sinus inveniri posse secundũ nostra præcepta quæ in libro de adscriptis edito, docuimus, quorum aliquot in hac tabella spectandos exhibemus.

gr.

grad.	min.	sec.
160		
80		
40		
20		
10		
5		
2	30	
1	15	
	37	30
	18	45
	9	22 1/2
	4	41 1/4
70		
35		
17	30	
8	45	
4	22	30
2	11	15
	5	37 1/2
	36	48 3/4
50		
25		
12	30	
6	15	
3	7	30
1	33	45
	46	52 1/2
	23	26 1/4
85		
42	30	
21	15	
10	37	30
5	18	45
2	39	22 1/2
1	19	41 1/4
87	30	
43	45	
21	52	30
10	56	15
5	28	7 1/2
2	44	3 3/4

grad.	min.	sec.
89	22	30
44	41	15
22	11	37 1/2
11	10	18 3/4
89	41	15
44	50	37 1/2
22	25	18 3/4
11	12	39 3/8
65		
32	30	
16	15	
8	7	30
4	3	45
2	1	52 1/2
1	0	56 1/4
77	30	
38	45	
19	22	30
9	41	15
4	50	37 1/2
2	25	18 3/4
1	12	39 3/8
83	45	
41	52	30
20	56	15
10	28	7 1/2
5	14	3 3/4
2	37	1 7/8
86	52	30
43	26	15
21	43	7 1/2
10	51	33 3/4
5	25	46 7/8
88	26	14
44	13	7 1/2
22	6	33 3/4
11	3	16 7/8
89	13	7 1/2
44	36	33 3/4
22	18	16 7/8

grad.	min.	sec.
89	55	18 3/4
44	57	39 3/8
22	28	49 11/16
55		
27	30	
13	45	
6	52	30
3	26	15
1	43	7 1/2
	51	33 3/4
	24	46 7/8
72	30	
36	15	
18	7	30
9	3	45
4	31	52 1/2
2	15	56 1/4
1	7	58 1/8
81	15	
40	37	0
20	18	45
10	9	22 1/2
5	4	41 1/4
2	32	20 5/8
1	10	18 5/16
85	37	30
42	48	45
21	24	22 1/2
10	42	11 1/4
5	21	5 5/8
87	48	45
43	54	22 1/2
21	57	11 1/4
10	58	35 5/8
5	29	17 13/16
47	30	
23	45	
11	52	20
5	56	15
2	58	7 1/2
1	29	3 3/4

grad.	min.	sec.
79	22	30
39	41	15
98	50	37 1/2
9	25	18 3/4
4	42	39 3/8
2	21	18 11/16
1	10	39 11/32
84	41	15
42	20	37 1/2
21	10	18 3/4
10	35	9 3/8
5	17	34 11/16
46	15	
23	7	30
11	33	45
5	46	52 1/2
2	13	26 1/4
1	26	43 1/8
68	7	20
34	3	45
17	1	52 1/2
8	30	56 1/4
4	15	28 1/8
2	7	44 1/16
1	3	52 1/32
79	3	54
39	31	52 1/2
19	45	56 1/4
9	52	58 1/8
4	56	29 1/16
2	28	14 17/32
1	14	7 49/64
45	37	30
22	48	45
11	24	22 1/2
5	42	11 1/4
2	51	5 5/8
1	25	32 13/16

grad.	min.	sec.
73	45	
36	52	30
18	26	25
9	13	7 1/2
4	36	32 3/4
2	18	16 7/8
1	9	8 7/16
53	45	
26	52	30
13	26	15
	43	7 1/2
	21	33 3/4
1	40	46 7/8.

88	45		89	36	$33\frac{3}{4}$	69	45		67	48	45
44	22	30	44	10	$16\frac{7}{8}$	34	52	30	57	30	
22	11	15	27	9	$1\frac{7}{8}$	17	26	15	28	45	
11	5	$37\frac{1}{2}$	89	50	$37\frac{1}{2}$	8	43	$7\frac{1}{2}$	14	22	30
5	32	$48\frac{3}{4}$	44	55	$18\frac{3}{4}$	4	21	$33\frac{3}{4}$	7	11	15
2	46	$24\frac{3}{8}$	32	27	$39\frac{1}{8}$	2	10	$49\frac{7}{8}$	3	35	$37\frac{1}{2}$
						1	5	$23\frac{7}{16}$	1	47	$48\frac{3}{4}$

Atq; hac via latera inscripti vigintiquinquanguli [illegible] qualium diameter 2, invenietur subtensa 14 gr. 24 min. & 7 gr. 12 min. & cæteræ inde bisectione alijsque modis supra expositis derivandæ.

Haud aliter dato latere inscripti vigintesseptanguli [illegible]. Hujus semissis est sinus 6 gr. 40 min. atq; hinc innotescent sinus 83 gr. 20 min. 3 gr. 20 min. 86 gr. 40 min. 1 gr. 40 min. 88 gr. 20 min. 44 gr. 10 min. & præterea 40, 20, 10, 5, $2\frac{1}{2}$ minutorum aliorumque complurium.

Latus inscripti quadragintaquinquanguli, quæ est 8 graduum subtensa valet [illegible], ejus semissis est sinus 4 graduũ, hinc porro invenientur sinus graduum 64, 32, 8, 4, 2, 1, & 26, 13, 77, 58, 29, 61, 74, 37, 53, 82, 41, 86, 43, 49 47, 88, 44, 46, 22, 23, 11, 67, 68, 34, 17, 73, 89, aliquequam plurimi qui non integris solum gradibus sed gradibus, minutis & secundis debentur.

Latus septuagintaquinquanguli valet [illegible], hujus semissis est sinus 1 gr. 12 min. atque hinc porrò dantur 1 gr. 12 min. & 36 min. & 18 min, & 9 min. & 1 gr. 4 min. 30 sec.

Latus polygoni 131 laterum valet [illegible] videlicet subtensa 2 graduum 40 min. hujus dimidium est sinus 1 gr. 40 min. hujus ope plurimarum rursum peripheriarum sinus invenientur, & haud operose sinus 40, 20, 10, 5, $2\frac{1}{2}$ minutorum.

Latus polygoni 225 laterum valet [illegible], tanta igitur est subtensa 1 gr. 36 min. hujus semissis est sinus 48 minutorum hinc itaque continua bisectione dabuntur sinus minutorum 24, 12, 6, 3, $1\frac{1}{2}$ & per horum complementa; alijq; præterea ultro citroque infiniti.

Latus polygoni 675 laterum est [illegible] minus & [illegible] majus vero, tanta igitur est inscripta 32 minutorum, cujus dimidiũ est sinus 16 min atque hinc continua bisectione datur inscripta minutorum 16, 8, 4, 2, 1, corundemque complementa, unde earum quoq; secantes & tangentes invenientur: Atque hæc omnia sola lateris quadrati analysi facillime expediuntur.

Ad hunc numerum istas quoque inscriptas aggregato.

Subtensa

Subtensa $\frac{1}{180}$ totius peripheriæ sive 2 graduum inscripta valet $\frac{349048128745670256388379}{10000000000000000000000000}$, hujus dimid. $\frac{174524064372835128194189}{10000000000000000000000000}$ est paulo amplius est sinus 1 gradus.

Subtensa 2 minutorum valet $\frac{5817764091268491927488}{10000000000000000000000000}$ minus vero, si 9 in fine statuas majus vero, hujus semissis $\frac{2908882045634245963744}{10000000000000000000000000}$ & sinus 1 minuti.

Cum autem diameter statuetur 2000000000000000000000000 erunt subtensæ minutorum 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 quantas in hac tabella vides.

Minut.

1	290888207640147342954
2	581776409126849192748
3	872664598306660186390
4	1163552769026135221228
5	1454440915131829585122
6	1745329030470299086609
7	2036217108888100185076
8	2327105144231790129320
9	261799313034792045771
10	2908881061083070148583

Hinc jam dabuntur subtensæ complementorum ad semicirculum, ut subtensa peripheriæ 179 gr. 58 min. quę cum 2 min. semicirculum explet est 99999997-884601255210283 5. Hinc porrò si expetas tangentem 89 gr. 54 min. posita diametro 20000000000 partium tum quidem sinus 1 min. est 2908882. atqui hinc eam tangentem accurate inveniri non posse supra ostendimus, quamobrē, sinus iste paulo debuit major assumi, videlicet 2908882$\frac{456342459}{10000000000}$. unde accurate concludes ut sinus 1 min. 2908882$\frac{456342459}{10000000000}$ ad sinum complementi 89 gr. 59 min. 9999999576$\frac{92025127}{100000000}$, ita rad. 10000000000 ad 343774667382222 accuratum verumque tangentem 89 gr. 59 min. atque inde dabitur quoque ejusdem peripheriæ secans 34377468192663.

Præterea si tangentem 89 gr. 59 min. 30 sec. inquiras. & sinum complementi videlicet 30 sec assumas tantū quātus est 1454441 posita diametro 20000000000, atque ita concludas ut sinus 30 sec. 1454441 ad sinum 89 gr. 59 min. 30 secund. ita radius 10000000000 ad 687549367351 44; unde & secans datur 6875493746-3946. In istis septem primi characteres dun axat veri erunt & ligitimi, quia primus terminus catenus duntaxat accurate verus , reliqui spurij & à vero alieni. Ideoque ut dicti numeri accurati & bene justi habeantur, accuratioribus & majoribus sinibus opus erit. atque eam ob causam pro istis hos sinus usurpavi 1454441$\frac{12200736}{1000000000}$ & 9999999894$\frac{2300527}{10000000}$. unde tangens dicta datur 687549349308 85, itemque secans 687549356581 05. Hac uia omnes illas quas in ista tabella exhibemus, à nobis indagatas, atque ideo veras & bene accuratas esse scito, posita diametro partium 2000000000000000.

Tang.

		Gr.	Min.	Sec.
Tang.	68754934930885			
Secan.	68754935658105	89.	59	30
T.	34377466738222	89.	59.	—
S.	34377468192663			
T.	22918310350791	89.	58.	30
S.	22918312532453			
T.	17188731914669	89.	58.	—
S.	17188734823552			
S.	13750988295174	89.	57	30
T.	13750984659071			
S.	11459157257057	89	57	—
T.	11459152993734			
T.	9822130237118	89.	56.	30.
S.	9822135327662			
T.	8594363048452	89.	56	—
S.	8594368866216			
T.	6875488693432	89.	55	—
S.	6875495965638			
T.	5729572133543	89.	54	—
S.	5729580860191			
T.	5728996163075942·	89 graduum.		
S.	5729868849850171			

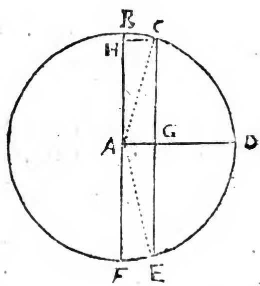
Longe plures ſubtenſas,aliaque hujus generis hic in medium proferre poſſem: Verum omnia hæc peculiarem ſibi locum depoſcunt in mea Algebra,quo in loco ſubtilem æquationum tabulam,in qua omnium polygonorum latera circulo in ſcripta Algebrica æquatione diſpoſita & diſtincta ſint deſcribam , & modum noſtrum quo 1æ valor inveſtigetur explicabo.

Appendicula de circulo data ratione secando.

PROBLEMA 1.

A dato circulo unica inscripta tertiam partem auferre.

Exponatur circulus BCDEE in quam inscribenda sit recta CE absumens sectionem C E D æqualem tertiæ parti totius circuli: sitque diameter BF 289 decempedarum. Respondeo inscriptã CE esse $278\frac{673198}{1000000}$ & peripheriam CDE $376\frac{449368}{1000000}$ decempedarum.



In eo quem de circulo & adscriptis edidi libro ostendi rectam inscriptam quæ $\frac{1}{3}$ totius circuli auferret secare é peripheria 149 gr. 16 min. 27 sec. ejus itaque dimidium CD erit 74 gr. 38 min. 13$\frac{1}{2}$ sec. & arcus complementi CD erit 15 gr. 21 min. 46$\frac{1}{2}$ sec. Horum itaque sinus dantur videlicet CG 9642671, HC 2649321, posita diametro partium 20000000. hinc jam per proportionem haud operosum erit concludere rectam GC $139\frac{336596}{1000000}$ decempedarum, ejusque duplam EG quantam supra notavimus. & HC vel AG, decempedarum $38\frac{282688}{1000000}$. Porrò hinc peripheria CD invenietur, hoc modo. Inquirito primũ peripheriam circuli secandum datam diametrum 289 decempedarum. id autem fiet multiplicatis 289 per $3\frac{14159265}{100000000}$, unde existent $907\frac{92027685}{100000000}$ pro quæsita peripheria. atq; hinc jam proportio, quemadmodum 360 gradus ad $74\frac{6371}{10000}$ gradus, ita $907\frac{92027685}{100000000}$ peripheria ad CD $188\frac{234781}{1000000}$, quæ in radium multiplicata dabit aream sectoris A C D E $27199\frac{925854}{1000000}$. Hinc si subducas aream trianguli ACE, quæ fit multiplicata AG in GC $5334\frac{179431}{1000000}$ decempedarum, relinquetur optata sectio CED $21865\frac{7464}{10000}$. ista igitur pars erit tertia totius circuli. Cujus periculum facies inventa totius circuli area. Namq; ut quadratum diametri 4 ad suam aream $3\frac{14159265}{100000000}$, ita quadratum á data diametro 83521 ad suam aream $65597\frac{2397}{10000}$, cujus pars tertia $21865\frac{7466}{10000}$ eadem est cum supra inventa, utriusque enim differentia ne quidem est $\frac{1}{1000}$ unius decempedæ quadratæ.

Poteris

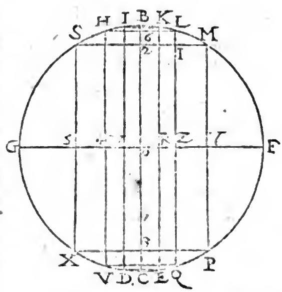
Poteris etiã hoc zetema per proportionẽ solvere, hoc modo. In libro quem de circulo et adscriptis edidi inveni O E lineam 19285^{142}, & peripheriam C D E 26053^{2569}, & peripheriam BC 2681^{3347}, & rectam HC 2649^{321} posita diametro partium 20000. Itaq; quemadmodum diameter 20000 ad inscriptam CE 19285^{142}, ita data diameter 289 ad suam inscriptam CE 278^{673163}, simili proportione peripheriam CDE & sinum HC vel AG concludes, atq; inde quæsito satisfacies.

Quomodo in quatuor, & quinque æquas partes inscriptis parallelis dividatur in eodem libro exhibimus; sed prçterea ibi quæritur.

PROBLEMA 2.

Quomodo circulus lineis parallelis in septem æquas partes dividi possit, cujus diameter sit partium 160.

Respondeo inscriptas $\left\{\begin{array}{l} KE\ 158^{985419} \\ LQ\ 150^{265783} \\ MP\ 128^{113592} \end{array}\right\}$ decempedarum esse.



Canonis hujus sinus & inscriptas subduximus secundũ tabulas *Valentini Ottonis* vel Rhetici posita diametro 20000000000 eas postea ad data diametri partes per proportionem reduximus. Exemplum esto inscripta MP. qualiũ diam. 20000000000 talum est MP 16014074062, itaque qualium diameter statuitur 160 decempedarum talium erit MP $128^{1135924}$ eodem modo ZM sive OY dabitur $47^{9243484}$. qua in dimidiam MP 64^{56296} multiplicata datur area trianguli O M P $3069^{8888467}$. Eadem analogia deprehendes peripheriam M F $74^{87751813}$

 decem

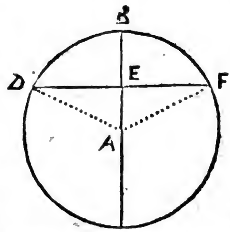
decempedarum, quæ per radium datum 80 multiplicata dabit aream sectoris MOP 5942 $\frac{3014408}{10000000}$, unde area trianguli PMO subducta reliquam facit aream sectionis PMF 2872 $\frac{31309}{100000}$ decempedarum. quæ est $\frac{1}{7}$ totius circuli. Cujus veritatẽ comprobabis totius circuli area inventa, ea enim posita diametro 160 deprehendetur 20106 $\frac{19298}{100000}$, cujus pars septima 2872 $\frac{31329}{100000}$ æquatur areæ supra inventæ. Reliqua é tabella proposita simili modo haud difficulter expediri poterunt.

	Gr.	Mi.	Se.	
BK	6.	27.	20 $\frac{9659}{10000}$	peripheriæ
KF	83.	32.	39 $\frac{389}{10000}$	
BL	20.	5.	20 $\frac{991}{10000}$	
LF	69.	54.	39 $\frac{9449}{10000}$	
MB	36.	48.	9 $\frac{9864}{10000}$	
MF	53.	11.	50 $\frac{4736}{10000}$	

NK	1124370962	Sinus.
TL	3434778248	
2M	5990605802	
KI	2248741924	subtensæ
LH	6869556496	
SM	11981211602	
KR	9936588448	Sinus.
Lv	9391607870	
MY	8007037031	
KE	19873176896	subtensæ
LQ	18783215740	
MP	16014074062	
BK	1126753605	peripheriæ
BL	3506175213	
BM	6423273500	
MF	9284689767	

PROBLEMA 3.

Data diametro & ratione peripheriæ ad suam subtensam, & subtensam & peripheriam invenire.



Exponatur circulus BFD, cujus radius AB sit partium 10000, sitque ratio peripheriæ DBF ad suam inscriptam DF data, quæ 7 ad 6, quæritur amplitudo peripheriæ DBF & rectæ DF eandem peripheriam subtendentis.

Hactenus autor quem in isto problemate mors oppressit.

Zetematis hujus solutio aleæ quam arti similior: dicatur tamen ne quid desideretur. Et regulam falsi quam vocant, huc in auxilium arcessamus. Primum itaque assumamus sinum quemcunque qui an ad suam peripheriam datam rationem habeat exploremus, is si habeat rationem majorem data quæ hic est 6 ad 7 sumatur sinus major, si minorem sumatur minor, quando ratio inscriptæ ad peripheriam non datur minor ea quæ est diametri ad semicirculum; si minor detur contra erit agendum. Fingamus itaque sinum eum ut proxime jam inventum 54 gr. qui est 8090170, *cujus peripheria in partibus radij* 10000000 *erit* 9424778. *hic si fiat ut* 6 *ad* 7, *ita sinus* 8090170 *ad* 9438532, *vides hunc numerum ab illa peripheria* 9424778 *deficere numero* 13754 : *itaque secundo tentemus & assumamus peripheriam* 54 *gr. &* 24 *min. ejus sinus* 8131008, *ejusdem peripheria in partibus radij* 10000000 *erit* 9494591. *atqui si fiat ut* 6 *ad* 7, *ita sinus* 8131008 *ad* 9486176 *vides hunc numerum ab illa peripheria excedi numero* 8415: *quare secundum regulæ falsi exegesin totius operis formula ita habet.*

8090170 — 13754	facti	111833884032
8131008 + 8415		68078780550
22169		179912664582

Iam

Iam divisis 179912664582 *per* 22169 *quotus erit* 8115506 *sinus optatus. cui debetur peripheria* 54 *gr,* 15 *min qui termini quæsito non usq; quaq; satisfaciunt. Namq; si fiat ut* 6 *ad* 7 *ita* 8115506 *ad* 94680490 *atqui peripheria* 54 *gr.* 15 *min. est* 9468411. *Et cũ hic numerus major sit illo argumento est sinum quæsitum minorem assumi debere. quare eodem modo falsi exegesin & opus secutus deprehendes sinũ* 54 *gr.* 14 *min.* 56 $\frac{9}{10}$ *secundorũ qui est* 8115652, *hujus duplũ* 16231304 *inscripta subtensa* 108 *gr.* 29 *min.* 53 $\frac{8}{10}$ *sec. qui in partibus radij sunt* 1893622. *Ajo itaq; inscriptã* 16231304 *ad suam peripheriam* 18936522 *habere rationem optatam quam* 6 *ad* 7.

Haud aliter circuli dati divisio in data segmenta quam superiore problemate autor quæsivit solvi solet.

Canonion quo peripheriæ amplitudine in gradibus & minutis definita, ejusdem magnitudo exhibetur secundum taxationem radij partium 10000000000 *hic subjiciam, ut ad hujus generis problematum solutionem, quasi* πρόχειρον *sit. In quo singulorum minutorum ad denarium usque quantitatem exhibeo, inde per denarij incrementum usque ad gradum integrum; tum rursum singulorum graduum ad denarium usque: denique per denos quosque quadrantis gradus.*

min.	
1	2,908,882
2	5,817,764
3	8,726,646
4	11,635,528
5	14,544,292
6	17,453,410
7	20,362,174
8	23271,057
9	26,17,9938
10	29,088,821
20	58,177,641
30	87,266,462
40	116,355,283
50	145,444,104

grad.	
1	174,532,925
2	349,065,850
3	523,598,776
4	698,131,701
5	872,664,626
6	1,047,197,551
7	1,221,730,476
8	1,396,263,402
9	1,570,796,327
10	1,745,329,252
29	3,490,658,504
30	5,235,987,756
40	6,981,317,008
50	8,726,646,260
60	10,471,975,512
70	12,217,384,764
80	13,962,634,016
90	15,707,963,268

Hujus canonij usus haud obscurus est ex ipsa constructione. Exemplo unico rem i..vabo. quæratur posita diametro 20000000000 *quanta sit in ijsdem partibus peripheria quæ* 54 *gr.* 15 *minutis respondeat hanc additione colliges, componas enim peripheriam* 50 *gr. &* 4 *gr. &* 10 *min. &* 5 *minutorum, summa omnium erit peripheria quæsita, ut hic vides.*

gr. 50

gr.	50	87266462627
gr.	6	698131701
min.	10	29088821
min.	5	14544410
		9468411192

Erit itaque peripheria optata 9468411192 *partium qualium diameter statuitur* 20000000000. *atque ita in cæteris analogia cirsimili. Reliquum hujus canonij usum lector per se facile animadvertet. Quamobrem hujus & Problematis & libri simul finis hic esto.*

FINIS.

30.

1-2.